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The Functional Dependence for the Estimation of Additional Losses of Active Power in a Double-Wound Power Transformer Caused by Asymmetric Active-Inductive Load with a Delta Connection

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ABSTRACT

This research investigates the problem of estimating additional losses in 6 (10)/0.4 kV voltage rating double-wound power transformers caused by asymmetric active-inductive load with a delta connection. This research used the symmetrical components method, modern methods of analysis and synthesis of electric circuits, the theory of electric circuits, full-scale experiment, and comparative experiment. The research found a functional dependence that enables estimating additional losses in the transformer caused by asymmetric load, which differs from similar ones in that it uses phase resistance. The amperage, voltage, and active power was measured in each phase of the “distribution transformer – asymmetric load” model to confirm the discovered functional dependence. The experiments showed that the loss of active power, calculated according to the standard formula, should be corrected with regard to the discovered functional dependence. The practical value of the offered functional dependence for the estimation of additional losses is that it enables estimating the losses of active power in transformers due to asymmetry by measured voltage, amperage, and active power for each phase. This makes it universal: there is no need to build equivalent circuits and carry out calculations according to the symmetrical components method.

KEYWORDS
Asymmetric active-inductive load, loss of active power, delta connection, double-wound power transformer, distribution network

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Introduction

Aggregate losses in the networks of power systems and users comprise a considerable part of power supplied to the network from power station buses (Shidlovsky, 1985). Most losses are attributed to distribution networks (Barker & Mello, 2000; Harlow, 2004). Optimizing modes and modernizing power grids is the priority way of cutting technical losses of power (Lathrop et al., 2011; Dmitriev & Kokin, 2010).

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According to foreign (Aoki et al., 2002; Rao et al., 2013) and Russian experts (Tropin, Savenko & Perepechin, 2005; Tropin, Savenko & Maleyev, 2008), the main reason behind losses in excess of industry standards is the asymmetric load of distribution transformers. In addition, the investigation of the operation of distribution transformers at agricultural and municipal facilities found that their installed capacity is underused (Dondi et al., 2002; Semenov, 2011).

A considerable growth of additional losses occurs when asymmetry exceeds the permissible limits. The main losses imply forced losses of power in symmetric, sinusoidal, even, and active nominal modes (Girshin et al., 2013). The research of R. Rao et al. (2013) shows that additional losses occur when power quality indexes deviate from standard values.

If the high voltages of distribution transformers differ, then their low voltages cannot be identical either. This generates a shift of the neutral point, which causes considerable additional active losses in transformers with a Y-connection low voltage winding and Y-connection high voltage winding with a the neutral available, despite the fact that in symmetric mode, the characteristics of idling and short circuit losses in these transformers are better than in transformers with other types of connection (Olivares et al., 2003).

For a transformer with a Y-connection high voltage winding and Z-connection low voltage winding with the neutral available to have similar characteristics of idling and short circuit losses, it is necessary to use a large amount of materials, since the Z-connection requires a greater number of coils (Kutin & Lagutin, 2008).

It is worth noting that statistical treatment of the protocols of distribution transformer tests in a city and rural district (Kostinsky, 2009), as well as the results of studies carried out using a TSZ-2.5/220 transformer, used in a "double-wound three-phase transformer – asymmetric load" physical model (Power Installation Design Manual, 2006), showed that in transformers that operated for an extended period of time, both idling and short circuit losses were greater than their rated values. At that, these indexes were higher in transformers of up to 1000 kV•A than in higher capacity transformers. These data match the results presented in (Tsyruk & Kireyeva, 2008).

According to the research of V. Zaugolnikov, A. Balabin & A. Savinkov (2006), after the rapid reduction of power consumption in the Russian Federation in the 1990s, this index has not yet reached its pre-crisis level in many regions.

Transformers operate under consideration underutilization, especially in rural areas. The economic efficiency of their operation is generally assessed by the energy conversion coefficient or relative losses, the charts whereof, depending on the load of a separate transformer, are essentially inverted charts of the energy conversion coefficient (Zaugolnikov, Balabin & Savinkov, 2006).

The service life of power transformers is 25 years; it is established based on the assumption that thermal wear of the winding insulation can take place during this period, since said insulation determines the resource of the power transformer (Serban, 2015). Experience of operation showed that transformers can sustain serious damage before the insulation resource is exhausted.

In addition to insulation, the magnetic structure is also exposed to wear, which is shown by increased idling losses. During maintenance tests of transformers with idling experiments, the idling losses should not differ from rated values by more than 5% (Shidlovsky & Kuznetsov, 1985).

Some studies give data, which show that the magnetic structure ages much earlier. Therefore, S. Tsyruk & E. Kireyeva (2008) offer doing repairs in accordance with the actual technical state based on diagnostic results, primarily using heat monitoring, instead of scheduled and preventive repairs, which are obligatory (Decree of the Ministry of Energy of the Russian Federation, 2003). With that, S. Tsyruk & E. Kireyeva argue that using additional monitored parameters in addition to conventional techniques is economically expedient (2008).

Asymmetric operating modes have begun drawing more attention in recent years, since the municipal power consumption in a number of power systems has exceeded the industrial power consumption, which disrupted the symmetry and balance of current and power systems (Damjanovic, Integlia & Sarwat, 2016). This shows the relevance of improving the estimation and the cutting of power losses in distribution networks (Dogru, 2008).

The purpose of this research is to determine the functional dependence for the estimation of additional losses of active power in a double-wound power transformer caused by asymmetric active-inductive load with a delta connection.

Methods

This research used the symmetrical components method, modern methods of analysis and synthesis of electric circuits, the theory of electric circuits, full-scale experiment, and comparative experiment.

The amperage, voltage, and active power was measured in each phase of the "distribution transformer – asymmetric load" model to confirm the discovered functional dependence. The following measuring devices were used:

- AR.5 CIRCUTOR portable power quality analyzer, factory No. 408612036. Measurement range: amperage – 0.05 … 5 A, 1 … 200 A; voltage 1 … 500 V;
- K-540 measurement kit, factory No. 1213: nominal voltage of built-in voltmeter – 15, 30, 75, 150, 300, 450, 600 V; nominal amperage of built-in ammeter – 0.1, 0.25, 0.5, 1, 2.5, 5, 10, 25, 50 A; nominal active power of built-in wattmeter – from 0 to 30 kW within the above limits of amperage and voltage measurement.

These measuring devices have 0.5 accuracy and certificates of calibration.

The research object was a TSZ-2.5 three-phase double-wound transformer with 2.5 kV·A nominal power, 220 V at the high voltage winding, and 127 V at the low voltage winding.

Loading rigs with active and inductive resistance were used to verity the discovered functional dependence.

The rigs enabled modeling various operating modes of the "distribution transformer – asymmetric load" model:

- active symmetric and asymmetric load;
- active-inductive symmetric and asymmetric load.

These rigs also enabled investigating the operating mode of the transformer with a delta-connection load.

Data, Analysis, and Results

Thes “distribution transformer – asymmetric load” model is presented in the form of a system of symmetric EMF sources, in which $\dot{E}_A = jU$, to which the asymmetric active-inductive load can be connected using a delta-connection (Figure 1), where the complex resistance of phases is

$$\underline{Z}_{AB} \neq \underline{Z}_{BC} \neq \underline{Z}_{CA}; \quad \underline{Z}_{AB} = R_{AB} + jX_{AB}; \quad \underline{Z}_{BC} = R_{BC} + jX_{BC};$$
$$\underline{Z}_{CA} = R_{CA} + jX_{CA}.$$

This scheme requires determining: losses from negative phase-sequence currents when compared with losses from positive phase-sequence currents; full, active, and reactive power; reactive power factor; pulsed power.

Due to the symmetry, electromotor forces of phases B and C, respectively, are: $\dot{E}_B = jU \cdot a^2$; $\dot{E}_C = jU \cdot a$. Here, $a = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$; $a^2 = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$ are unit vectors of 120° and 240° counterclockwise rotation.

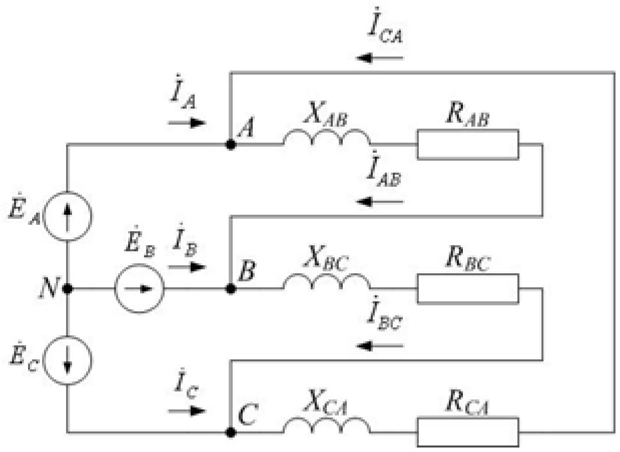

Figure 1. Three-phase network with a symmetric system of EMF sources and asymmetric active-inductive load with a delta connection.

Due to symmetric electromotor force sources,

$\dot{E}_B = \dot{E}_A \cdot \boldsymbol{a}^2 = jU \cdot \boldsymbol{a}^2$, $\dot{E}_C = \dot{E}_A \cdot \boldsymbol{a} = jU \cdot \boldsymbol{a}$,

where $\boldsymbol{a} = e^{j120^0} = -\frac{1}{2} + j\frac{\sqrt{3}}{2}$; $\boldsymbol{a}^2 = e^{j240^0} = -\frac{1}{2} - j\frac{\sqrt{3}}{2}$ – are rotation operators.

The scheme in Figure 1 can use all the results of the study for the Y-connection with an isolated neutral point, given a transition from this scheme to a delta-connection scheme. In this case,

$$\underline{Z}_A = \frac{(R_{AB} + jX_{AB})(R_{CA} + jX_{CA})}{R_{AB} + R_{BC} + R_{CA} + j(X_{AB} + X_{BC} + X_{CA})} = \frac{\underline{Z}_{AB} \cdot \underline{Z}_{CA}}{\underline{Z}_{AB} + \underline{Z}_{BC} + \underline{Z}_{CA}}; \quad (1)$$

$$\underline{Z}_B = \frac{(R_{AB} + jX_{AB})(R_{BC} + jX_{BC})}{R_{AB} + R_{BC} + R_{CA} + j(X_{AB} + X_{BC} + X_{CA})} = \frac{\underline{Z}_{AB} \cdot \underline{Z}_{BC}}{\underline{Z}_{AB} + \underline{Z}_{BC} + \underline{Z}_{CA}}; \quad (2)$$

$$\underline{Z}_C = \frac{(R_{CA} + jX_{CA})(R_{BC} + jX_{BC})}{R_{AB} + R_{BC} + R_{CA} + j(X_{AB} + X_{BC} + X_{CA})} = \frac{\underline{Z}_{CA} \cdot \underline{Z}_{BC}}{\underline{Z}_{AB} + \underline{Z}_{BC} + \underline{Z}_{CA}}. \quad (3)$$

According to Figure 1,

$$\dot{U}_{AB} = \dot{E}_B - \dot{E}_A = jU(\boldsymbol{a}^2 - 1); \quad (4)$$

$$\dot{U}_{BC} = \dot{E}_C - \dot{E}_B = jU(\boldsymbol{a} - \boldsymbol{a}^2); \quad (5)$$

$$\dot{U}_{CA} = \dot{E}_A - \dot{E}_C = jU(1 - \boldsymbol{a}). \quad (6)$$

$$\dot{I}_{AB} = \frac{\dot{U}_{AB}}{\underline{Z}_{AB}} = \frac{jU(\boldsymbol{a}^2 - 1)}{\underline{Z}_{AB}}; \quad (7)$$

$$\dot{I}_{BC} = \frac{\dot{U}_{BC}}{\underline{Z}_{BC}} = \frac{jU(\boldsymbol{a} - \boldsymbol{a}^2)}{\underline{Z}_{BC}}; \quad (8)$$

$$\dot{I}_{CA} = \frac{\dot{U}_{CA}}{\underline{Z}_{CA}} = \frac{jU(1 - \boldsymbol{a})}{\underline{Z}_{CA}}. \quad (9)$$

According to equations for nodal currents of the scheme shown in Figure 1,

$$\begin{cases} \dot{I}_A + \dot{I}_{CA} - \dot{I}_{AB} = 0; \\ \dot{I}_B + \dot{I}_{AB} - \dot{I}_{BC} = 0; \\ \dot{I}_C + \dot{I}_{BC} - \dot{I}_{CA} = 0; \end{cases} \rightarrow \begin{cases} \dot{I}_A = \dot{I}_{AB} - \dot{I}_{CA}; \\ \dot{I}_B = \dot{I}_{BC} - \dot{I}_{AB}; \\ \dot{I}_C = \dot{I}_{CA} - \dot{I}_{BC}. \end{cases}$$

Considering equations (7) … (9), the expressions for the calculation of linear currents will be as follows:

$$\dot{I}_A = jU\left(\frac{\boldsymbol{a}^2 - 1}{\underline{Z}_{AB}} - \frac{1 - \boldsymbol{a}}{\underline{Z}_{CA}}\right) = jU\,\frac{\underline{Z}_{CA}(\boldsymbol{a}^2 - 1) + \underline{Z}_{AB}(\boldsymbol{a} - 1)}{\underline{Z}_{AB}\,\underline{Z}_{CA}}; \quad (10)$$

$$\dot{I}_B = jU\left(\frac{\boldsymbol{a} - \boldsymbol{a}^2}{\underline{Z}_{BC}} - \frac{\boldsymbol{a}^2 - 1}{\underline{Z}_{AB}}\right) = jU\,\frac{\underline{Z}_{AB}(\boldsymbol{a} - \boldsymbol{a}^2) + \underline{Z}_{BC}(1 - \boldsymbol{a}^2)}{\underline{Z}_{AB}\,\underline{Z}_{BC}}; \quad (11)$$

$$\dot{I}_C = jU\left(\frac{1 - \boldsymbol{a}}{\underline{Z}_{CA}} - \frac{\boldsymbol{a} - \boldsymbol{a}^2}{\underline{Z}_{BC}}\right) = jU\,\frac{\underline{Z}_{BC}(1 - \boldsymbol{a}) + \underline{Z}_{CA}(\boldsymbol{a}^2 - \boldsymbol{a})}{\underline{Z}_{CA}\,\underline{Z}_{BC}}. \quad (12)$$

Considering the values of equations for linear currents (10) … (12), using the Fortescue transformation,

$$\dot{I}_{1A} = \frac{1}{3}\left(\dot{I}_A + \boldsymbol{a}\cdot\dot{I}_B + \boldsymbol{a}^2\cdot\dot{I}_C\right) = -jU\left(\underline{Y}_{AB} + \underline{Y}_{BC} + \underline{Y}_{CA}\right),$$

$$\dot{I}_{2A} = \frac{1}{3}\left(\dot{I}_A + \boldsymbol{a}^2\cdot\dot{I}_B + \boldsymbol{a}\cdot\dot{I}_C\right) = jU\left(\underline{Y}_{BC} + \boldsymbol{a}\cdot\underline{Y}_{CA} + \boldsymbol{a}^2\cdot\underline{Y}_{AB}\right).$$

The asymmetry factor with negative sequence is

$$\dot{K}_2 = \frac{\dot{I}_{2A}}{\dot{I}_{1A}} = -\frac{\underline{Z}_{AB}\underline{Z}_{CA} + \boldsymbol{a}\cdot\underline{Z}_{AB}\underline{Z}_{BC} + \boldsymbol{a}^2\cdot\underline{Z}_{BC}\underline{Z}_{CA}}{\underline{Z}_{AB}\underline{Z}_{BC} + \underline{Z}_{AB}\underline{Z}_{CA} + \underline{Z}_{BC}\underline{Z}_{CA}} = -\frac{\underline{Y}_{BC} + \boldsymbol{a}\cdot\underline{Y}_{CA} + \boldsymbol{a}^2\cdot\underline{Y}_{AB}}{\underline{Y}_{AB} + \underline{Y}_{BC} + \underline{Y}_{CA}}.$$

Additional losses of power for the scheme presented in Figure 1 are in proportion to the squared modulus of the current asymmetry factor with negative sequence (Troitsky, 2001):

$$\Delta P^* = \left|\dot{K}_2\right|^2 = \left|-\frac{\underline{Y}_{BC} + \boldsymbol{a}\cdot\underline{Y}_{CA} + \boldsymbol{a}^2\cdot\underline{Y}_{AB}}{\underline{Y}_{AB} + \underline{Y}_{BC} + \underline{Y}_{CA}}\right|^2. \quad (13)$$

By comparing the right-hand side of equation (13) with the similar equation for the Y-connection scheme with an isolated neutral point, one can conclude that phase conductance is used hear instead of phase resistance.

The following designations are introduced:

$\varepsilon = Y_{BC}^2 + Y_{CA}^2 + Y_{AB}^2 - G_{BC}G_{CA} - G_{BC}G_{AB} - G_{CA}G_{AB} - B_{BC}B_{CA} - B_{BC}B_{AB} - B_{CA}B_{AB};$

$\epsilon = \sqrt{3}G_{BC}(B_{CA} - B_{AB}) + \sqrt{3}G_{CA}(B_{AB} - B_{BC}) + \sqrt{3}G_{AB}(B_{BC} - B_{CA});$

$\zeta = Y_{BC}^2 + Y_{CA}^2 + Y_{AB}^2 + 2(G_{BC}G_{CA} + B_{BC}B_{CA}) + 2(G_{BC}G_{AB} + B_{BC}B_{AB}) +$
$+2(G_{CA}G_{AB} + B_{CA}B_{AB})$.

With the above designations, the equation for the estimation of additional losses of active power in arbitrary units (AU) for asymmetric active-inductive three-phase load with a delta-connection is a follows:

$$\Delta P^* = \frac{\varepsilon + \epsilon}{\zeta}. \tag{14}$$

In the special case when the inductive resistances of phases are equal, but active resistances differ, i.e. $X_{AB} = X_{BC} = X_{CA} = X$; $R_{AB} \neq R_{BC} \neq R_{CA}$, according to equation (14),

$$\Delta P_1^* = \frac{G_{BC}^2 + G_{CA}^2 + G_{AB}^2 - G_{BC}G_{CA} - G_{BC}G_{AB} - G_{CA}G_{AB}}{G_{BC}^2 + G_{CA}^2 + G_{AB}^2 + 2G_{BC}G_{CA} + 2G_{BC}G_{AB} + 2G_{CA}G_{AB} + 9B^2}. \tag{15}$$

If additionally, the reactive load in phases is compensated ($X = 0$), then

$$\Delta P_2^* = \frac{G_{BC}^2 + G_{CA}^2 + G_{AB}^2 - G_{BC}G_{CA} - G_{BC}G_{AB} - G_{CA}G_{AB}}{(G_{BC} + G_{CA} + G_{AB})^2};$$

$$\Delta P_2^* = \frac{\frac{1}{R_{BC}^2} + \frac{1}{R_{CA}^2} + \frac{1}{R_{AB}^2} - \frac{1}{R_{BC}R_{CA}} - \frac{1}{R_{BC}R_{AB}} - \frac{1}{R_{CA}R_{AB}}}{\frac{1}{R_{BC}^2} + \frac{1}{R_{CA}^2} + \frac{1}{R_{AB}^2} + \frac{2}{R_{BC}R_{CA}} + \frac{2}{R_{BC}R_{AB}} + \frac{2}{R_{CA}R_{AB}}}. \tag{16}$$

By taking $R_{BC} = 1, r_{ca} = \frac{R_{BC}}{R_{CA}}, r_{ab} = \frac{R_{BC}}{R_{AB}}$, from equation (16),

$$\Delta P_2^* = \frac{1 + \frac{1}{r_{ca}^2} + \frac{1}{r_{ab}^2} - \frac{1}{r_{ca}} - \frac{1}{r_{ab}} - \frac{1}{r_{ca}r_{ab}}}{\left(1 + \frac{1}{r_{ca}} + \frac{1}{r_{ab}}\right)^2}. \tag{17}$$

Figure 2 shows the diagram of function (17) in r_{ab} and r_{ca} variation interval from 0.01 to 1, in increments of 0.01.

During the operation of networks, failures of one phase or phase-to-ground short circuits can occur.

For an asymmetric active-inductive three-phase load with a delta-connection and the abovementioned limitations, if the reactive load in phases is compensated ($X = 0$) and $R_{BC} = R_{CA} = 1$, a $R_{AB} = \infty$, excess loads will be

$$\Delta P_2^* = \frac{\frac{1}{1} + \frac{1}{1} + \frac{1}{\infty} - \frac{1}{1} - \frac{1}{\infty} - \frac{1}{\infty}}{\left(\frac{1}{1} + \frac{1}{1} + \frac{1}{\infty}\right)^2} = \frac{1}{4} = 0.25.$$

In addition to $X_{AB} = X_{BC} = X_{CA} = X$, consider that the active load in only one phase differs from the load of other phases $R_{AB} \neq R_{BC} = R_{CA}$, then, according to formulas (15) and (16),

$$\Delta P_1^* = \frac{\left(\frac{1}{R_{BC}} - \frac{1}{R_{AB}}\right)^2}{\left(\frac{1}{R_{BC}} + \frac{2}{R_{AB}}\right)^2 + \frac{9}{X^2}} = \frac{(G_{BC} - G_{AB})^2}{(G_{BC} + 2G_{AB})^2 + 9B^2}; \ P_2^* = \frac{\left(\frac{1}{R_{BC}} - \frac{1}{R_{AB}}\right)^2}{\left(\frac{1}{R_{BC}} + \frac{2}{R_{AB}}\right)^2},$$

and according to formula (17),

$$\Delta P_2^* = \frac{\left(1 - \frac{1}{r_{ab}}\right)^2}{\left(1 + \frac{2}{r_{ab}}\right)^2}. \qquad (18)$$

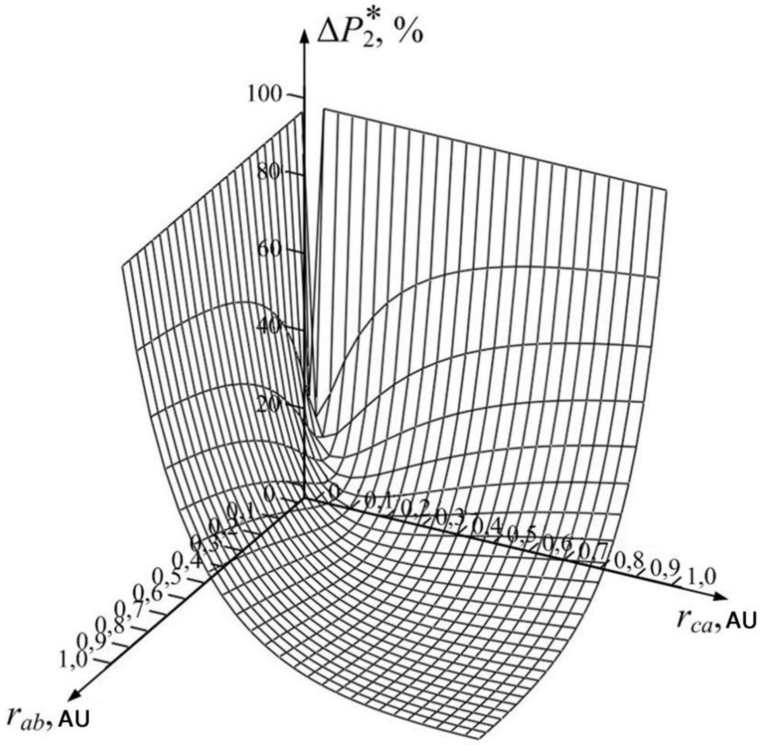

Figure 2. Function (17) diagram.

Assume $R_{BC} = n \cdot R_{AB}$, then

$$\Delta P_2^* = \frac{\left(\frac{1}{n} - 1\right)^2}{\left(\frac{1}{n} + 2\right)^2}. \qquad (19)$$

With equal active ($R_{AB} = R_{BC} = R_{CA} = R$) and unequal inductive ($X_{AB} \neq X_{BC} \neq X_{CA}$) phase resistances, formula (14) is as follows

$$\Delta P_1^* = \frac{B_{BC}^2 + B_{CA}^2 + B_{AB}^2 - B_{BC}B_{CA} - B_{BC}B_{AB} - B_{CA}B_{AB}}{B_{BC}^2 + B_{CA}^2 + B_{AB}^2 + 2B_{BC}B_{CA} + 2B_{BC}B_{AB} + 2B_{CA}B_{AB} + 9G^2}. \qquad (20)$$

If resistances in all phases are equal, then $\Delta P_1^* = 0$.

When the system of symmetric electromotive forces is connected only to an inductive load, then a formula for ΔP_2^* is obtained, which is similar to formula (16), the only difference being that it uses inductive phase conductance instead of active ones:

$$\Delta P_2^* = \frac{B_{BC}^2 + B_{CA}^2 + B_{AB}^2 - B_{BC}B_{CA} - B_{BC}B_{AB} - B_{CA}B_{AB}}{(B_{BC} + B_{CA} + B_{AB})^2};$$

$$\Delta P_2^* = \frac{\dfrac{1}{X_{BC}^2} + \dfrac{1}{X_{CA}^2} + \dfrac{1}{X_{AB}^2} - \dfrac{1}{X_{BC}X_{CA}} - \dfrac{1}{X_{BC}X_{AB}} - \dfrac{1}{X_{CA}X_{AB}}}{\dfrac{1}{X_{BC}^2} + \dfrac{1}{X_{CA}^2} + \dfrac{1}{X_{AB}^2} + \dfrac{2}{X_{BC}X_{CA}} + \dfrac{2}{X_{BC}X_{AB}} + \dfrac{2}{X_{CA}X_{AB}}}. \quad (21)$$

Assume $X_{BC} = 1, x_{ca} = \frac{X_{BC}}{X_{CA}}, x_{ab} = \frac{X_{BC}}{X_{AB}}$. In this case, formula (21) is identical to formula (17), i.e.

$$\Delta P_2^* = \frac{1 + \dfrac{1}{x_{ca}^2} + \dfrac{1}{x_{ab}^2} - \dfrac{1}{x_{ca}} - \dfrac{1}{x_{ab}} - \dfrac{1}{x_{ca}x_{ab}}}{\left(1 + \dfrac{1}{x_{ca}} + \dfrac{1}{x_{ab}}\right)^2}. \quad (22)$$

When the reactive load in only one phase, for instance, BC, differs from the reactive loads of two other phases, then, according to formulas (20) and (21),

$$\Delta P_1^* = \frac{\left(\dfrac{1}{X_{BC}} - \dfrac{1}{X_{AB}}\right)^2}{\left(\dfrac{1}{X_{BC}} + \dfrac{2}{X_{AB}}\right)^2 + \dfrac{9}{R^2}} = \frac{(B_{BC} - B_{AB})^2}{(B_{BC} + 2B_{AB})^2 + 9G^2}; \ \Delta P_2^* = \frac{\left(\dfrac{1}{X_{BC}} - \dfrac{1}{X_{AB}}\right)^2}{\left(\dfrac{1}{X_{BC}} + \dfrac{2}{X_{AB}}\right)^2},$$

and according to formula (22),

$$\Delta P_2^* = \frac{\left(1 - \dfrac{1}{x_{ab}}\right)^2}{\left(1 + \dfrac{2}{x_{ab}}\right)^2}. \quad (23)$$

If $X_{BC} = n \cdot X_{AB}$, then equation (19) is obtained. The diagram of function (12) is similar to that of function (17), the only difference being that the X-axis shows inductive resistances.

If the load is present only in one phase, for instance, *BC*, i.e. $R_{BC} + jX_{BC} \neq 0$, $\underline{Z}_{AB} = 0, \underline{Z}_{CA} = 0$,, then (see formula (14))

$$\Delta P^* = \frac{Y_{BC}^2}{Y_{BC}^2} = 1.$$

This special case confirms the "validity" of the general formula for additional losses from negative phase-sequence currents.

The full power of the asymmetric load under consideration equals the sum of phase powers:

$$\dot{S} = \dot{S}_A + \dot{S}_B + \dot{S}_C = \left|\dot{I}_A\right|^2 \cdot \underline{Z}_A + \left|\dot{I}_B\right|^2 \cdot \underline{Z}_B + \left|\dot{I}_C\right|^2 \cdot \underline{Z}_C. \quad (24)$$

The following designations are introduced:

$l = Z_{AB}^2 + Z_{CA}^2 + R_{AB}R_{CA} + X_{AB}X_{CA} + \sqrt{3}(R_{CA}X_{AB} - R_{AB}X_{CA})$;

$m = Z_{AB}^2 + Z_{BC}^2 + R_{AB}R_{BC} + X_{AB}X_{BC} + \sqrt{3}(R_{AB}X_{BC} - R_{BC}X_{AB})$;

$t = Z_{BC}^2 + Z_{CA}^2 + R_{BC}R_{CA} + X_{BC}X_{CA} + \sqrt{3}(R_{BC}X_{CA} - R_{CA}X_{BC})$;

$v_2 = Z_{AB}^2 + Z_{BC}^2 + Z_{CA}^2 + 2R_{AB}R_{BC} + 2R_{AB}R_{CA} + 2R_{BC}R_{CA} + 2X_{AB}X_{BC} + 2X_{AB}X_{CA} +$
$+2X_{BC}X_{CA}$;

$\alpha_2 = R_{AB}^2R_{CA} + R_{AB}R_{CA}^2 + R_{AB}X_{CA}^2 + R_{CA}X_{AB}^2 - X_{AB}X_{CA}(R_{AB} + R_{BC} + R_{CA}) +$
$+R_{AB}R_{BC}R_{CA} + +X_{AB}X_{CA}(R_{AB} + R_{CA}) + R_{AB}X_{BC}X_{CA} + R_{CA}X_{AB}X_{BC}$;

$\beta_2 = X_{AB}^2X_{CA} + X_{AB}X_{CA}^2 + R_{AB}^2X_{CA} + R_{CA}^2X_{AB} - R_{AB}R_{CA}(X_{AB} + X_{BC} + X_{CA}) +$
$+X_{AB}X_{BC}X_{CA} + R_{AB}R_{CA}(X_{AB} + X_{CA}) + R_{AB}R_{BC}X_{CA} + R_{BC}R_{CA}X_{AB}$;

$\gamma_2 = R_{AB}^2 R_{BC} + R_{AB} R_{BC}^2 + R_{AB} X_{BC}^2 + R_{BC} X_{AB}^2 - X_{AB} X_{BC}(R_{AB} + R_{BC} + R_{CA}) +$
$+R_{AB} R_{BC} R_{CA} + X_{AB} X_{BC}(R_{AB} + R_{BC}) + R_{AB} X_{BC} X_{CA} + R_{BC} X_{AB} X_{CA};$
$\delta_2 = X_{AB}^2 X_{BC} + X_{AB} X_{BC}^2 + R_{AB}^2 X_{BC} + R_{BC}^2 X_{AB} - R_{AB} R_{BC}(X_{AB} + X_{BC} + X_{CA}) +$
$+X_{AB} X_{BC} X_{CA} + R_{AB} R_{BC}(X_{AB} + X_{BC}) + R_{AB} R_{CA} X_{BC} + R_{BC} R_{CA} X_{AB};$
$\lambda_2 = R_{BC}^2 R_{CA} + R_{BC} R_{CA}^2 + R_{CA} X_{BC}^2 + R_{BC} X_{CA}^2 - X_{BC} X_{CA}(R_{AB} + R_{BC} + R_{CA}) +$
$+R_{AB} R_{BC} R_{CA} + X_{BC} X_{CA}(R_{BC} + R_{CA}) + R_{CA} X_{AB} X_{BC} + R_{BC} X_{AB} X_{CA};$
$\mu_2 = X_{BC}^2 X_{CA} + X_{BC} X_{CA}^2 + R_{CA}^2 X_{BC} + R_{BC}^2 X_{CA} - R_{BC} R_{CA}(X_{AB} + X_{BC} + X_{CA}) +$
$+X_{AB} X_{BC} X_{CA} + R_{BC} R_{CA}(X_{BC} + X_{CA}) + R_{AB} R_{BC} X_{CA} + R_{AB} R_{CA} X_{BC}.$

By using the complex values of phase currents from equations (10) – (12) and the above designations, the squares of their moduli can be determined:

$$\left|\dot{I}_A\right|^2 = \frac{3U^2 \cdot l}{Z_{AB}^2\, Z_{CA}^2}; \qquad (25)$$

$$\left|\dot{I}_B\right|^2 = \frac{3U^2 \cdot m}{Z_{AB}^2\, Z_{BC}^2}; \qquad (26)$$

$$\left|\dot{I}_C\right|^2 = \frac{3U^2 \cdot t}{Z_{CA}^2\, Z_{BC}^2}. \qquad (27)$$

The full resistances of phases from equations (1) – (3), with regard to the introduced designations, are presented in the form of a real and imaginary part:

$$\underline{Z}_A = Re\underline{Z}_A + jIm\underline{Z}_A = \frac{\alpha + j\beta}{\nu}; \qquad (28)$$

$$\underline{Z}_B = Re\underline{Z}_B + jIm\underline{Z}_B = \frac{\gamma + j\delta}{\nu}; \qquad (29)$$

$$\underline{Z}_C = Re\underline{Z}_C + jIm\underline{Z}_C = \frac{\lambda + j\mu}{\nu}. \qquad (30)$$

According to equation (24), with regard to equations (25) – (30), the full power of the asymmetric load is

$$\dot{S} = \frac{3U^2}{Z_{AB}^2\, Z_{BC}^2 Z_{CA}^2 \cdot \nu_2}\left(Z_{BC}^2 \cdot l(\alpha_2 + j\beta_2) + Z_{CA}^2 \cdot m(\gamma_2 + j\delta_2) + \right.$$
$$\left. + Z_{AB}^2 \cdot t(\lambda_2 + j\mu_2)\right), \qquad (31)$$

while its orthogonal components – active and reactive powers, respectively, are

$$P = \frac{3U^2}{Z_{AB}^2\, Z_{BC}^2 Z_{CA}^2 \cdot \nu_2} \cdot (Z_{BC}^2 \cdot l \cdot \alpha_2 + Z_{CA}^2 \cdot m \cdot \gamma_2 + Z_{AB}^2 \cdot t \cdot \lambda_2); \qquad (32)$$

$$Q = \frac{3U^2}{Z_{AB}^2\, Z_{BC}^2 Z_{CA}^2 \cdot \nu_2} \cdot (Z_{BC}^2 \cdot l \cdot \beta_2 + Z_{CA}^2 \cdot m \cdot \delta_2 + Z_{AB}^2 \cdot t \cdot \mu_2). \qquad (33)$$

Equations (32) and (33) are used to obtain the expression of the reactive power factor for the special case of asymmetric active-inductive three-phase load:

$$\tan\varphi = \frac{Z_{BC}^2 \cdot l \cdot \beta + Z_{CA}^2 \cdot m \cdot \delta + Z_{AB}^2 \cdot t \cdot \mu}{Z_{BC}^2 \cdot l \cdot \alpha + Z_{CA}^2 \cdot m \cdot \gamma + Z_{AB}^2 \cdot t \cdot \lambda}. \qquad (34)$$

Functions (14), (31) - (34) in expanded form, expressed via their six arguments, are too bulky. For a specific network, it is possible to use them with numerical methods. Finding the global extreme points of these functions is a complicated task, which is why investigations were carried out for special cases encountered during operation.

The pulsed power of a three-phase asymmetric system equals the sum of the pulsed powers of its phases:

$$\dot{N} = \dot{U}_{AB} \cdot \dot{I}_{AB} + \dot{U}_{BC} \cdot \dot{I}_{BC} + \dot{U}_{CA} \cdot \dot{I}_{CA}. \quad (35)$$

The values of phase-to-phase voltage and amperage from equations (4) - (9) were inserted into formula (35). After certain transformations, one obtains the following:

$$\dot{N} = 3U^2 \cdot \frac{\underline{Z}_{AB}\underline{Z}_{CA} + \boldsymbol{a} \cdot \underline{Z}_{AB}\underline{Z}_{BC} + \boldsymbol{a}^2 \cdot \underline{Z}_{BC}\underline{Z}_{CA}}{\underline{Z}_{AB}\underline{Z}_{BC}\underline{Z}_{CA}}. \quad (36)$$

According to equation (36), $N = 0$ if

$$\underline{Z}_{AB}\underline{Z}_{CA} + \boldsymbol{a} \cdot \underline{Z}_{AB}\underline{Z}_{BC} + \boldsymbol{a}^2 \cdot \underline{Z}_{BC}\underline{Z}_{CA} = 0,$$

which is equal to $\underline{Z}_{AB} = \underline{Z}_{BC} = \underline{Z}_{CA}$.

A set of experiments was conducted on the “distribution transformer – asymmetric load” model to confirm the discovered functional dependence for the estimation of additional losses of active power in a double-wound power transformer caused by asymmetric active-inductive load with a delta connection.

The dependences of active power losses and errors of measurement of active power loss on the load factor were built based on the measurements and calculations. Below is the analysis of experimental data for a delta-connection load scheme (Figure 1). Obtained experimental dependences approximate well with polynomials of degree 5 (Figures 3 - 8).

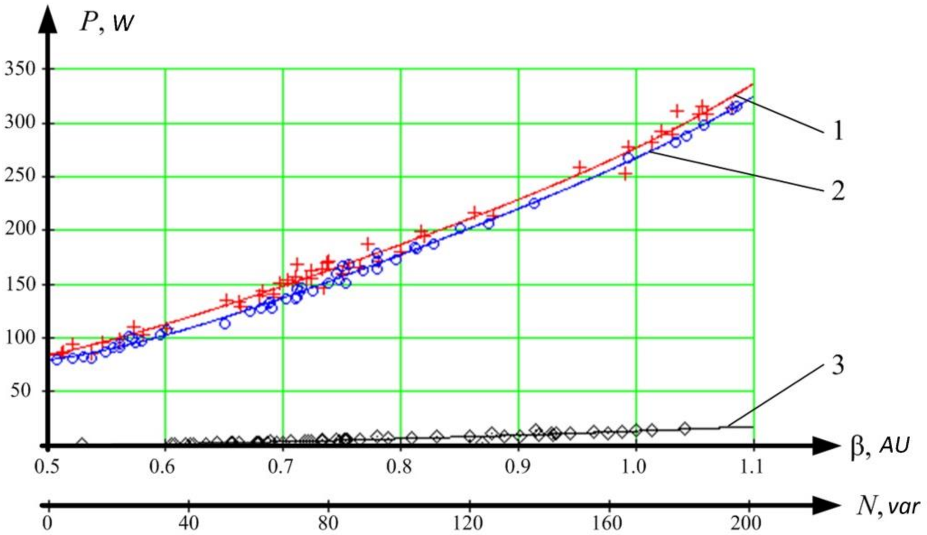

Figure 3. Dependence of active power loss in the transformer on the load factor: 1 - with asymmetric active-inductive load; 2 - with symmetric active-inductive load; 3 - dependence of active power loss in the transformer with asymmetric active-inductive load on the pulsed power of three phases.

In the 0.5 – 1.1 interval, the difference of losses with asymmetric and symmetric modes remains relatively constant – 4.9% on average.

Calculations according to the standard formula ($\Delta P = \Delta P_x + K_c \cdot \Delta P_k$) give underestimated losses rather than actual ones; the offered functional dependences for the estimation of active power losses from asymmetric active-inductive load have the smallest error (Figure 4).

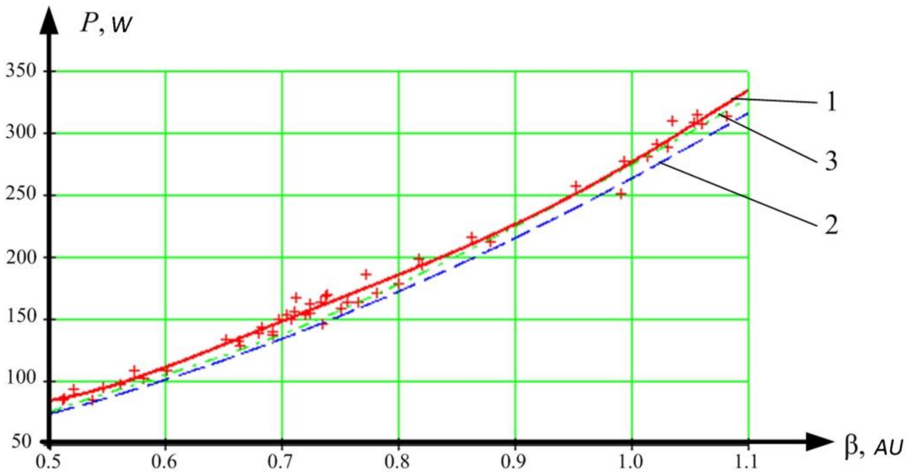

Figure 4. Dependence of active power loss in the transformer with asymmetric active-inductive load on the load factor: 1 – experimental; 2 – calculated according to the standard formula; 3 – with regard to the asymmetric active-inductive load (the discovered functional dependence was used).

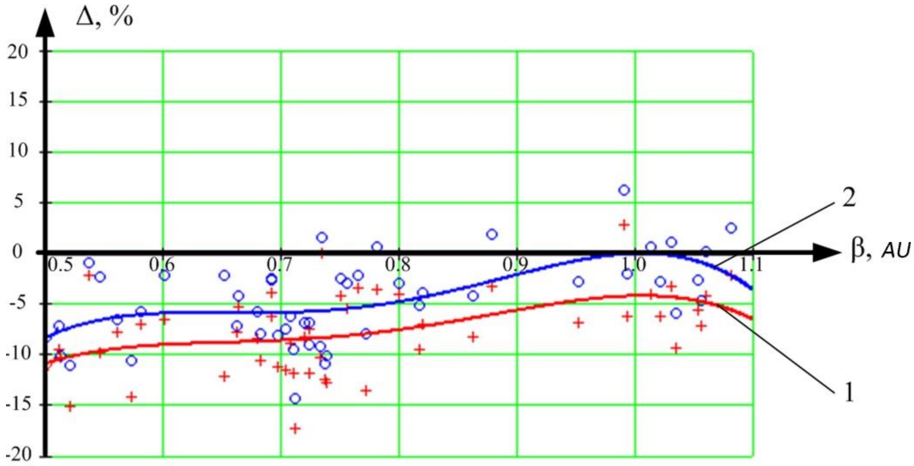

Figure 5. Dependence of the errors of measurement of active power loss on the load factor with asymmetric active-inductive load: 1 – according to the standard formula; 2 – according to the standard formula, with regard to the discovered functional dependence.

In the entire measurement interval from 0.5 to 1.1, the measurement error according to the standard formula, with regard to the discovered functional dependence, was negative; at that, the maximum error of -8.23% was with 0.5 load factor (Figure 5). The mean error with a load factor of 0.5-1.1 with the standard formula is 6.24%; with the offered functional dependences, it is -3.42%.

The diagram of dependences of active power loss on pulsed power (Figures 3 and 6, Curve 3) on a certain scale matches the diagram of the dependence of active power loss on the load coefficient with asymmetric active load, i.e. it is a characteristic of the asymmetric mode.

When the load factor changes in the interval of 0.5 – 1.1 (Figure 6), the difference of losses in symmetric and asymmetric modes is 4.2% on average.

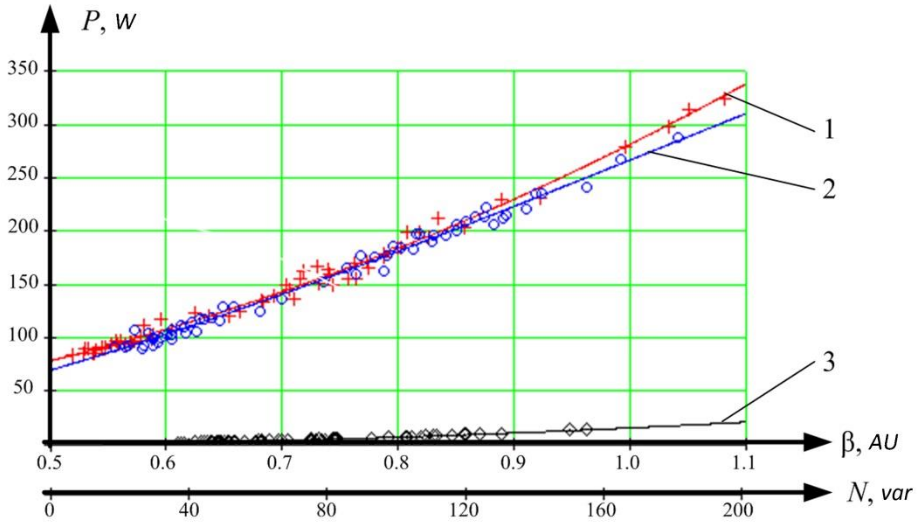

Figure 6. Dependence of active power loss in the transformer on the load factor: 1 – with asymmetric active load; 2 – with symmetric active load; 3 – dependence of active power loss in the transformer with asymmetric active load on the pulsed power of three phases.

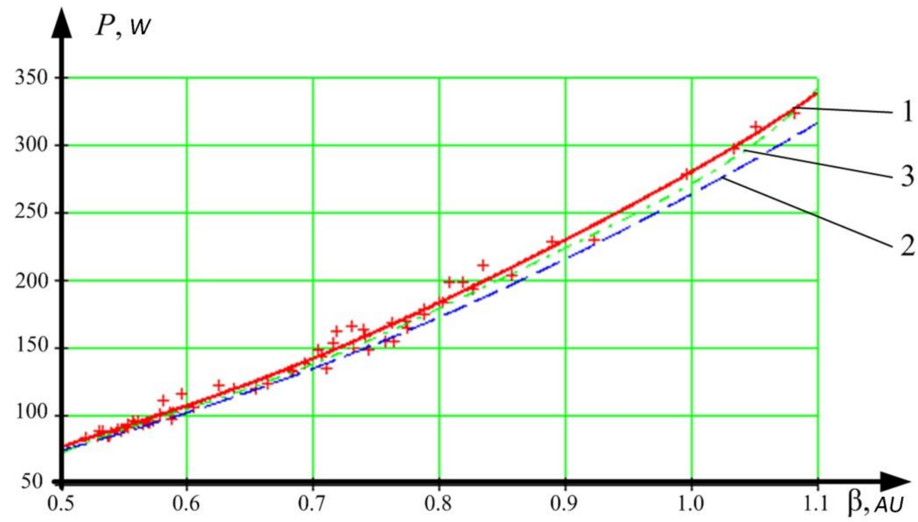

Figure 7. Dependence of active power loss in the transformer with asymmetric active load on the load factor: 1 – experimental; 2 – calculated according to the standard formula; 3 – with regard to the asymmetric active load (the discovered functional dependence was used).

Calculations according to the standard formula give underestimated losses rather than actual ones; the offered functional dependences for the estimation of active power losses from asymmetric active load have the smallest error (Figure 7).

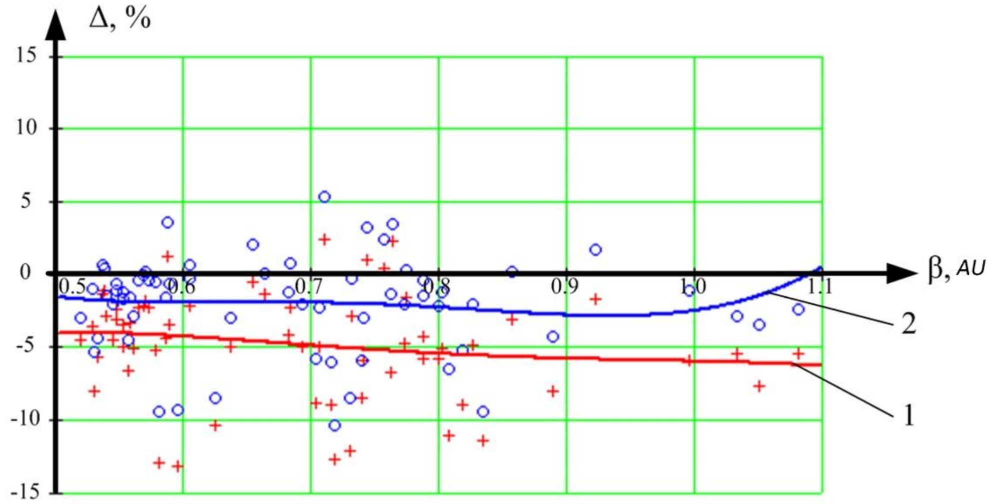

Figure 8. Dependence of the errors of measurement of active power loss on the load factor with asymmetric active load: 1 – according to the standard formula; 2 – according to the standard formula, with regard to the discovered functional dependence.

In the entire measurement interval from 0.5 to 1.1, the measurement error according to the standard formula, with regard to the discovered functional dependence, was negative; at that, the maximum error of -2.9% was with 0.93 load factor (Figure 8). The mean error with a load factor of 0.5-1.1 with the standard formula is -4.59%; with the offered functional dependences, it is -1.8%.

Thus, the loss of active power, calculated according to the standard formula, should be corrected with regard to the discovered functional dependence.

Discussions

The accuracy of obtained results was confirmed through experiments and by using modern highly accurate measuring devices.

Research (Zaugolnikov, Balabin & Savinkov, 2006) shows diagrams of relative losses, obtained experimentally after repairs and based on reference data for TM-250/10 and TM-400/10 transformers, as well as systematic data on the increase of idling losses for various types and lifetimes of transformers. During operation, idling losses increase significantly when compared with rated characteristics. By taking this factor into consideration, it was possible to specify their values by 15% on average for a power network facility. In some cases, they were two times higher. The sampling was representative: performed for 2425 distribution transformers from 11 districts of power networks. Almost 80% of distribution transformers continue to operate after exceeding the standard service life period of 25 years. The highest relative increase in losses is found in low-power transformers. This can be explained by the fact that the technological errors in the manufacturing of winding coils for three-leg low-power transformers creates 2-3% asymmetry of phase voltage.

V. Tropin, A. Savenko, and O. Maleyev (2008) analyzed an adequate model of a 0.4 kV power network with set variable load parameters and obtained a practically convenient equation for the amount of power losses of one phase in symmetric mode, depending on the load power factor, degree of compensation for reactive power load, and amount of power losses in the network.

In the equivalent scheme under consideration, the parameters of one phase are substituted by parameters of one phase of a transformer combined with the resistance caused by the load. In order to determine the effect on the load with a bridging capacitor bank and, ultimately, the losses in the transformer, the absolute value of active power loss that was prevented by the compensation of reactive power is calculated according to the following equation:

$$\Delta P_{\mathrm{C}} = (\tan\varphi)^2 \cdot \delta U \cdot (1 + \delta U) \cdot (2\beta - \beta^2) \cdot P_{\mathrm{L}},$$

where P_{L}, $\tan\varphi$ are the active component and load power factor;

δU is the loss of power in the network phase;

$\beta = \frac{Q_{\mathrm{C}}}{Q_{\mathrm{L}}}$ is the factor of reactive load power compensation, which equals the ratio of the capacitor bank power (Q_{C}) that is activated in parallel with the load to the reactive power of the load (Q_{L}). At $\beta \geq 0.6$, the sensitivity to loss reduction is insignificant (Tropin, Savenko & Maleyev, 2008).

The research of A. Arutyunian (2012) offers a method for determining additional losses in 6 (10)/0.4 kV transformers with regard to changes in high voltage caused by uneven distribution of loads across phases. The calculations that were performed for 400 and 630 kV according to this method confirmed the increase in losses with asymmetric load, when compared with the symmetric load.

When using the method for estimating additional losses of active power (Shidlovsky & Kuznetsov, 1985), it is necessary to measure phase power at the high voltage winding. However, this is technically impossible when it comes to main lines, because measuring transformers are installed only in finished lines.

It is also worth noting that in research (Arutyunian, 2012), the absolute value of additional losses in the distribution transformer caused by asymmetric loads is calculated according to the following equation:

$$\Delta P = (P_{\mathrm{XX}} + P_{\mathrm{SC}}/U_{\mathrm{SC}}^2)(K_{2u}^2 + K_{0u}^2),$$

where $P_{\mathrm{XX}}, P_{\mathrm{SC}}, U_{\mathrm{SC}}$ are the rated characteristics of the transformer;

K_{2u}, K_{0u} are the values of 6(10) kV power asymmetry factor moduli with negative and zero phase-sequence.

For distribution transformers with a delta-connection scheme, it is no longer necessary to calculate the voltage and asymmetry factor for the zero component of the winding; the equation for the absolute value of additional losses in distribution transformers caused by asymmetric loads is simpler: $\Delta P = (P_{\mathrm{XX}} + P_{\mathrm{SC}}/U_{\mathrm{SC}}^2)\, K_{2u}^2$.

The idea set forth in the above research is relevant for practical engineering; however, the algorithm of calculations is bulky. Furthermore, the authors do not give information about methodological errors.

Using databases based on measurements with portable devices and repeated measurements on their basis are bound to cause errors in measurements, which have to be assessed by programming specialists.

Thus, the conclusion is that further research of asymmetric modes of power transformers is required.

Conclusion

Obtained experimental data from measurements in the “distribution transformer – asymmetric load” model confirmed the need for correcting the standard formula for power loss in the transformer.

This research offers a functional dependence for the estimation of additional losses of active power in a double-wound power transformer caused by asymmetric active-inductive load with a delta connection.

The error of the measurement of active power loss in the transformer when using the offered functional dependence for a delta-connection scheme ranges from 0 to -5%.

It is expedient to take into consideration the idling and short circuit losses in transformers with regard to additional losses caused by asymmetric loads both when calculating process losses and when substantiating the economic effect of transformation replacement.

The practical value of the offered functional dependence for the estimation of additional losses is that it enables estimating the losses of active power in transformers due to asymmetry by measured voltage, amperage, and active power for each phase. This makes it universal: there is no need to build equivalent circuits and carry out calculations according to the symmetrical components method.

The developed functional dependence, which is based on the “distribution transformer – asymmetric active-inductive load” model, can be used by organizations that design, replace, and modernize transformer substations in city and industrial distribution networks of power systems to increase their energy efficiency.

Disclosure statement

No potential conflict of interest was reported by the authors.

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