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Reflections from planning and implementing a modelling task

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ABSTRACT

The aim of this study is to investigate both the planning and implementation process of a modelling task by a middle school mathematics teacher. The participant of the study, which was conducted as a single case study, is a middle school mathematics teacher who have knowledge and experience in modelling. The data collecting tools of this study were the lesson plans of the teacher, the observation notes taken by the researchers during the implementation of the modelling task, the video records of the implementation, students' solution papers and the voice records of the pre- and final-interview conducted with the teacher. Whole data were analyzed according to a framework for modelling implementations developed by the authors through descriptive analysis. The teacher planned his in-class implementation to consolidate the circle concept and revised an existing modelling task by developing appropriate materials for the task. He asked students to arrange their desks appropriate to group work, decided to enable students to evaluate both the implementation and their peers, and announced their students that they would present their group solution and evaluate each other at the end of the lesson. During the implementation, he first reminded prior knowledge about circle and then presented the materials. He shared instructions about group work in the context of sharing social norms and gave information about the peer assessment form, which the students filled at the end of the lesson. During the students were solving the task, he intervened about enabling self-assessment in the groups, considering realistic values and comparing mathematical results with the problem context. In addition, during the solution, he tried to establish socio-mathematical norms by making explanations. After the solution process ended, student groups presented their solution approaches and discussed different solutions by comparing each solutions. Peer assessment form and affective evaluation form regarding the implementation were filled by the students at the end of the implementation. This study contributed the literature regarding modelling implementations and presented a different view about teacher actions in planning and implementing a modelling task.

Keywords: mathematical modelling, middle school mathematics teacher, modelling implementation, modelling task

INTRODUCTION

Using mathematics in solving real life problems is considered as a necessary topic for mathematics learning and teaching by both the common core state standards for mathematics (Common Core State Standards Initiative, 2010) and the principles and standards for school mathematics (National Council of Teachers of Mathematics [NCTM], 2000). Kaiser et al. (2010) and Lesh et al. (2010) emphasized integration of mathematical modelling into the mathematics lessons to enable students to solve real life problems by using mathematical knowledge. In parallel with these considerations, most countries standards and curricula include learning and teaching of mathematical modelling (Borromeo Ferri, 2018). Teachers have important responsibilities in the learning and teaching of mathematical modelling.

Considering existing studies regarding learning and teaching of mathematical modelling, different researchers focus on teachers' skills and knowledge during modelling implementations. Borromeo Ferri and Blum (2010) emphasized that teachers should have appropriate skills and competencies to teach modelling effectively. Burkhardt (2006) explained teachers' skills in modelling as handling in-class discussion in a supportive way for students to feel responsible for their decisions and others' reasoning, giving students necessary time and confidence to explore, offering help only students felt exhausted, providing strategic guidance and support, and supporting students with supplementary questions according to each student's progress. In addition to these skills, Blum (2011) focused on the fact that mathematics teachers should have knowledge of modelling tasks, using these tasks with different purposes, different interventions modes and using these interventions, and should have abilities to diagnose students' difficulties while working in modelling processes, and acquire appropriate beliefs. Kuntze et al. (2013) stated that in-class professional knowledge involved knowledge about mathematical modelling, modelling process and implications for teaching, specific classroom settings, technology use, supporting students, and views related to modelling tasks.

Although the requirements of teachers to carry out modelling implementations are clearly stated in the literature, it is inevitable for teachers to experience some difficulties when it comes to practice. Borromeo Ferri and Blum (2010) explained this situation clearly by saying that mathematical modelling is difficult for teachers, and this difficulty stemmed from both the nature of modelling and teachers' deficiencies. Because structure of modelling problems is open, complex and realistic (Maaβ, 2006) and the teaching process is unpredictable and demanding (Borromeo Ferri and Blum, 2010), teachers can have lots of problems. Lack of knowledge about having various competencies, mathematical and extra-mathematical issues, modelling tasks, and beliefs about modelling (Blum, 2015) impede the efficient modelling implementations. Borromeo Ferri and Blum (2010) focused on the obstacles during modelling implementations as organizational, student-related, teacher-related, material-related, systemic, and research-related according to their literature review. de Oliveira and Barbosa (2013) explained these difficulties as deciding what to do, students' involvement and students' domination of mathematical content. Chan et al. (2015) discussed that teachers had challenge in designing an appropriate modelling task, building confidence in students' handling modelling problems, and balancing the demands of the syllabus with the modelling implementations in view of limited curriculum time.

To eliminate difficulties of teachers, Henn (2007) stated that "an adequate modelling pedagogy is necessary" (p. 322) and focused on that the necessity of adequate problems, adequate teaching methods and instructional modes, adequate tools, and adequate modes of assessment should be well defined. Therefore, some researchers developed models about modelling implementations and explained teachers' activities according to these models. For example, Maaß and Gurlitt (2009) presented that teachers should know what modelling is and how it is used, choose from existing tasks or develop new tasks according to their needs, identify appropriate teaching methods, know how to use technology, predetermine teacher and student actions, and be aware of using suitable formative and summative assessment, and feedbacks. Borromeo Ferri (2018) explained teachers' competencies in teaching mathematical modelling in four dimensions based on a previous study (Borromeo Ferri & Blum, 2010), as follows:

(1) **Theoretical dimension:** Knowledge about modelling cycles, goals/perspectives for modelling and types of modelling tasks.

(2) **Task dimension:** Abilities to solve modelling tasks in multiple ways, to cognitively analyze the tasks and develop different tasks.

(3) **Teaching competency dimension:** Abilities to plan and perform lessons with modelling tasks and knowledge of appropriate interventions, support and feedback during pupils' modelling processes.

(4) **Diagnostic dimension:** Abilities to identify phases in pupils' modelling processes, diagnose their difficulties during such processes and mark tests.

Ang (2015) stated that teachers should determine in which modelling level their students were, which skills and competencies would be developed in students, which subject of mathematics related to modelling task was necessary, how the modelling task could be solved, and which students' difficulties might be arisen, and how to check whether the planned implementation was successful or not. Greefrath et al. (2021) suggested a modelling-specific pedagogical content knowledge framework based on Borromeo Ferri and Blum's (2010) research with four dimensions as knowledge about intervention, modelling processes, modelling tasks, and modelling aims and perspectives. In another study, Geiger et al. (2021) concentrated on teachers' instructional competency in modelling in the dimensions of pre-engagement, modelling process review, initial problem presentation, and body of lesson, conclusions and report.

While models for modelling implementations continue to be developed, studies on examining teachers who conduct mathematical modelling implementations in their classes have also increased in recent years. The majority of these studies, which focus on teachers or student teachers in classroom practices, are on teachers' interventions (e.g., Cakmak Gurel & Bekdemir, 2022; Didis Kabar & Inan Tutkun, 2021; Leiß et al., 2010; Passarella, 2021; Stender & Kaiser, 2015; Tropper et al., 2015). Although the interventions of the teachers in modelling give us an idea about the content of the modelling implementations, there is need for detailed examinations regarding different issues such as conducting in-class discussions, constructing norms and identifying assessment methods, etc. In this context, we aimed to investigate both the planning and implementation process of a modelling task by a middle school mathematics teacher in this study to take a deep look into a real-class modelling implementation. For this purpose, we try to contribute the modelling studies by discussing the considerations paid attention by the teacher in the planning stage; and the reflection of planning process into the real-class modelling implementation by examining the warm-up activities, implementation process, socio-mathematical norms, teacher interventions, and preferred assessment tools. In this context, the research questions are, as follows:

1. How does the teacher plan the modelling implementation process?
2. How does the teacher conduct the modelling implementation process?

METHOD

Since it was aimed to examine a teacher's planning and implementation process thoroughly, a single case study design (Yin, 2018) was utilized.

Participants

The participant of the study is a ten-year experienced middle school mathematics teacher whose fictitious name is Mert. He realized the modelling implementation with 6th grade students. He was participated in two modelling workshops organized by the

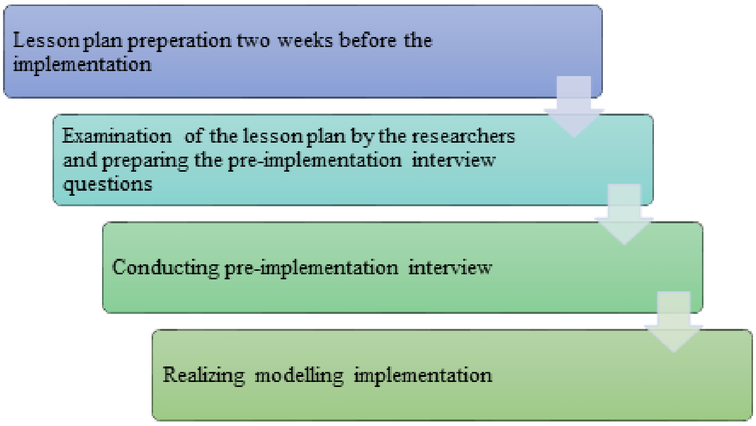

Figure 1. Data collection process of the study (Source: Authors' own illustration)

researchers. These workshops lasted five days and the participant teachers were chosen by the provincial directorate of national education. In the first workshop, we aimed for teachers to comprehend the nature of modelling, to work on the solution process of various modelling tasks and to design model eliciting activities. Three years after the first workshop, the second one was organized to enhance the modelling knowledge and competencies of the teachers, who had already participated in the first workshop. Our aim was to ensure that they had knowledge and experience about the implementations of modelling in their lessons in the second workshop. Some of the teachers conducted modelling implementations after the workshops in their classes. In addition to these workshops, some of these teachers also participated in the workshops organized by the experts in the field of modelling such as Rita Borromeo Ferri, Andreas Meister and Akio Matsuzaki between the two workshops. Mathematics teachers need to have modelling competencies before they can realize modelling implementations (Borromeo Ferri & Blum, 2010). It is thought that Mert gained modelling competencies thanks to these workshops and modelling experience during in-class modelling implementations. Therefore, the criterion for determining him as the participant of the study was on existence of modelling knowledge and being volunteer. After being volunteered, a written consent was obtained from him that he participated in the study voluntarily. Although Mert had modelling knowledge and experience, he chose a class in which students had not been involved in any modelling implementation before.

Data Collecting Tools

The data collecting tools of this study were the lesson plan of the teacher, the observation notes taken by the researchers during the implementation of the modelling task, the video recording of the implementation, students' solution papers and the voice recording of the pre-interview and final-interview conducted with the teacher. Data collection process of the study is explained in the **Figure 1**.

The researchers asked for the lesson plan nearly two weeks before the implementation. Mert explained duration of the implementation, students' and teachers' possible actions, the reason of choosing the task, and the materials to be used during the implementation, and to include a possible solution of the task in his plan. Then, the researchers examined the lesson plan and prepared interview questions based on these examinations. The questions of the semi-structured interview were, as follows: "Which aspects did you consider while planning the modelling implementation?", "What is the purpose of choosing this task?", "Could you explain entirely how you will conduct your lesson plan?", "What do you expect students to do in this process?", and "How do you plan to evaluate students' works?" This pre-implementation interview was conducted before the lesson and recorded with a voice recorder.

Mert decided to implement the bed problem with a supporting material (**Figure 2**) at own request. With this material, he aimed his students to visualize and reduce the actual bed dimensions by 1:10 using proportional reasoning.

The whole implementation process took two lesson hours and was recorded with a video camera. After the implementation, students' written works were collected. The researchers conducted an unstructured interview with Mert to determine his views at the end of the implementation. The questions were, as follows: "Could you make an evaluation of the implementation in general?", "When you compare your ideas before the implementation with your current ideas, is there a change? Please, explain it." and "What do you think about your students' modelling skills?"

Researcher Role

When Mert conducted the modelling implementation, both of the researchers participated in the implementation. They took observation notes individually to comprehend student-teacher interactions, questions, mimics, etc. especially outside the camera angle. In addition, one of the researchers dealt with the issues about the camera, for example capturing the teacher and the students' actions and the students' bed sketches and models. They initially sat in an empty desk at the back of the classroom and walked around the students with the camera to capture current instances during the solution process. They did not directly interact with the students. Before the implementation, Mert informed his students about the implementation and the researchers participated in an hour lesson with Mert so that the students would not be unfamiliar.

While Deniz's parents were examining the catalogue of a furniture store, they liked the circular bed (210 cm in diameter) model and decided to buy it. However, they were not sure whether they would be comfortable in this bed. How much space will there be between Deniz's parents when they lie on this bed without any of their arms and legs sticking out?

Figure 2. Bed problem (Borromeo Ferri, 2014) and supporting material

Table 1. Framework for modelling implementations (Tekin Dede & Bukova Güzel, 2016)

Planning (P)	Implementation (I)	Assessment (A)
P1. Identifying the goals P1.1. Identifying the grade, subject, objectives/goals, & concept P1.2. Forming/consolidation/evaluating the concept P.1.3. Identifying the implementation type (in-class/out-class, performance task, & project work)	I1. Conducting warm up activities I1.1. Discussing on the newspaper article & answering readiness questions/the collected data/the context of the modelling task I1.2. Having students watch videos I1.3. Reminding prior knowledge I1.4. Presenting materials	A1. Presenting solution (oral/written presentation & poster preparation)
P2. Deciding the modelling task P2.1. Choosing/revising the existing task P2.2. Developing a new task P2.3. Solving the task	I2. Sharing norms I2.1. Social norms I2.2. Socio-mathematical norms	A2. Discussing on solutions & doing necessary revisions A2.1. Self-assessment A2.2. Peer-assessment A2.3. Teacher assessment
P3. Making preliminaries P3.1. Identifying necessary materials P3.2. Arrangement of the implementation environment P3.3. Specifying student tasks P3.4. Deciding the students' ways of work P3.5. Identifying the assessment criteria P3.6. Deciding the ways of solution presentations	I3. Sharing assessment criteria (self-assessment/peer assessment/assessment with rubric, etc.)	A3. Assessing cognitive/affective/social competencies
	I4. Presenting modelling task	A4. Sharing assessment results & arriving at decisions
	I5. Tracking students' works I5.1. Taking observation notes I5.2. Teacher interventions	

Data Analysis

The data analyzing process was conducted in two stages in line with the research questions as analyzing of lesson planning and analyzing of implementation. In the first stage regarding lesson planning, the lesson plan and the semi-structured interview were transcribed and analyzed by using a framework for modelling implementations (**Table 1**) through descriptive analysis (Creswell, 2008).

The codes that Mert preferred to use in planning his lesson were identified by taking into consideration the planning stage (P) of the framework (**Table 1**).

The second stage of the data analyzing consisted of the implementation stage. The video recording of the implementation was transcribed by using the implementation (I) and assessment (A) stages of the framework (**Table 1**) through descriptive analysis (Creswell, 2008). In this context, the codes that Mert preferred to use during implementation were identified, and researchers' observation notes and students' solution papers were utilized to support the implementation process.

The transcript of the final-interview was analysed by conducting the content analysis (Strauss & Corbin, 1990) at the last stage of the data analysing.

Table 2. Identified codes regarding Mert's planning process

Planning (P)
P1. Identifying the goals
P1.1. Identifying the grade, subject, objectives/goals, & concept
P1.2. Consolidation the concept
P.1.3. Identifying the implementation type (in-class)
P2. Deciding the modelling task
P2.1. Revising the existing task
P3. Making preliminaries
P3.1. Identifying necessary materials
P3.2. Arrangement of the implementation environment
P3.4. Deciding the students' ways of work
P3.5. Identifying the assessment criteria
P3.6. Deciding the ways of solution presentations

Both researchers coded the whole data together and simultaneously. If they conflicted about any coding, they discussed and reached a shared decision. During data analysing, the lesson plan, interviews' transcripts, implementation transcript and the researchers' observation notes were constantly compared with each other, thus providing a holistic perspective for the planning and implementation process.

To increase the validity of the study, we applied the data triangulation by using different data collection tools. In addition, we used the detailed description method both in the methods and results section. To increase the reliability, both of the researchers conducted the data analysing process by comparing the codes and inferences. When they conflicted about the analysing of any instance, they discussed to reach an agreement about the instance and so completed the data analysing.

RESULTS

The results section is addressed under in three sub-titles as planning and implementation processes, and the general views about the implementation in the context of the research questions.

Results About Mert's Planning Process

The codes identified in Mert's lesson plan, and the transcript of the interview are presented in bold in **Table 2**.

Mert planned an in-class implementation of the Bed problem, which he revised by adding appropriate material, to consolidate the circle topic he had just taught (P1.1, P1.2, P1.3, and P2.1). Although he did not include the possible solution of the modelling task in his lesson plan, he determined in advance what information could be used in the solution. He decided to concretize the problem by estimating that the students did not know the concept of a chord in a circle, and that they would not be able to calculate with the Pythagorean equation by using half of the chord length and the radius. For this, he prepared a material based on students' previous knowledge of the center, radius and diameter of the circle. This material is circular cardboards with a diameter of 21 cm by reducing the diameter of the bed given in the problem by 1:10 to enable students to use proportional reasoning (P3.1). He explained the reason of choosing this material in his pre-implementation interview, as follows:

> Mert: The fact that the bed is 210 cm, the height of a person is something, which is easy to reduce by dividing by 10, such as 160 cm or 180 cm, led me to prepare such a material. In addition, my students know the easy way of multiplying by 10 and can make calculations with decimal numbers.

Mert decided to have his students work in groups during the modelling implementation. For this, he emphasized that students with different mathematics achievements should be in different groups and that he would prepare the class environment suitable for group work (P3.2 and P3.4).

> Mert: In some lessons, for example, if I am going to do activities or play mathematics games, I prefer group work. I try to form mixed groups, but since students always want to be with their close friends, I let them to form their groups in some lessons. But I did not get much efficiency because there is a competitive environment in the classroom, good students are grouped with good students, those with low grades are generally excluded. Therefore, I will make sure that there are students with different mathematics grades in each group. So, I am going to form mixed groups. Before the lesson, I will ask them to prepare the class environment suitable for group work by moving the desks.

Mert stated that he would ask students to evaluate both each other and the modelling implementation during the planning phase (P3.5). For this purpose, he prepared forms containing evaluation criteria for each student. In the first of these forms, he aimed to evaluate the affective aspects such as whether the students liked the modelling implementation, whether they wanted the continuation of such implementations, and whether they liked group work or not (**Figure 3**).

Another assessment form that Mert planned to use was for peer assessment. Mert especially stated that he considered peer assessment important to enable students to participate in group work effectively. In the form he used for this, he wanted to determine whether each member of the group participated in the group work effectively, whether they distributed the tasks, and whether the tasks were fulfilled (**Figure 4**).

	Yes	No
Do you like the implementation? Why?..		
Do you want to be taught with this kind of implementations? Why?..		
Do you like to solve the problem by group work? Why?..		
Is the lesson attractive? Why?..		

Figure 3. Affective evaluation form (Source: Authors' own illustration)

	Yes	Partly	No
… participated in the group work effectively. (Please fill in the gap with your group friend's name)			
… participated in the group work effectively. (Please fill in the gap with your group friend's name)			
… participated in the group work effectively. (Please fill in the gap with your group friend's name)			
… participated in the group work effectively. (Please fill in the gap with your group friend's name)			
Did you distribute the duties in the solution process?			
… did her/his duty. (Please fill in the gap with your group friend's name)			
… did her/his duty. (Please fill in the gap with your group friend's name)			
… did her/his duty. (Please fill in the gap with your group friend's name)			
… did her/his duty. (Please fill in the gap with your group friend's name)			

Figure 4. Peer evaluation form (Source: Authors' own illustration)

Table 3. Identified codes regarding Mert's implementation & assessment process

Implementation (I)	**Assessment (A)**
I1. Conducting warm up activities I1.3. Reminding prior knowledge I1.4. Presenting materials	A1. Presenting solution (oral presentation) A2. Discussing on solutions and doing necessary revisions A2.2. Peer assessment A2.3. Teacher assessment A3. Assessing cognitive/affective/social competencies
I2. Sharing norms I2.1. Social norms I2.2. Socio-mathematical norms	
I3. Sharing assessment criteria (peer assessment)	
I4. Presenting modelling task	
I5. Tracking students' works I5.2. Teacher interventions	

Mert also stated in the interview that students should not only solve the modelling problem, but also share the solutions. Therefore, he explained that students should present their solutions in the classroom (P3.6) and the reason for this is, as follows:

> Mert: They will all solve it in some way, but it is important for them to share these, see different solutions, and notice each other's deficiencies. Especially when we saw the solutions of different groups in the workshops, we were discussing the solutions we liked or needed to be developed. This was very beneficial for the group and the audience. That is why I wanted to give this opportunity to my students. Let them see that the solution in modelling is not just belong to them.

Results About Mert's Implementation Process

The codes identified in the transcript of Mert's implementation and assessment process are presented in bold in **Table 3**.

At the beginning of the implementation, Mert divided 18 students into four groups, two of which were four and two were five. After the groups were ready, he reminded the prior knowledge about the concepts of center, radius and diameter to be used by drawing on the board (I1.3).

> Mert: … A very simple reminder: This is circle, is not it? [He draws a circle on the board] However, we had a few components. We called it the center right in the middle. What is the name of the line segment we drew from the center to the border?
>
> Students: Radius.
>
> Mert: We were denoting the radius with the letter r. If I combined this on the other side, what would we call both?
>
> Students: Diameter.

After giving the necessary preliminary information, Mert reminded the instructions of the group work and motivated the students to work collaboratively in the group (I2.1.). These instructions were considered as explanations about the social norms that he constructed before in his lesson.

> Mert: Please follow my instructions well. You are working as a group; you are not piling the tasks on one person. At the end of the second lesson, I will distribute forms to you. In these forms, I will ask you to evaluate each other. That is why it is so important for everyone to participate in group work.

Mert then distributed the problem text to each group (I4) and asked them to first read the problem and then ask about the unclear parts, if any. Therefore, he wanted to make sure the students understood the problem. He also drew attention to the peer assessment to be made at the end of the implementation (I3).

> Mert: Read and try to understand the problem. Tell each other later what is required. One of you can read aloud if you want. At the end of the solution, you will do a peer review. You will evaluate each other appropriately.

He distributed to the groups the 21 cm diameter circular cardboards that he had prepared before, handcraft papers of different colors, scissors, rulers and glues (I1.4) and made explanations that would enable them to relate these materials with the given in the problem. Although he did not express it directly, he tried to make the students feel that the diameter of the bed in the problem was 10 times the diameter of the material he gave and directed the students to proportional reasoning. Therefore, he enabled students to have an opinion about transition from the real model to the mathematical model that they will construct.

> Mert: Now I am giving you this material. What is this?
>
> Students: Circle.
>
> Mert: No, it is a disc since it is filled. But you can think of it like a circle. How can you use it?
>
> Students: As a bed.
>
> Mert: In addition, the diameter of these circles is exactly 21 cm. The diameter of the bed in the problem is 210 cm. Do not forget 210 cm and 21 cm. How will you use them in problem solving? Now I'm going to give you colorful papers, you can cut and draw them as you wish.

After the students read the problem and examined the materials, when a student asked Mert what was required in the problem, Mert made statements that would encourage his students to work in groups (I5.2). With these statements, he motivated students to take responsibility of the modelling process themselves. This was a teacher intervention to enable students to self-assess their modelling approaches during the group work.

> Student: What does the problem ask for?
>
> Mert: Now take a good read of the problem. First of all, try to understand yourself. What has it given, what is being asked for, what can you do? First, try to understand yourself. After that, I will help you a bit if necessary. I will not say you will solve it like that directly. Okay? It will be your product; you will finalize it. So just reread the problem carefully. Discuss with your group members what it means. What does it want from us? What can you do? It's a group work after all. So, try to finish this job first with your group members. Okay? You can use the materials as you wish.

While the students were solving the problem, Mert walked around the groups and asked them if there were any issues they did not understand and made necessary explanations without directing them to the solution. Students in some groups stated that they could not understand what to do by saying that the height of the parents was not given in the problem. Mert said that since their solution would be peculiar to their solution approaches (I2.2), they should decide the height of the parents. He focused on the realistic assumptions that the students should make, and this was considered as he tried to establish a socio-mathematical norm. He stated that while making this decision, they should consider realistic values, think about their parents' height if they know, or make some estimations based on their own height if they do not know their parents' (I5.2). With these explanations, it was seen that Mert intervened his students about showing regard to realistic assumptions.

All groups expressed the height of the parent by making assumptions about the height of their own parents. Considering the relationship between the circular cardboard and what is given in the problem, they also took one-tenth of the mother's and father's height, based on the 1:10 ratio. They started to draw the mother and father on colorful paper according to the heights they found. In the meantime, they noticed that they could not make a stickman and that both mother and father should have a body width. Some students considered the width of the mother and father in real life to be 30 and 40 cm, while others considered the widths of the mother and father figures they randomly drew on their handcraft papers. In other words, they first drew the mother and father, whose height they had determined, and in the meantime, they tried to draw the father thicker than the mother, and then they measured the width of these drawings with a ruler. They found the real value by multiplying the length they found by 10. Therefore, the students used proportional reasoning to construct their mathematical models from real models. Considering their parents' widths as real as possible caused their mathematical models more appropriate than the other models in which the students did not consider the widths.

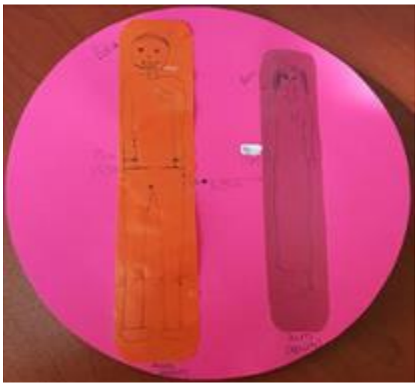

Figure 5. The design of one of the groups regarding the solution (Source: Authors' own illustration)

Mert often reminded his students that at the end of the second lesson, they would present their ideas for solutions, the models they constructed and the results they reached to their classmates. These explanations enabled students to reconsider their solution approaches.

> Mert: After deciding on the height of mom and dad, cutting them and saying we are ready, paste them. Show your measurements on it. Because at the end of the second lesson, I will ask each group to come up on the board and explain how they solved it. So be prepared. Here you will share what you are doing and how you are solving the problem with other groups.

Having completed their drawings of the mother and father, the students began to discuss how they should cut the colorful papers and place it on the bed. While making these placements, they took the radius length of the bed into consideration and made sure that the mother and father did not protrude out of the bed. The design of this layout showed a real model of the problem. Since the students firstly drew the parents and then cut and paste the appropriate papers, they could review their both real and mathematical models. In other words, while they were cutting papers for the parents' height and width, they validated parents' place in the bed and the necessary lengths. The design of one of the groups is given in **Figure 5**.

When they completed the design, they calculated the distance by measuring the gap between the mother and father with a ruler by mathematizing the situation and working mathematically on it. Here, the remarkable situation in the solution approach of most groups is that they expressed the distance they measured with the ruler as a real value, not in terms of problem situation. Realizing it, Mert intervened with the aim of making the students notice on the fact that their real results were not suitable for the problem (I5.2). This intervention was on the difference between real and mathematical results and caused to revise their mathematical result, as follows:

> Student: There is 1.5 cm between mom and dad.
>
> Mert: [He shows the distance corresponding to 1.5 cm on the ruler] Is there that much?
>
> Student: Yes.
>
> Mert: Too little do not you think? It looks more where you paste it. Look, this distance is nearly close to the width of your mother or father.
>
> Student: Quite so! So, what are we going to do now? Have we done wrong?
>
> Mert: Look, how tall did you find the parents?
>
> Student: The mom is 160 cm, and the dad is 170 cm tall.
>
> Mert: How many centimeters are these papers you cut?
>
> Student: 16 cm and 17 cm.
>
> Student: Well, 1.5 is really too little. Let's measure it again, we definitely did something wrong.
>
> Mert: Why did you take the actual height of the mother and father 16 and 17?
>
> Student: To fit this paper [circular cardboard].
>
> Student: The bed is 210 cm, but this cardboard is 21 cm.
>
> Mert: Exactly. So, you made it suitable for the material you have, right? What action did you take?

> Student: Remove 0.
>
> Student: We divided by 10.
>
> Mert: Good, removing 0 means dividing by 10 anyway. So, is this 1.5 cm you found the actual length or is it the appropriate length for the material?
>
> Student: We found it according to the cardboard. Ok, now I guess I understand.
>
> Mert: What are you going to do then?
>
> Student: Are we going to divide by 10?
>
> Mert: What do you think? Are you dividing by 10? You are going backwards. You divided by 10 at first, what about the inverse of the division?
>
> Student: We are going to multiply by 10 [multiples 1.5 by 10] Ok I found it, 15 cm.

Students in another group claimed that the distance between the parents was 350 cm after they pasted papers by solving their mathematical models wrong. Mert intervened about comparison of their mathematical solution and the diameter of the bed in the context of interpreting the solution, and this intervention enabled students to validate their mathematical operations.

> Mert: How did you find 350 cm?
>
> Student: We measured the gap and found 3.5 cm. We multiplied it by 10 and found 350.
>
> Mert: What was the diameter of the bed?
>
> Student: 210 cm.
>
> Mert: 350 is much bigger than 210, is not it?
>
> Student: Really. It's even bigger than the bed [laughs].
>
> Student: When multiplying by 10, 0 is not added, there is a comma, it will go away. [Multiples 3.5 by 10.] Okay 35 cm. Not 350.

Mert stated that there were about 20 minutes left until the solution was completed, and that they would present their solutions with 2-3 minute presentations. Meanwhile, students in a group asked Mert whether their solution was right or not. Thereupon, Mert said that there is no single correct solution, and that if it is logical and realistic, theirs will be the most realistic solution (I2.2). In this way, he focused on the accuracy of the result is related to the fact that it is realistic and suitable for the problem in the context of a socio-mathematical norm.

When the groups completed their solutions, Mert distributed the assessment forms (A3) and made the following explanation:

> Mert: Please fill the forms carefully by yourself. I want you to write your own thoughts, your own expressions. Your opinion is important and valuable. Everybody's opinion can be different. In this, I want you to evaluate only your group mates, not yourself. You write the name of your group mate here and then evaluate him/her.

Mert told the students who filled out the checklist and peer assessment papers to be prepared for the presentation of solution. For this, he asked each group to determine who would make the presentation and gave information about how they would present their solution.

> Mert: We will start the presentation in 5 minutes. You will make 2-3 minute presentations. You will describe how you solved the problem on the board. You will tell us what result you found. This presentation need not be very long. Simply tell your friends how you solved it. Maybe they have questions, then we can get one or two questions. Now we are quietly listening to how your friends solved the problem. Make your critiques after your friends tell us, we are not interfering, okay?

Upon these statements of Mert, all groups started to discuss what they would say about the solution when they came to the board. These discussions caused some groups to validate and revised their solutions if necessary. All members of a volunteer group came to the board and explained how they solved the solution (A1). When the group finished their presentation, a student from another group stated that their solution was different and Mert explained that this was a normal situation and focused on the fact that each solution was peculiar to each group (I2.2). Therefore, his statements were regarded as an attempt to establish a socio-mathematical norm about the nature of mathematical modelling.

> Student: We found something different. Should be the solution of this problem the same for all of us?

> Mert: Look guys, your friend made a very good point. He said, should the solution of this problem be the same for all of us? No! Of course, your solution is unique to you. There is no single solution for this problem anyway. It may be different in each group. Because you all think differently, okay?

After this explanation, the groups voluntarily continued to present their solutions. Each group explained how many centimeters they took the mother and father's height realistically, what they consider for their width, and then explained how they placed them on a circular cardboard. Finally, they expressed how much the distance between them was on the material and in reality. While making their presentations, the members of the third group only expressed how tall the parents were in reality, how many centimeters they were when they had taken one tenth of it, and how they pasted it on the material. Mert asked how much they found the distance between them. When the students said that they did not calculate the length of the space, Mert asked the group to make the necessary corrections when a student from the other group emphasized what was wanted in the problem (A2.2. and A2.3).

> Mert: How many centimeters are there between mom and dad?
>
> Student: We have not calculated it.
>
> Student [from another group]: You have to find the space between them [The group members on the board reread the problem].
>
> Student: It asks how much space will be between them.
>
> Mert: Now take your place and try to calculate the distance as a group.

While the group members took their places and tried to calculate the distance between them, the last group came to the board and presented their solution. After the presentation and revision of the solutions were finished, Mert made a general evaluation of the modelling implementation, took the students' views on the implementation through the affective evaluation form, and ended the lesson.

Results About Mert's Views After the Implementation

Mert's views in the final-interview were categorized under three themes, namely general evaluation, self-experience and students' experience. Mert evaluated the implementation by focusing on the students' active participation, success in group working, and engagement in an affective modelling process. In addition, he indicated that some of his thoughts had changed after the implementation. Those thoughts were examined under the self-experience theme. For instance, he stated that he had worry for being redundant interventionist since his students could have difficulties due to the lack of modelling experience. Nevertheless, he expressed his students had less difficulty than he expected because they worked affectively in their groups and the circular cardboards given with the problem facilitated reaching the solution.

> Mert: Since they have never tried to solve a modelling problem before, I honestly thought I would intervene a lot. This scared me a little in case I over-directed them. But they worked well with their group and used the cardboards well. I do not need to intervene much.

Mert explained that his students could work at all stages of the modelling process to some extent although they did not have any modelling experience under the students' experience theme. For instance, he stated that the students considered their parents while making assumptions. However, he also indicated that the solution process was extended due to having difficulty in reducing by tenth.

> Mert: Frankly, from making assumptions to validating their solution, they performed better than I expected. Even though they have not solved a modelling problem. It was effective that they thought of their own parents. But they found the distance between mom and dad 1.5 cm, which surprised me. And we lingered there for a long time, wasting quite a bit of time getting them to interpret it according to the cardboards.

In addition, Mert stated that the students were able to fulfil the requirements of group work, and that they were able to refine their models and solutions thanks to the peer and self-assessments they made after the presentations.

DISCUSSION

This study, aiming at investigating both the planning and implementation process of a modelling task by a middle school mathematics teacher, contributed the modelling implementation literature by presenting an example. Blum (2015) stated that some problems could be encountered in reflecting modelling knowledge to class implementation due to the demanding structure of teaching modelling and teachers' lack of competencies to implement modelling. Because the teacher had modelling knowledge and experience to some extent, he could reflect his knowledge and experience to his planning and implementation process. This situation prevented possible problems stated by Blum (2015), and supported other studies (e.g., Blomhøj & Kjeldsen, 2006; Blum, 2015; Borromeo Ferri, 2018) indicating the importance of having modelling knowledge and experience.

This study presented an alternative way to plan and implement a modelling task according to the framework. When Mert's planning process was considered in detail, various similarities were seen when comparing the planning of modelling implementations in the literature. Ang (2015) stated that modelling implementations should be planned according to the skills and competencies that students have. Similarly, Mert planned his implementation to consolidate the circle concept in line with his students' skills and competencies about the circle concept. To consolidate the circle concept, he chose the Bed problem, which he had already known from the workshops. Since he thought about his students' pre-knowledge and identified their lack of knowledge about chord of the circle and Pythagorean equation, which were necessary for the problem, he prepared additional materials. This situation was in parallel with the researchers' explanations about the necessary of students' knowledge in selecting or developing a suitable modelling task (Blum, 2015; Borromeo Ferri, 2018; Greefrath et al., 2021; Maaß & Gurlitt, 2009) and determination of possible student behaviors during planning (Maaß & Gurlitt, 2009). In addition, Geiger et al. (2021) stated that students should be supported with necessary materials to familiarize them with the initial modelling process and Mert's materials served this purpose.

At the beginning of the implementation process, Mert reminded prior knowledge and presented the materials in the context of conducting warm up activities. These warmup activities prepared the students the context of the problem as Geiger et al. (2021) stated. At the same time, he focused on sharing social norms since their students were inexperienced in modelling. Sharing these norms was efficient in providing effective and learner-oriented classroom management, as Blum (2015) states. The social norms were more specific to group work such as motivating students working collaboratively or sharing tasks equally within the group. With the social norms, Mert tried to establish socio-mathematical norms. These were focusing on realistic assumptions, being more than one solution in the class depending on the reality and suitability of the real world context of the problem and being group-specific solutions. These situations were seen as the actions of Mert to create norms based on the knowledge and experience he gained in the workshops, not jointly in the classroom. Bonotto (2007) stated in his study that one of the requirements for the implementation of modelling in the classroom is to change the classroom culture by establishing new socio-mathematical norms in the classroom. In this context, Mert's frequent emphasis on socio-mathematical norms in our study to create a culture suitable for modelling supported Bonotto's (2007) thought. In particular, Mert's explanations that there may be more than one solution supported Blum's (2015) view about the fact that teachers should promote multiple solutions during modelling implementation. In addition, all these norms were stemmed from the realistic nature of modelling and so we can consider them as modelling-specific socio-mathematical norms. Mert also considered how he evaluate and assessed his students both in the planning and during the implementation. He preferred to enable the students assess each other according to the performance in group work. In the literature, it was not identified any study about assessment during modelling implementations, so Mert's preference was regarded as significant. During the implementation, Mert tracked his students' works and intervened by presenting necessary scaffolding, if necessary. As Blum (2015) stated, Mert tried to provide a balance between his students' independence and his guidance. His interventions were enabling the students to self-assess their modelling approaches, to show regard to realistic assumptions, to notice whether the real results were suitable for the problem or not, and to compare the mathematical result, which they reached and the problem context. Leiβ (2005) explained that teachers could intervene students to enable them to work independently in the modelling process and stated that teachers should develop an attitude to support students to reach realistic information or data. Mert's guidance about self-assessment and making realistic assumptions supported Leiβ's (2005) explanations. In addition, Stender and Kaiser (2015) also focused on the teacher interventions during making realistic assumptions. When Mert's interventions were considered in general, he tried to enable students to reach decisions independently about their mathematical results in the context of the bed given in the problem by comparing the material and real-life situation. In other words, he intervened his students in constructing and solving mathematical models, and in validating the solution. Therefore, this showed that the interventions that Mert used supported mathematization, working mathematically and validation, which is one of the sub-competencies of modelling. In the literature, it is stated that students have the most difficulty in validation in modelling implementations (Blum, 2011; Borromeo Ferri, 2018; Ji, 2012; Maaβ, 2006) and that teachers have difficulties in supporting their students in the validation phase (Chan et al., 2015). Therefore, it can be concluded that Mert's appropriate interventions facilitated the students' work in validation.

After the students completed solutions, Mert encouraged them to share and discuss each group's solutions. During this process, it was remarkable that the students tended to compare their solutions with others'. Mert coped with this situation by reminding that the solution of each group would be unique depending on realistic assumptions. He did not prefer to assess his students but guided them to assess every solution. This situation caused that the students assessed their solution approaches during problem solution to defend better in the solution presentation. Thus, Mert enabled the use of the self-assessment principle of modelling (Lesh et al., 2000).

There are many empirical studies in the literature (e.g., Ang, 2015; Borromeo Ferri, 2018; Galbraith et al. 2021; Greefrath et al., 2021; Maaß & Gurlitt, 2009) describing teacher knowledge, teacher interventions, and teacher actions in modelling implementations. We aimed to investigate a teacher's planning and implementation of a modelling task. Although this study has some limitations due to the data of a single teacher, it is thought to contribute to the field as it is a study that deals with the planning process of a teacher with modelling knowledge and experience, actions during implementation, interventions, assessment approaches and students' contributions in the implementation process. Blum (2015) describes modelling implementations that aim to enable students to be cognitively active in modelling successful by providing an effective and student-centered classroom management. In this context, we can conclude that this implementation is successful because teacher creates a content suitable for a framework developed for planning and realization of modelling implementations and tries to implement content by ensuring maximum student interaction. Since limitation about one teacher's modelling implementation, we can advise examination of different mathematics' teachers modelling planning and implementations in future. Also, long-term implementations in modelling can enable us to elicit possible patterns during learning and teaching process.

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