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Underprepared College Students' Understanding of and Misconceptions with Fractions

Hea-Jin Lee [1*], Irina Boyadzhiev [1]

[1] Ohio State University, Lima, USA

* CORRESPONDENCE: Lee.1129@osu.edu

ABSTRACT

This study investigated understanding of and misconceptions with fractions in college students enrolled in a remedial mathematics course. Data were collected from 22 college students for one semester. The analysis of 41 fraction problems revealed that participants' common misconceptions were associated with a lack of understanding of basic definition of fractions, least common denominators/least common multiples, and order of operations. In addition, some students were able to recall the procedures but could not compute fractions accurately due to the misconceptions listed above.

Keywords: remedial class, underprepared students, fraction computational skills, misconceptions about fraction

INTRODUCTION

Although a majority of students continue their education after high school graduation, over 40% find they are not prepared for college level work and some gaps on their mathematics preparation (Achieve, 2005, 2015). In 2011–2012, about one-third of all first- and second-year bachelor's degree students—29 percent of those at public 4-year institutions and 41 percent of those at public 2-year institutions—reported having ever taken remedial courses. According to Chen (2016), approximately 15% of students in the remediation courses did not complete the course. Even with the support from developmental courses, some students still experience disappointing outcomes in college mathematics classes.

The mathematics department at the researchers' institution gives one of two different mathematics placement tests based on a student's American College Testing (ACT) mathematics score. Placement test for students with an ACT mathematics score less than 25 contains 40 multiple choice questions. Approximately 18% of the placement test require knowledge of fractions to answer. Examples of such questions include: finding the lowest common denominator for the five fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, and $\frac{1}{6}$; dividing $3\frac{4}{5}$ by $2\frac{8}{15}$; changing the fraction $\frac{6}{15}$ to an equivalent fraction with a denominator of 40; and simplifying $\frac{\frac{1}{2}-\frac{2}{3}}{\frac{3}{4}+\frac{4}{5}}$. The remaining test questions cover more advanced numeric and algebraic content. Students who answered less than 16 of the 40 questions correctly are placed in the lowest remedial mathematics course. With such a large portion of the exam requiring fraction knowledge, it might be worth evaluating the fraction knowledge and the type of fraction errors they are making.

Most mathematics topics of remedial courses have already been taught in middle school and high school. Misconceptions and errors observed in remedial courses are similar to what middle school and high school teachers strive to overcome. For example, students in remedial courses have common and persistent

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misconceptions involving operations with fractions or decimals, possibly due to overgeneralizing computation strategies with whole numbers to fractions and decimals (DeWolf & Vosniadou, 2015; Irwin, 2001). Knowledge of fraction concepts and computational skills with fractions are essential skills for students' success in other topics and eventually for college level mathematics readiness (Garfield & Ben-Zvi, 2007; Siegler & Lortie-Forgues, 2015). However, half of middle and high school students and even many college students lack fraction sense and struggle with fraction applications to mathematics (Siegler & Lortie-Forgues, 2015). Misconceptions can persist for a long time, become rooted deeply, and adverse to change (Eryilmaz, 2002; McNeil & Alibali, 2005). In order to help underprepared students for college mathematics readiness, it is important to have knowledge of students' mathematical understanding and diagnose misconceptions.

The purpose of the study is to investigate the nature of the misconceptions with fractions in the students who were enrolled in a remedial mathematics course. Fraction is a basis for decimals, rational numbers, algebra, and more advanced mathematical concepts. Also, fractions is the first unit of the 1st of two remedial course series. Thus, how fractions are understood by this specific group of students is important in understanding the nature of their mathematical understanding and comprehension and its development. Although the immediate work was done on solving fraction operation problems, the real aim of the study was to better prepare the students for work with rational expressions, complex fractions, and more advanced concepts. This study provides important information for instructors of developmental mathematics classes about how to help underprepared students improve their understanding of mathematics and performance. As a result of this study, researchers hope to gain insights into effective ways of preparing students in remedial courses succeed in college mathematics. These results may also guide students themselves to enact effective learning strategies to succeed in future mathematics courses.

MATHEMATICS UNDERSTANDING

Kilpatrick, Swafford, and Findell (2001) define five components of mathematical proficiency as successful mathematics learning: (1) conceptual understanding (an integrated and functional grasp of mathematical ideas); (2) procedural fluency (knowledge of procedures, knowledge of when and how to use them appropriately, flexibly, accurately, and efficiently); (3) strategies competence (the ability to formulate mathematical problems, represent them, and solve them); (4) adaptive reasoning (the capacity to think logically about the relationships among concepts and situations); and (5) productive disposition (the tendency to see sense in mathematics, to perceive it as both useful and worthwhile, and to see oneself as an effective learner and doer of mathematics) (p. 116).

One observation of entering college students is their lack of conceptual understanding in mathematics (Richland, Stigler, & Holyoak, 2012). Misconceptions and errors with fractions have been found in both conceptual understanding and procedural fluency (Panaoura et al., 2009). According to Conley (2008) "college-ready students possess more than a formulaic understanding of mathematics. They have the ability to apply conceptual understandings in order to extract a problem from a context, solve the problem, and interpret the solution back into the context" (p. 8). It appears that many students acquire only a procedural understanding of mathematics at the high school level and be able to get by with this limited knowledge (Kajander & Lovric, 2005; Richland, et al., 2012). Often students who enrolled in a basic algebra course in college see mathematics not as concepts but as rules and procedures of things to do in a particular order. Students who have not developed the conceptual understanding are the ones who have not formed the cognitive links between conceptual understanding and procedural knowledge (Hiebert & Wearne, 1986). The depth of basic mathematical knowledge and mathematical flexibility is often missing in students who arrive in a remedial or basic mathematics course in college. College instructors find that students have gone through high school mathematics classes without really understanding the mathematics involved (Hoyt & Sorensen, 2001).

Students first develop their mathematical understanding with natural numbers, including the definition and strategies with four operations (Vamvakoussi & Vosniadou, 2010). When new information is introduced, students attempt to "assimilate new information into their existing conceptual structures" (Stafylidou & Vosniadou, 2004, p. 505). However, the new information or concepts do not always fit into their existing conceptions, and as a consequence their existing knowledge structure becomes disintegrated and misconceptions can be generated (Kurkin & Rittle-Johnson, 2014). Thus, understanding new concepts sometimes requires a restructuring of the existing concepts. Restructuring existing concepts could take a long time, and newly structured concepts co-exist with existing concepts. For example, adults with a fully developed number system first look at whole number parts of fractions and whole number ordering may still interfere in comparing fractions (DeWolf & Vonsniadou, 2015).

These studies give us an insight into some of the reasons why students could form misconceptions with fractions. The findings of the current study could help educators look for methods for redirecting whole number bias in fractions and restructuring existing knowledge to facilitate the learning of more advanced mathematical concepts.

MISCONCEPTION WITH FRACTIONS

According to the National Assessment of Educational Progress (NAEP) report, only 50% of U.S. 8th graders could correctly order three fractions (Martin, Strutchens, & Elliott, 2007). Even adults compare fraction values by using their whole number parts (numerator and denominator) rather than the whole fraction value (Bonato, et al., 2007). According to Ni and Zhou (2005), students' common misconceptions about fractions derive from the generalizations of natural numbers to fractions.

Some examples of common misconceptions are as follows (Lee & Boyadzhiev, 2015; Mack, 1995; Stafylidou & Vosniadou, 2004; Vamvakoussi & Vosniadou, 2010)

- properties of whole numbers can be applied to fractions,
- the longer is larger (the larger numerators and denominators) is the larger fractions
- numerator and denominator are separate values, two separate whole numbers
- the numbers in numerator and denominator should be compared separately rather than considering the whole fraction
- operation rules for natural numbers can be applied to operations with fractions
- the value of the fraction increases when either the numerator or the denominator increase
- the unit is the smallest fraction
- multiplication always makes the number bigger and division always makes the number smaller
- fractions have unique successors (does not understand infinity and density)

Another point that mathematics educators should note is that students often hold a variety of correct concepts and misconceptions at the same time (Vamvakoussi & Vosniadou, 2010; Vosniadou & Verschaffel, 2004). Students make errors or exhibit misconceptions even though they have the correct concept, depending on the task. For example, students often ignore the fractional parts focusing only on the whole numbers when computing with mixed numbers (Fazio & Siegler, 2011). Strategies and mistakes for comparing fractions also differ depending on the given fractions, e.g., fractions with common denominators, fractions with common numerators, or fractions with no common components (Meert, et al., 2010). Fractions used in the current study have various forms: like denominators, unlike denominators, proper fractions, improper fractions, mixed numbers, etc. Also, simple computational errors were not marked as misconceptions.

METHOD

The research design includes both quantitative and qualitative data. Scores for quizzes and tests (quantitative data) were analyzed first using descriptive statistics, and qualitative data (students' solutions) were used follow upon the quantitative results. In order to get in-depth information about underprepared college students' misconceptions of fractions, the case study method was used. In this context, the case study method of qualitative research methods was used to examine the situation as a whole and in a comprehensive way. This mixed methods design (Creswell, 2015) integrates both data types and draws interpretations using the strengths of both sets to understand the research questions.

Participants

Participants of the study were enrolled in the first of two remedial mathematics series at a small university in Midwest. Twenty-two students were enrolled in the class, and 10 students agreed to participate in the case study. The 10 students' solutions were analyzed and reported in the paper. The group included eight Caucasian and two Black students; three female and seven male students, one student with a documented learning disability who was given extended time on the exams and quizzes. The final course grades ranged from A to E. Three of the 10 students had to retake the course and seven students were able to move on to the 2nd course in the sequence.

Data Collection and Analysis

Daily fraction questions were given at the beginning of each class, 2-3 days a week. Students who turned in a correct solution received one bonus point in addition to an attendance check. The aim was to create a safe environment for reviewing basic operations with fractions, without any negative effect on the grade of the students. Usually when a new type of fraction operation was introduced, it was repeated on two consecutive days before a different type was introduced. After collecting students' work, the instructor solved the problem, discussing different strategies and typical mistakes. Initial analysis of the study informed us that when faced with a problem taken out of the context of a particular section, students often had difficulties analyzing the problem and developing a strategy for solving it. In an effort to remediate this difficulty, we asked the students before they attempt any calculations to write a very brief outline of their strategy, starting by clearly stating what are the operations and the order in which they will be executed.

As can be seen in **Table 1**, different types of fractions and operations were used; like denominators, unlike denominators, problems with single operation, problems requiring more than one operation, or complex fractions. According to Common Core math standards (NGA, 2010), strategies needed to solve daily fraction problems were covered before graduating from high school. Student solutions were analyzed to investigate how their understanding of the core concepts progress. Our goal of collecting the qualitative data was to add insight and provide additional description and context found in scores for daily fraction problem scores and their incorrect responses.

Quantitative data were analyzed using descriptive statistics. Descriptive statistics and frequency distributions were examined to determine students' learning progress. As for solutions for daily fraction problems, students' work was coded using standard qualitative analysis techniques. Students' solution analysis involved four processes: (1) an initial reading of each response; (2) identifying correctness of the responses; (3) exploring the type of errors or strategies used; and (4) interpreting the data quantitatively and qualitatively (Creswell, 1994). For the purpose of the study, this paper shares findings from the in-depth analysis of qualitative data.

RESULTS

Fractional Understanding and Misconceptions: Whole Group

Reviewing the whole class data (22 students), most students solved fraction problems in a purely symbolic, notation format, using a traditional standard algorithm. On average 47% of students present correctly solved the fraction problems. Students particularly struggled with complex fraction problems. Almost half of the class were able to solve fraction problems even for multiple operations, but only 37% of students on average were able to solve complex fraction problems correctly. There were 7 problems that less than 25% of the students were able to solve correctly, and 5 of them were complex fraction problems.

Common error types in student solutions for fraction computations were treating numerators and denominators separately; errors with whole number computations; errors with fraction computations; misconception with LCM/LCD; misunderstanding (or errors) with simplifying fractions; misconception with equivalent fractions; lack of understanding the order of operations; confusion the minus and negative signs; incorrectly using cross multiplication. **Table 1** summarizes errors and misconceptions observed for different problem types.

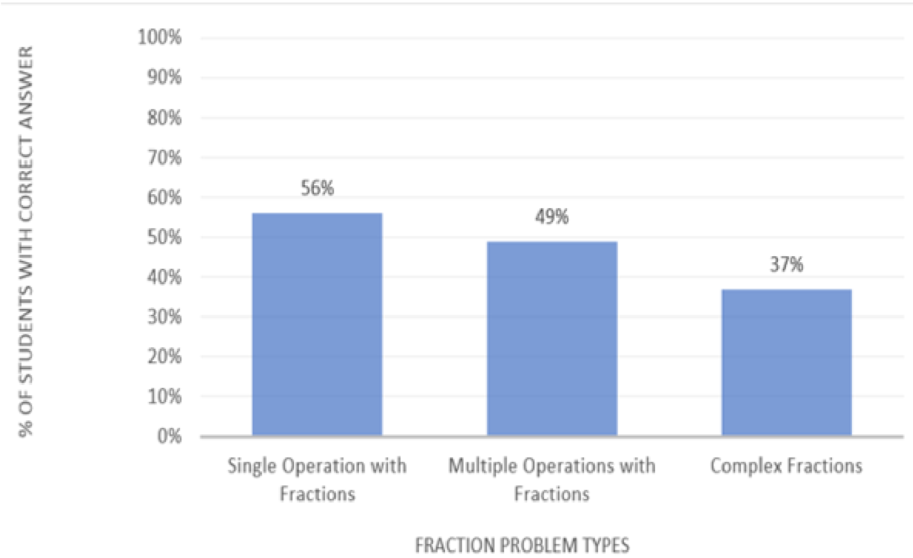
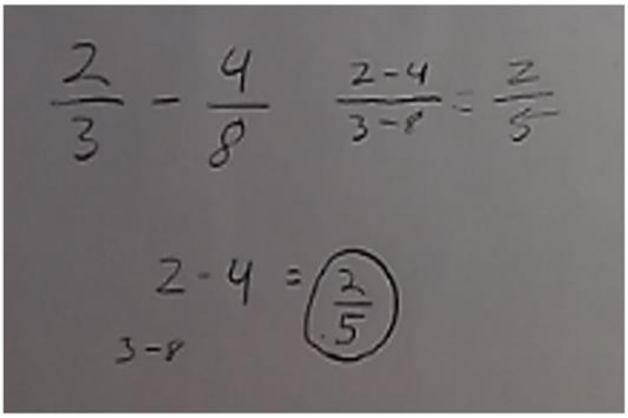


Figure 1. Students' Computational Skills with Fractions

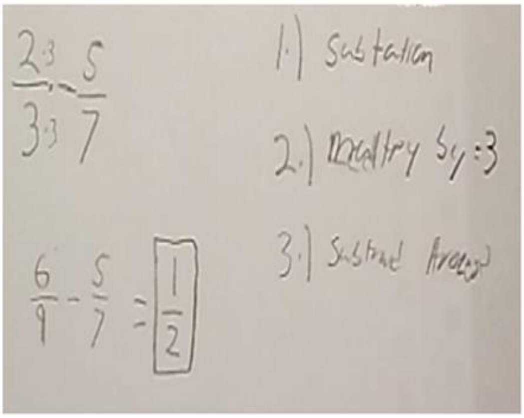
Table 1. Errors and Misconceptions

Problem Types	Examples	Errors and Misconceptions
Single operation with fractions	addition and subtraction problems with unlike denominators $\frac{5}{12}+\frac{3}{4}$ or $\frac{2}{3}-\frac{4}{8}$	• finding the Least Common Denominator (LCD) or Least Common Multiple (LCM) • writing equivalent fractions • did not attempt to find the LCD but combined separately the numerators and the denominators • operating numerators and denominators separately
	multiplication problems $\frac{3}{20}\times\frac{12}{40}$	• whole number multiplication errors • conceptual errors (e.g., writing the fractions with common denominators first) • errors from mimicking procedures unrelated to the problem at hand (e.g., cross-multiplying, or flipping one of the fractions and then multiplying) • could not simplify the answer
	division problems $\frac{5}{12}\div\frac{15}{60}$	• rewrote the division problem as a multiplication, but then made the mistakes associated with multiplication • false use of cross multiplication
Multiple operations with fractions	problems with more than one operation $\frac{2}{5}-\frac{3}{4}\div\frac{9}{2}$ $\frac{3}{25}-\frac{1}{2}\times\left(-\frac{3}{5}\right)$	• incorrect order of operations • false use of cross multiplication • lack of understanding of divisor and dividend. • differentiating negative fraction and subtraction sign
Complex Fractions	$\frac{\frac{3}{4}-\frac{5}{6}}{2-\frac{1}{3}}$ $\frac{2+\frac{3}{5}}{3+\frac{4}{7}}$	• incorrect order of operations • whole number computation errors • errors with simplifying fractions • finding the Least Common Denominator (LCD) or Least Common Multiple (LCM) • operating numerators and denominators separately

Misconceptions and errors with fraction computations were (1) lack of understanding the nature of fractional numbers, (2) lack of computational skills with whole numbers; (3) lack of understanding the meaning of fraction operations; (4) too much emphasis on finding a common denominator, and (5) misunderstanding order of operations. These misconceptions possibly resulted from insufficient understanding of operations or lack of interest in conceptually understanding fraction computations.



2a



2b

Figure 2. Treating Denominator and Numerator Separately

Close Analysis of Fraction Understanding and Misconceptions

Findings presented in this section are based on the 10 students who volunteered to participate in the study. As the course progressed and solving a fraction problem became a routine activity, some students who initially could not identify a correct strategy for solving the problem learned to list correctly the steps of the solution, but were still unable to execute the solution process. This section shares sample student solutions for various error types.

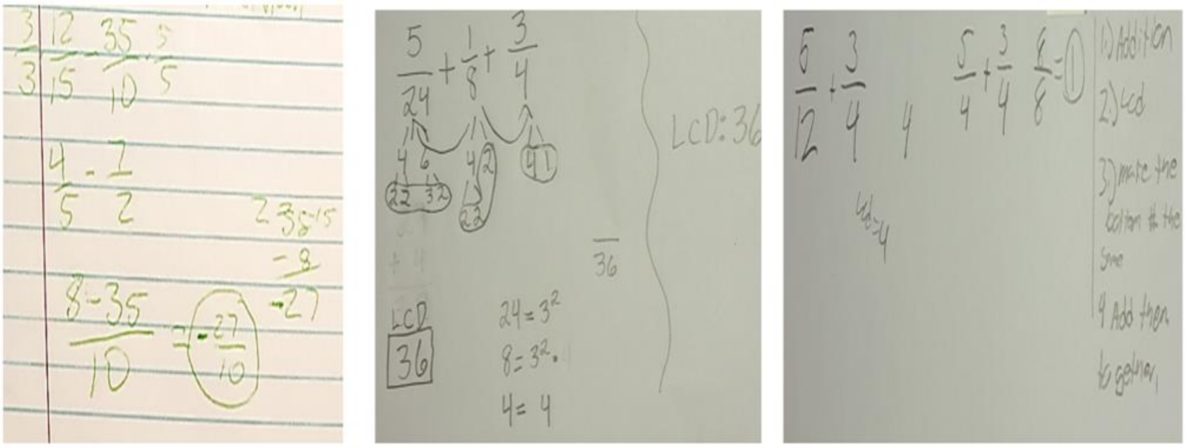
Treating denominator and numerator separately

One of the first problems on the daily practice was to calculate 2a) $\frac{2}{3}-\frac{4}{8}$ and 2b) $\frac{2}{3}-\frac{5}{7}$. Less than half of the students in the class solved it correctly. The most typical mistake was to treat the numerator and the denominator as separate numbers (**Figure 2**). None of the students who sloved the first problem correctly thought about simplifying the fraction $\frac{4}{8}$ before finding the LCD.

Lack of understanding of least common denominator (LCD)

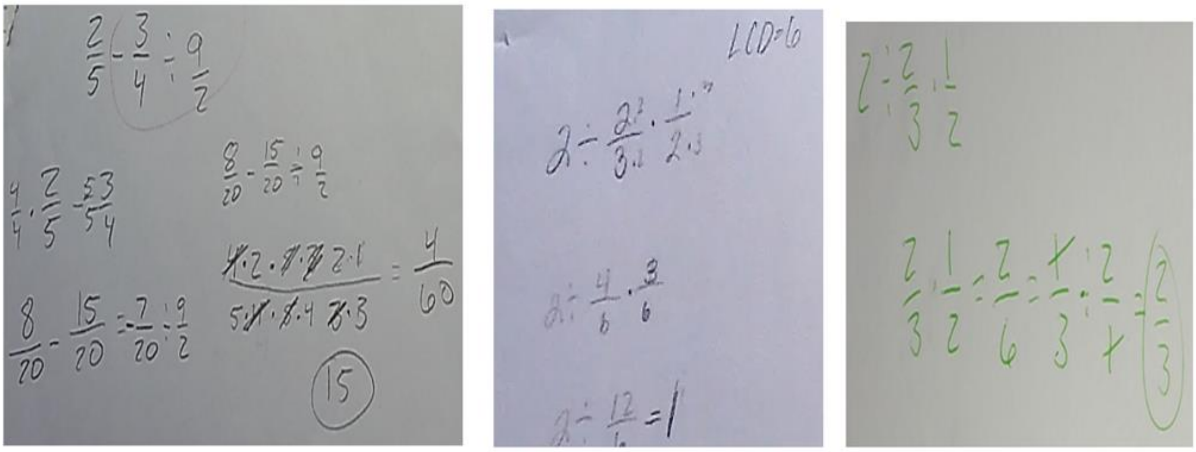
Figure 3 shows differents types of mistakes related to the LCD. In a follow-up problem a student (Student solution 3a) simplified the fractions before finding the LCD, but he puts a multiplication sign instead of division. This shows that he intuitivelly knows that he wants to simplify the numbers and was able to get the correct answer. However, the procedure he learned demanded multiplying the numerator and denominator, so he put a multiplication sign. Textbooks often overemphasize the procedure of finding the equivalent fraction as "multiplying the numerator and the denominator by an appropriate number" instead of explaining the concept.

Student solution 3b shows a lack of understanding the LCD in addition to a basic computation and a procedural error. However, it was not easy to interpret her incorrect method in finding the LCD rather than the solution was showing the student's lack of understanding the LCD. Conducting an interview with this student might help us understand her incorrect reasoning. Student solution 4c shows a confusion of LCD with the Greatest Common Factor (GCF), which is not uncommon, "make the bottom numbers the same." This error possibly resulted from misunderstanding equivalent fractions. Student solutions 3a – 3c show that there is too much emphasis on the procedure of finding the LCD as "multiplying by an appropriate number" rather than the concept of finding the "smallest possible number, multiple to all denominators." Students' errors with the LCD are possibly due to their misunderstanding of the definition of LCD, lack of understanding about the use the LCD, or a lack of computational skills with whole numbers.



3a 3b 3c

Figure 3. Lack of understanding of Least Common Denominator (LCD)



4a 4b 4c

Figure 4. Misunderstanding of Order of Operations

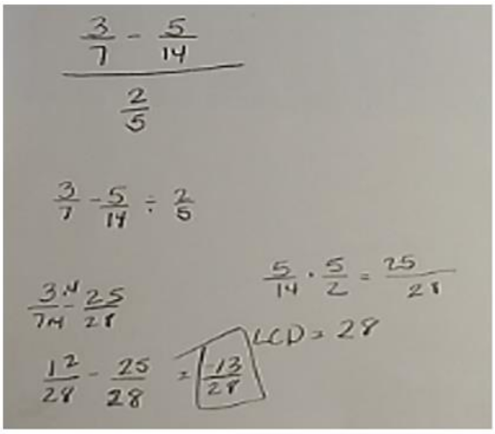
Computational and procedural errors

Student solutions 3b and 3c show some other common mistakes. Student solution 3b shows a basic computation and a procedural error in addition to misconceptions with exponents. The student presenting solution 3c has some conceptual understanding, but lacks the ability to perform each procedure correctly (e.g. errors in expanding fractions and in addition of fractions with the same denominator). The outline of this solution presents an interesting aspect. Step number (3) reads "make the bottom numbers the same", and she did exactly that, without changing the numerators. She changed the 1st addend $\frac{5}{12}$ to $\frac{5}{4}$ instead of changing the 2nd addend $\frac{3}{4}$ to $\frac{9}{12}$. This error is probably due to the way this topic is usually taught, overemphasizing the requirement of equal denominators in the addition and subtraction problems instead of emphasizing the equivalency of the fractions with equal denominators and the original fractions.

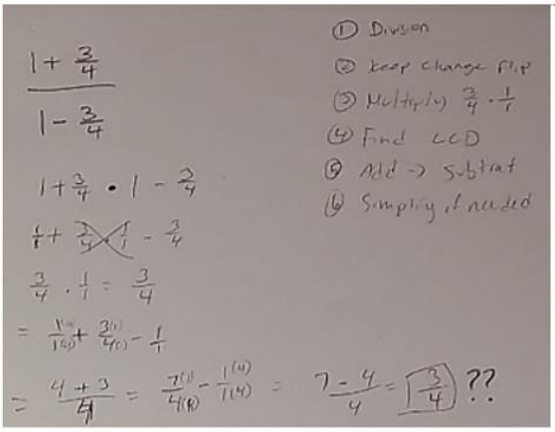
Misunderstanding of order of operations

One of the most common mistakes or misunderstanding observed from student solutions was incorrect use of the order of the operations (**Figure 4**).

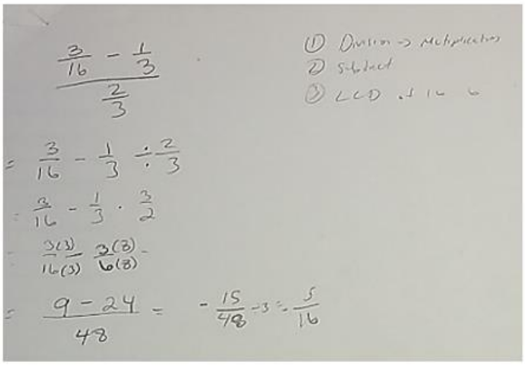
Mistakes and misconceptions associated with order of operations as in student solution 5a can be corrected easily. Most students know about PEMDAS and like to use this mnemonic rule. However, sometimes the problem is PEMDAS itself. We see this in student solutions 5b and 5c. There are other computational errors in both of them, but these solutions have a common order of operations error due to following PEMDAS without



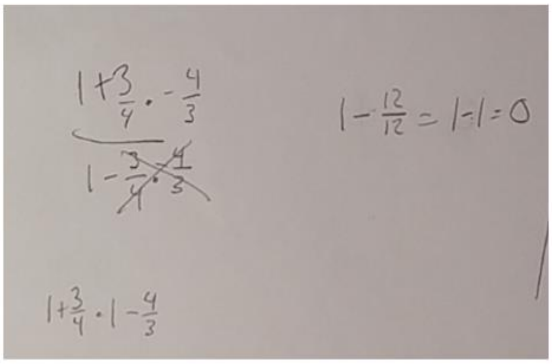
5a



5b



5c



5d

Figure 5. Lack of Understanding the Definition of Fractions

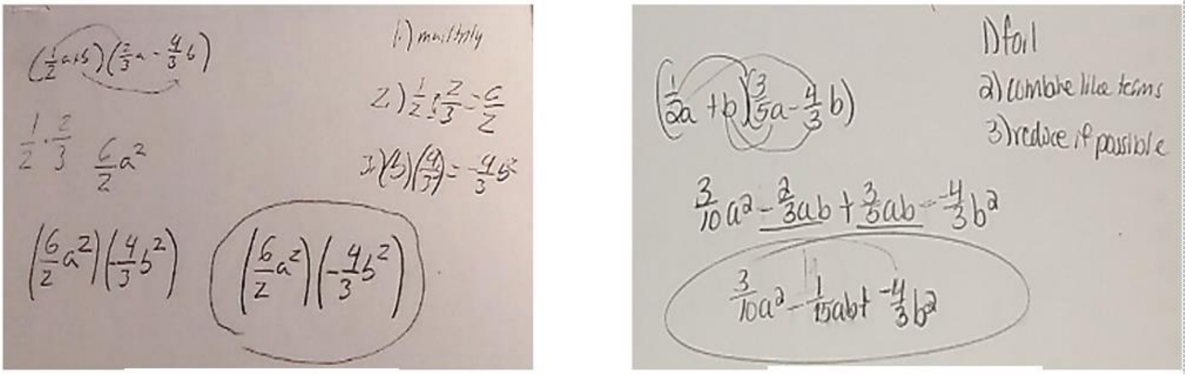
a conceptual understanding of the order of the operations. By putting M to the left of D, PEMDAS implies that multiplication has higher precedence than division, which of course is not true! We think that the practice of substituting mathematical rules based on the meaning of operations and definitions by mnemonic rules, provides only short-term gain and should not be encouraged.

Lack of understanding the definition of fractions

Students in the remedial mathematics classes often do not realize that the fractional bar, in addition to being a sign for division, is also a symbol of grouping (**Figure 6**). The same type of error persists later in work with rational expressions and it is usually difficult to correct. Student solution 6d has another interesting mistake, which propagates later to algebra problems involving negative exponents. The student flips the fraction in the denominator (1-3/4) and this is enough to justify rewriting the division problem as a multiplication. No symbols of grouping, of course.

Lack of conceptual understanding of the problem

Students often did not have a clear plan for solving a mathematical problem. Instead they tried a series of calculations that often did not lead to a solution. In an effort to remediate this issue, throughout the semester, we asked our students to outline briefly the steps of their solutions of the daily fraction problem, starting by clearly stating what are the operations and the order in which they will be executed, before they attempt any calculations. The next work samples compare two different approaches, procedural vs. conceptual understanding based (**Figure 6**). Student 6a focused on procedures thinking in terms of particular calculations whereas student 6b approached the problem with a conceptual understanding of the problem and a clear plan of problem solving strategy.



6a 6b

Figure 6. Lack of Conceptual Understanding of the Problem

DISCUSSION AND IMPLICATIONS

One cannot underestimate the importance of improving the mathematical readiness of students entering college or the university. We can trace the reasons for mathematical unpreparedness of some students all the way back to elementary school. Typical remedial math classes briefly review basic numerical operations and quickly move to algebra, only on to find that close to 50% of the students are often unable to complete the classes successfully.

Although many studies investigated children's understanding of fractions, a relatively small number of studies addressed the problem on the college level. In this regard, our findings provide specific information for the problems encountered in pre-college algebra courses. This study provides important information for instructors of developmental mathematics classes about how to help underprepared students improve their mathematical understanding and performance. The immediate work was done on solving problems involving operations with fractions, but the real aim of the study was to better prepare the students for work with rational expressions and complex fractions. Misconceptions and errors found in this study suggest that none of these problems can be corrected in the span of a one-week review. We may have to rethink the way we plan the remedial courses so that they really remediate the problem, and not contribute to it. As a result of this study, college mathematics instructors could gain insights into effective ways of preparing students in remedial courses so that they can succeed in college mathematics.

Students that were previously unsuccessful in acquiring basic mathematics skills can be successful in their mathematics courses through positively changing experiences, attitudes, and learning strategies. Identified learning strategies for successful achievement in mathematics are attendance, taking notes, students' ability to ask questions effectively, and accurate homework completion (Howard & Whitaker, 2011). The study found that most students solved fraction problems in a purely symbolic, notation format, using a traditional standard algorithm rather than using the context of the problem or solving problems conceptually. Relying on memorized algorithms and procedures and lacking conceptual understanding of fractional concepts caused the same mistakes repeatedly and lost confidence in operating with fractions. Underprepared students often rely on memorizing procedures rather than understanding basic math concepts because they are in a constant hurry to complete a homework assignment, to get ready for a quiz, to cram for the exam, and to reach the coveted C that will let them take the next class.

Another problem is introducing mnemonics without conceptual understanding (Ameis, 2011; Rambhia, 2002). As discussed in the Results section, some students in the study used PEMDAS, completing the multiplications and then the divisions or vice versa, regardless of the operation orders in the task. This prevalent use of incorrect strategy is observed in other studies (Hooslain & Naraine, 2012). The lack of conceptual understanding limits students' ability to master basic mathematics concepts and hinders their chances to succeed in college mathematics courses (Conley, 2008; Hoyt & Sorensen, 2001; Kajander & Lovric, 2005).

A common weakness in the work of the underprepared college student is the inability to write an organized solution. Often their work is a collection of separate scattered calculations rather than coherent mathematical statements. Students are focused only on finding the answer and treat the steps of the solution as a secondary and unimportant part of it. This could be traced back to school assignments that consists of a large number of similar problems printed on a one-page handout where students have only a small amount of space for some occasional intermediate calculations, not enough for writing a complete solution. Typically, only the answers are checked. This might be a good way to prepare students for a multiple-choice standardized test, but it doesn't contribute to students' readiness for algebra. Teaching arithmetic should never be done for the sake of learning only simple computations. Everything we teach in these classes should have as an ultimate goal preparing students for the more abstract symbolic operations in algebra. An emphasis should be placed on using correct terminology, organized writing, correct use of symbols of grouping, correct use of the equality sign, correct order of operations, etc.

To address these problems, we may have to reevaluate the content of the remedial math courses. At least half of the semester should be spent on building basic arithmetic skills that lead to seamless transition to algebra.

CONCLUSION

This study focused on fractional concepts, because misunderstanding and incomprehensive skills with fractions will hinder students' learning of operations with rational expressions, exponents, and other algebraic operations. The study investigated the nature of the misconceptions of fractions in the students enrolled in a remedial mathematics course. Most common misconceptions were not only due to the inability to compute with fractions but also a lack of understanding of basic fraction concepts. In addition, errors with whole number computations, misconception of LCM/LCD, a lack of understanding with the order of operations, or confusion with the minus and negative sign were also found in students' solutions for fraction computations.

Common errors and misconceptions associated with fraction operations possibly resulted from three issues, students' lack of understanding about fractions, their lack of number sense and computational skills with whole numbers, and relying too much on procedures without conceptual understanding. Students in remedial mathematics course, including the participants of this study, do not fully understand the nature of fractional numbers and the meaning of fraction operations. This leads students to rely on memorizing procedures rather than understanding the procedures conceptually. Students often remember what they learned last about fractions, e.g. 'find a common denominator', 'cross multiply' or 'flip' without understanding why and what the action means. As shown in student work samples, students automatically look for a common denominator without understanding the problem, rewrite the division problem as a multiplication, or use cross multiplication incorrectly.

In addition, students' lack of understanding of numbers in general or computational skills with whole numbers was observed of many students in remedial courses. Our findings concur that students are often confused about divisor and dividend, do not fully understand order of operations, or struggle finding common multiples. Issues discussed in the study can be a critical barrier for operating fractions or understanding terminology/concepts associated with fractions, such as equivalent fractions, common denominator, fraction as an operator, etc.

Further studies are needed in determining other sources of mathematics unpreparedness of some college students. For example, the lack of mathematical writing skills, the inability to correctly use the equal sign and parentheses, or the inability to understand and use correctly mathematical terminology should be investigated. In addition, this study did not aim to find the effectiveness of certain interventions for underprepared college students. The students gained a bonus point if they solved a problem correctly, but there were no consequences if they failed. Some students might have not been very motivated to succeed. Students practiced operating with fractions 3 times a week, and the study found that students' quiz scores with daily fraction problems and their overall grade were positively related. Future research should explore what types of approaches or opportunities can be provided in order to address the problem of college mathematics readiness for all students.

Disclosure statement

No potential conflict of interest was reported by the authors.

Notes on contributors

Hea-Jin Lee – Ohio State University, Lima, USA.

Irina Boyadzhiev – Ohio State University, Lima, USA.

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