International Electronic Journal of Mathematics Education
2024, 19(2), em0772
e-ISSN: 1306-3030
https://www.iejme.com

MODESTUM

Research Article **OPEN ACCESS**

Professional collaboration in a lesson study with university mathematics professors

Adriana Richit [1,2]*, Neila Tonin Agranionih [3], Tania Teresinha Bruns Zimer [3], Ranúzy Borges Neves [4]

[1] Universidade Federal da Fronteira Sul, Chapecó, BRAZIL
[2] Universidade de Lisboa, Lisboa, PORTUGAL
[3] Universidade Federal do Paraná, Curitiba, BRAZIL
[4] Instituto Federal Catarinense, Blumenau, BRAZIL
***Corresponding Author:** adrianarichit@gmail.com

Citation: Richit, A., Agranionih, N. T., Zimer, T. T. B., & Neves, R. B. (2024). Professional collaboration in a lesson study with university mathematics professors. *International Electronic Journal of Mathematics Education, 19*(2), em0772. https://doi.org/10.29333/iejme/14290

ARTICLE INFO

Received: 27 Oct. 2023

Accepted: 13 Feb. 2024

ABSTRACT

The article analyzes professional collaboration in a lesson study involving university mathematics professors, with the objective of contributing to professional development. The research was guided by the question, What principles promoted professional collaboration among university mathematics professors participating in a lesson study? The research focused on a lesson study involving university professors of mathematics and mathematics education over twelve weekly meetings of two hours each. The empirical material included the researchers' field notes and the transcripts of the recordings of the lesson study sessions. A qualitative analysis, based on content analysis, revealed that collaboration involved: *professional sharing* and *decision-making*. The research shows that by fostering collaboration the lesson study mobilized different principles of teaching professionalism, such as knowledge that is basic to teaching, values and ways of conducting university teaching, a social and moral commitment to teaching, and teaching autonomy, contributing to the professional development of higher education professors.

Keywords: professional collaboration, lesson study, higher education, professional development, mathematics

INTRODUCTION

Because the work of professors in higher education is restricted to specific subjects that are often compartmentalized within the programs of undergraduate courses, professors wind up working in isolation. Nevertheless, we believe that teaching at all levels of education, especially university teaching, presupposes dialog among professors, the sharing of experiences and professional practices, reflection on teaching and its aims, and the need to promote interrelationships between topics from different disciplines. University teaching interdepends on the work of other professionals (Lima & Fialho, 2015), and is thus a highly collaborative activity.

In addition, changes in university education require teachers' involvement and reflection about each dimension of their practice and compel them to make professional decisions (Brodie, 2021). Teaching in higher education involves spontaneous or intentional collaborative practices that range from sharing ideas about curriculums and students to joint planning of teaching (Pischetola et al., 2023). These actions involve dialogue, shared decisions, mutual support, diversification of teaching strategies, the proposal of changes, and reflection on practice, as well as other aspects (Richit, 2023). According to Bødker and Kyng (2018), collaborative professional practices have meaningful and lasting impacts on teachers' professional lives and on the teaching processes of professors and can elicit collectively generated shifts in their practices.

These processes become evident in lesson study, a collaborative and reflective approach to professional development (Burroughs & Luebeck, 2010; Murata, 2011; Richit & Tomkelski, 2020). Collaboration permeates the main steps of a lesson study, which include:

(a) delimiting the topic and objectives for a lesson (the research lesson), considering students' learning needs and difficulties;

(b) planning the research lesson, based on the study of the curriculum guidelines and directives related to teaching the curriculum topic addressed–anticipating the steps to be followed–delving deeper into the curricular topic, discussing research results related to the teaching of the topic, as well as preparing a task, and anticipating the students' difficulties and actions when solving it;

(c) teaching the research lesson by a teacher participating in the lesson study, while observed by the other members of the group, who register the students' actions; and

(d) reflecting on the research lesson based on the notes taken and the students' activities and reactions (Richit et al., 2019).

Cavadas and Branco (2023) emphasize that collaboration among teachers is essential for professional development. They analyze two cycles of collaborative teaching research to characterize the dynamics among four professors in higher education involved in an experience related to the creation, implementation and evaluation of interdisciplinary learning spaces. The collaborative nature of teaching, although sometimes absent in higher education due to the curricular structure of some courses, requires that professors try to promote interaction between the subjects they teach and other curricular components, and between their professional practice and that of their peers. Participating in a lesson study is one way to engage in this interaction.

In this article we analyze professional collaboration in a lesson study, which involved eight university mathematics professors and a prospective mathematics teacher. This qualitative research can support discussions about teaching at a university level, considering the new principles of teaching professionalism in this context, and to discussions about the contributions of lesson study to this field.

PROFESSIONAL COLLABORATION AMONG TEACHERS

Collaboration is understood as a relational system among people, or as an emerging social process characterized by unpredictability that involves negotiations and *decision-making*, and focuses on resources, goals, products, and results (Morris & Miller-Stevens, 2016). The fundamental principles of collaboration are dialog among peers, interaction between colleagues, shared *decision-making*, and mutual support (Hargreaves, 1998). Moreover, collaboration among teachers involves interdependence and shared responsibility towards a common goal (Romeu et al., 2016). Professional teacher collaboration characterizes the predominant ways of being, acting and thinking in learning communities composed of teachers (Day, 2001) in which are valued dialogue, shared reflection, negotiation, joint work, cooperation, mutual support and encouragement, overcoming hierarchies and *professional sharing* (Richit et al., 2021a).

Professional sharing encourages professional learning to the extent that teachers have the opportunity to socialize teaching experiences, observe each other's practices, learn new practices, study and plan teaching with peers, and discuss and reflect together (Plauborg, 2009). In this article, we discuss *professional sharing* and *decision-making*, which are collaborative principles that complement each other in the different steps of the lesson study.

Conceived in this way, professional collaboration mobilizes collective efforts to improve teaching, promote student learning and foster educational change because it creates opportunities for groups of teachers to learn about teaching and reflect on their role in an institution and in society (Robutti et al., 2016). Assuming collaboration as a dimension of the professional development of teachers, Day (2001) proposes a reflection on the professional partnerships through which teachers and instructors in higher education institutions constitute learning communities. According to the author, these partnerships are nothing new, since the school and university careers of these professionals enable them to develop "an affinity of moral purpose and complementarity of practice" (Day, 2001, p. 236).

Day (2001) emphasizes that there is a diverse range of partnerships among professors in higher education, which involves continuing teacher education actions for the internal and external community of professors, collaborative research projects, educational consultancies, production and dissemination of educational knowledge, and institutional cooperation agreements, and others. Partnerships such as these, and others not mentioned by the author, establish a context for professional collaboration among university professors when conducting their academic and social mission, demonstrating their moral commitment to improving teaching and learning in collaboration with their peers, schools and teachers.

In conclusion, Day (2001) points out that partnerships among university professors, in addition to creating opportunities for reflection, enable these professionals to recognize teaching as a moral activity. These actions are realized through the formation of "networks, critical friendships, peer training, reflection in and out of the workplace, analysis and planning" (p. 264). This aspect is supported by Hargreaves (1995, p. 261), who says that "by reflecting on their own practice, observing and analyzing the practice of other teachers or studying case studies, teachers can clarify the dilemmas they face and develop thoughtful ways of dealing with them, based on principles, practices[1]" and on skills that are increasingly developed.

Through a study focused on the perspective of interdisciplinary collaboration, which involved higher education professors of science and mathematics, Cavadas and Branco (2023) emphasize that collaboration among professionals from different fields of knowledge favors the development of content knowledge (in this specific case mathematics and science) and pedagogical knowledge in relation to interdisciplinary learning spaces. Collaboration also encourages reflection on practice and its effects on student learning. They add that professional collaboration among professors takes place through dialogue, *decision-making*, implementation of teaching actions and reflection on components for evaluation (Cavadas & Branco, 2023).

Collaboration is thus a basic dimension of professional culture that is supported by a set of beliefs, values, habits and ways of acting within teaching communities, which have had to deal with similar demands and constraints over the years that confer "meaning, support and identity to professors and their work" (Hargreaves, 1998, p. 186).

Thus, the various elements that characterize collaboration–the ways in which professors relate to each other, considering the intersubjectivities that permeate these relationships, values, practices, knowledge, and ways of acting–constitute elements of teacher professionalism, which is marked by some idiosyncrasies due to the particularities of the context and the subjects who interact in it (Richit, 2023). Collaboration is therefore a dimension of teachers' professional development (Richit, 2021), a process,

[1] Cited by Day (2001, p.264).

which is influenced by teachers' personal and professional lives, educational policies, working conditions and the contexts in which they work (Day, 2001).

Professional Collaboration Conceived as a Basic Principle of a Lesson Study

Efforts to modify educational practices, resulting from social evolution and the emergence of new issues and new values, are requiring teachers to teach in ways that differ substantially from how they were taught and how they learned to teach (Borko & Putnam, 1995). This calls for new models of professional development that can improve classroom teaching practices. Professional development is characterized by the systematic efforts made to promote changes in teachers' practices, attitudes, and beliefs, and in students' learning (Guskey, 1997). Among the approaches to professional development that have gained relevance in this context of change are lesson studies (da Ponte et al., 2016; Richit & Tomkelski, 2022).

Originating in Japan, lesson study is one of the main approaches to the professional development of teachers in that country. The basic principles of lesson study are reflection and collaboration (Stiegler & Hiebert, 2016). According to Lewis (2012), Japanese primary school teachers have a strong culture of collaboration, through which they dialog and plan classroom activities together. In addition, they jointly engage in social programs, walks, trips, sporting events, and other activities. The author adds that teachers in Japan, and Japanese citizens in general, approach/assume feedback and criticism as a stimulus for future improvements and believe that such changes come about through collective efforts. In this sense, teachers understand that lesson study makes it possible to make incremental changes that lead to greater changes.

Research has indicated the potential of lesson study to promote teacher learning and development, such as improving knowledge of curriculum content and how to teach it (Fujii, 2016; Lewis, 2002), improving classroom practice (da Ponte et al., 2016), improving teachers' didactic competence (Huang et al., 2011), collaboratively developing research into lessons (Takahashi & McDougal, 2016), enabling change through incremental improvements (Lewis, 2012) and creating interdisciplinary spaces for professional learning (Cavadas & Branco, 2023).

In a study on the contributions of lesson study to developing teacher professionalism, Najikhah et al. (2023) point out that by participating in this process, teachers have the opportunity to: think carefully about learning, teaching objectives, curriculum topics and fields of study; revisit and develop student learning in the best possible way; improve knowledge of the subject to be taught; reflect deeply on the long-term objectives to be achieved in relation to students; plan investigative learning paths; carefully examine the forms and processes of student learning and action; develop 'powerful' pedagogical knowledge; and see the results of professional learning through study and colleagues. In addition, lesson study allows teachers to work collaboratively by sharing objectives, discussing ideas, and developing teaching resources (Burroughs & Luebeck, 2010), and to overcome the isolation characteristic of the profession and achieve collaborative forms of professional learning (Richit & Tomkelski, 2020).

Examining the implementation of lesson studies in Italy, Mellone et al. (2019) affirm that when teachers import this approach into the cultural contexts of their countries, the experience can cause them some estrangement. They add that by participating in a lesson study teachers gain the opportunity to reflect on and rethink their own culture, a process called cultural transposition. For the authors, this transposition is activated by researchers, educators of teachers, and teachers through the deconstruction of established practices by reconsidering issues of educational intentionality at different levels and contexts (Mellone et al., 2019).

In this sense, Nguyen and Tran (2022) point out that different cultural backgrounds generate different possibilities for constructing meaning and perspectives. Thus, when implementing lesson study in different cultures, it is crucial to

(a) emphasize cultural transposition,

(b) make explicit the idealized, received and intended curricula, and

(c) contemplate the power relations between teachers-students, teachers-teachers and teachers-researchers (Nguyen & Tran, 2022).

The aspects highlighted by Mellone et al. (2019) and Nguyen and Tran (2022) draw attention to the cultural transposition intrinsic to the process of conducting a lesson study in contexts other than Japan, where collaboration is a central characteristic of the culture.

In this context, the literature has reported different forms of collaboration in lesson study (Quaresma & Ponte, 2021) and indicates that cultural elements influence the development of this approach (Richit & Ponte, 2017). Studies also indicate that in professional cultures marked by isolation, the implementation of lesson study must initially develop collaboration based on dialog and trust among peers (Richit et al., 2023).

Examining professional collaboration in the Brazilian scenario, Richit et al. (2021a) show that professional collaboration in a lesson study is promoted through different but connected aspects including, dialog, *professional sharing*, incentive, and mutual support, cooperation and shared reflection. They conclude that through lesson study, teachers can experience a collaborative and reflective form of teacher education, overcoming the individualism that is still prevalent in teaching, as it is in the university context in some cases.

Meanwhile, Cerbin (2011) identifies various contributions of lesson studies in higher education by analyzing a series of experiences conducted between 2003 and 2010 by a team from the University of Wisconsin, USA, which involved the fields of Biology, Economics, English and Psychology. The author points out that lesson studies with university professors not only improve student learning, but also promote the growth of professors and positively influence the participants' teaching practices. The reflections of the project members indicate that the lesson study influenced them in the following aspects: it develops pedagogical thinking, favors changes in teaching practices, provides opportunities to understand student learning and develop knowledge about students, promotes a deeper understanding of the curriculum topic covered, and broadens collaboration with colleagues (Cerbin, 2011). The author highlights that the most significant and beneficial aspects for the participants were collaboration with

Table 1. Participants

Field	State & institutions					
	Paraná	Santa Catarina		Rio Grande do Sul		
	UFPR	UFFS	IFC	URI	UFRGS	UFFS
Mathematics education	Ester & Tatiana	Estrela	Amy & Catarina			Alice
Calculus	Michel			Christopher	Natan	

Note. IFC: Instituto Federal Catarinense; UFFS: Universidade Federal da Fronteira Sul; URI: Universidade Regional Integrada; UFPR: Universidade Federal do Paraná; & UFRGS-Universidade Federal do Rio Grande do Sul

peers, the study of different strategies and alternative forms of teaching and the observation and analysis of student learning (Cerbin, 2011).

METHODOLOGY

Nature & Objective

This research, which is qualitative in nature (Denzin & Lincoln, 2006), corroborates that the professional development of university professors has an eminently social and dynamic nature, and is a process that is promoted through collaboration among peers, and examined in the light of the different meanings and values that this process involves for each participant in a lesson study. The objective of this qualitative research was to analyze professional collaboration in a lesson study with university mathematics professors to contribute to their professional development.

Context & Participants

The research was conducted of a lesson study carried out between April and July 2021, which involved eight professors (Amy, Alice, Catarina, Christopher, Esther, Estrela, Michel, and Tatiana–fictitious names), who teach mathematics and/or mathematics education at universities and federal institutes in southern Brazil, and an undergraduate student working towards teaching certification in mathematics (Natan–fictitious name), who contributed to preparing the materials for the research lesson and carrying out the research. The team was established through the initiative of Alice, Amy, and Esther, who determined that each of them should invite two higher education mathematics teachers to take part in the lesson study, and that one should have experience in calculus. We thus formed a team of teachers with experience in calculus and mathematics education. **Table 1** shows the field in which the participants work, their higher education institutions, where they are from and the state in which they are based.

All the professors have more than 10 years' experience in university teaching, working mainly in teaching certification courses in mathematics. Some professors work in mathematics education and calculus, as shown in **Table 1**.

The lesson study was conceived and conducted by the first author of the article, and the activities of each step were planned in collaboration with the participants. The lesson study was organized in four stages; definition of objectives for the research lesson (three meetings), planning (six meetings), teaching the lesson (one meeting) and post-lesson reflection (two meetings). It was realized in twelve remote, weekly meetings of two hours each, and was dedicated to deepening the curriculum topic 'maximums and minimums', covered in the subject of calculus.

Empirical Material & Analysis

The empirical material consists of the researchers' field notes and the transcripts of the recordings of all lesson study sessions. We emphasize that although the research assumes the principles of collaboration established by the literature in the field, the analysis was conducted inductively, in such a way that the categories emerged during the process. The analysis established as reference units the group of excerpts extracted from the empirical material that revealed the basic principles of professional collaboration in the group. We then grouped these reference units, considering their proximity to each other, forming the categories of analysis, which were named: *professional sharing* and *decision-making.*

RESULTS

Professional Sharing

This principle of collaboration was favored in the lesson study with higher education professors due to the diversity and duration of the participants' professional experiences, their academic trajectories, and the fact that the team worked in calculus (and other disciplines) and in mathematics education. Thus, the sharing allowed reflecting on aspects related to university teaching from two complementary perspectives. The main sharing among the group involved readings, classroom activities, mathematics tasks, academic and professional experiences, academic achievements and events.

Spontaneously, from the very first meeting, the *sharing of readings* (articles, books, theses, and dissertations) was consolidated in the dynamics of the lesson study, so that some suggestions were incorporated into the planning of activities and others served as support material for other teaching and research activities by the participants. In the second session, after presenting the structure of the lesson studies, a discussion emerged on the concept of an exploratory approach and the distinction between exercise, problem, investigation, and exploratory task. As part of this discussion, Esther recommended reading the text

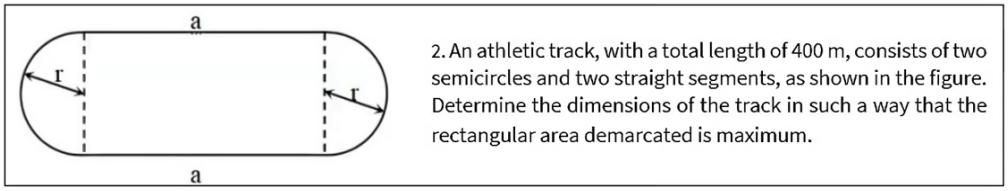

Figure 1. Illustration of an athletic track (Source: Personal archive of Christopher, reprinted with permission)

"Investigar, ensinar e aprender", (investigate, teach and learn), by da Ponte (2003), which was essential for developing the task for the research lesson and, above all, for explaining the nature of each type of task, as was seen in the 4th session of the cycle.

> Catarina: [When sharing on a screen the article mentioned above, she says:] Do you see this chart, [in the form of a] Cartesian plane? [...] on the right side are the open tasks, which are explorations and investigations. And on the left are the closed tasks, exercises, and problems. What defines each one is the level of challenge, whether it's low or high. Exploration has a lower level of challenge, investigation is higher. The exercise is closed and has a low level of challenge, and the problem is closed and has a slightly higher level of challenge. Thus, this would be the main difference between them, right? (4th session).

In relation to the readings shared, the experience with this group allowed reflecting on the organization of lesson study by socializing some of the predominant models in the field. In the context of a discussion on lesson study models developed by researchers from various countries, and considering the risks of losing the essence of the approach due to how it has been adapted in cultural contexts other than Japan, it was highlighted that:

> *Catarina:* And there's this English model that Alice mentioned, which is used in basic education. They [set up] focus groups, they choose [themes], they select students–students with more skills, students with more difficulties–[and so they set up] a group that represents the class, which is a sample of the class. And then they do a focus group and discuss in more depth what was done in the research lesson, to try to find more elements. And in higher education, in the article I read, they have also adopted this model, of doing focus groups with higher education students as well. So, if we have the opportunity to do this focus group, it would also be interesting. [Otherwise], I think a well-designed questionnaire [...] will give us a lot of interesting answers (6th session).

Next, Catarina shared the results of the study by Amanda Alvine et al. (2007), which presents an experience of a lesson study conducted in calculus at Harvard University. This reading was essential for preparing the observation protocol for the research lesson and, above all, for carrying out the post-lesson reflection.

In the eighth session of the lesson study, after socializing a reading done by Esther, the group continued to work on the task for the research lesson. After discussing the structure of the task and its potential to encourage mathematical discoveries from the perspective of an exploratory approach, some readings were suggested, as recorded below:

> After discussing the particularities of exploratory tasks and the care we should take when designing an exploratory task for higher education, Catarina mentioned an article by Alessandro Jacques Ribeiro and Juliana Paulin Viol, published in 2020 in the Revista Acta Scientiae, which discusses Exploratory Tasks in university teaching. In addition, Catarina said that she had contacted the authors and asked for access to the tasks presented in the text so that we could compare them with the one we were developing, which is also related to calculus (field notes, 8th session).

Some of the readings were socialized and compared in the context of the discussions held during the planning stage of the research lesson. Others were shared via the communication devices used by the group (Classroom, Google Drive, and Whatsapp), providing support for the teaching and research activities carried out by the participants, such as the work of Cerbin (2011), which became one of the main references for our research on lesson study in higher education.

Sharing tasks involving calculus also promoted collaboration in the group. At the beginning of the lesson study, when the group began to think about the task for the research lesson, the participants who taught Calculus presented some problems as a starting point. For example, in the 3rd session Christopher shared some tasks that he often presents in calculus lessons of which he highlighted an introductory problem on the topic of 'maximums and minimums' (see **Figure 1**), based on which he promotes the construction of a definition and then carries out the tests of the first and second derivatives.

After Christopher explained how he often presents the task to his classes, Amy asked him about the discoveries/conclusions made by the students from these problems.

> Amy: That's an interesting problem [...]. What are the main mathematical discoveries and conclusions that students make from problems like this?

> Christopher: This is a problem they like, although some of them find it more difficult. At first we use a table so they can see the possibilities of lengths that satisfy the fixed value. [With the] table, fixing the length, because it has to be fixed, they vary x and y, to see if the length gets bigger or smaller. So there are several possibilities. But you will not be able to solve by [table] for the rest of your life. And then they realize that they can solve faster with the tools of calculus, based on derivatives for maxima and minima.

> Amy: I'd like to propose these problems to my students if you allow me, of course.

At the end, Amy asked Christopher if she could use the tasks he shared with her calculus students, which he readily agreed to. Furthermore, the sharing of activities was important in defining the context and structure of the task prepared for the research lesson, as the following excerpt shows.

> Michel: There's a problem [...] in James Stewart's book, which I think is also interesting to raise. It has a ready-made model, so you would not need to create a problem like […] the geometry ones we were doing. However, the problem provides some very interesting interpretations of what we were talking about in [4th session], about what a critical point is. That a critical point is a candidate for a maximum or a minimum. It's not necessarily going to be a global maximum, nor is it necessarily going to be a global minimum, sometimes it's just going to be a maximum in the region of the graph (6th session).
>
> Esther: I also found interesting the first idea of working with something more open and related to their context (referring to the agricultural context of the IFC region).
>
> Natan: we could begin with a task in this contextual sense and at some point steer it towards this specific objective, related to the concept [of maximum and minimum].

The activities socialized and analyzed in the lesson study influenced the elaboration of the task and promoted collaboration at higher levels. Similarly, *sharing of academic achievements* was a particular feature of this cycle. For example, in the seventh meeting of the cycle, after welcoming all the participants, Michel shared an important academic achievement during the time dedicated to general reports and events of the week.

> Michel: Guys, I'd like to share some news with you. It may seem like a small thing, but for me it's very important and means a lot. This week, an article of mine, and some of my colleagues, including Tatiana, was published. I'm very proud, very happy with this work [...]. The article is a discussion, based on a literature review, about the meaning of concept from Vygotsky's perspective (7th session).

This article was then read by some of the lesson study participants, who made some comments at the next meeting (8th session) in which Alice pointed out that the article provides important reflections on how the central concepts of Vygotsky's theory are assimilated into teaching processes. In addition, *sharing of academic experiences* was highly valued in the group and became part of the process. Before beginning the activities of each planning session, in the space dedicated to brief news and general sharing, they spontaneously began reporting on the events they had taken part in that week.

> Catarina: [Referring to the doctoral seminars held that day at the Institute of Education of the Universidade de Lisboa] This is an internal seminar, where doctoral students talk a bit about how their research is going. And today we had six presentations and on Wednesday we'll have four more if I'm not mistaken. So there are doctoral students from various classes, not all from the same lesson [...]. Each one talks a little about their research. Today's were lesson studies with teachers in-service, and I was the last one. And of the [doctoral students] who presented today, I'm the only one who has not begun collecting data yet because my lesson studies have not started yet, due to the pandemic [...]. So, it was very interesting (4th session).

The professional and academic sharings were important for the participants, because they provided support to understanding the lesson study and its dynamics, for planning the task of the research lesson, and for deepening conceptual aspects of the field of mathematics education and calculus. In addition, the exchanges were a context for professional learning and academic training, as Natan highlighted.

> Natan: The exchanges were enriching. During the development of the LS [lesson study], we had moments of [sharing]. And not just about the theoretical framework of the LS. All these moments contributed to our education in different ways […]. They mentioned events and activities they had taken part in, for example. Some of them took part in LS-related events and brought back some of what they had learned, presented, or read (participant's field notes, July/2021).

Therefore, the sharing was highly valued by the participants because they created a space that contributed to the discussions that were conducted in the lesson study, and that helped to recognize and value experiences and achievements (both individual and collective). They also provided input for planning the lesson and preparing the task, for private reading and reflection, and fostered new partnerships within the group through co-authored work for events and projects.

Decision-Making

Decision-making was a basic principle of the collaboration experienced by the group. In the first stage of the lesson study cycle, *deciding on the topic* to be covered was a process of dialog and valuing everyone's suggestions and arguments, which led the group to define the topic 'maximum and minimum', which provides opportunities for mathematical investigations in different situations related to maximization or minimization.

> The group talked about some topics that could be covered in the lesson study. After Amy said she would begin the study of derivatives in a few weeks, Christopher and Michel suggested tackling the application of derivatives. This initial proposal

> was refined over the course of the meeting so that, in the end, the group settled on the topic of 'maximum and minimum' (researcher field notes, 2nd session).

Likewise, *deciding on the objective for the research lesson* was a negotiated, reflective process that was well founded. The objective for the research lesson was defined through discussions about elements of the students' context, their familiarity with more open tasks (such as exploratory tasks), the objectives of the calculus component and the research question (problem) established for the lesson study (What is the role/importance of calculus and the concepts of maximums and minimums in the training of future mathematics teachers?).

> Esther: Amy said that the students are from rural areas, most of their parents are farmers. I [think] it would be interesting if we worked on a task related to their reality, because although they may not have mastered calculus, they may have mastered some other knowledge that could even help them solve the task (6th session).

> Catarina: I think that if we were going to use a problem like this, an agricultural problem, I think it would be interesting for Amy to talk to the people in the field [...]. There's also the issue of motivation [...]. Perhaps something related to their reality would also motivate them more, and arouse more curiosity, something along those lines (6th session).

> Christopher: It's necessary to consider other things that influence the activity, such as the weather, the technical standards that regulate this activity [...]. The students need to have this information to think about the problem (6th session).

Based on this discussion and with the objective of considering the reality of the students who would take part in the lesson, both in terms of the social and cultural context in which they were inserted and the prior knowledge that would be mobilized, the group defined the following objectives for the research lesson.

- Relate the study of maximums and minimums to everyday situations or real contexts.
- Approach a contextual situation by motivating student involvement in modeling and representing the mathematical objects of the problem.
- Verify and articulate the algebraic and geometric domain of the objects described in the problem and conclude with an investigation of the central object of the task (analysis of minimization of the object function–total length of the electrical wire).
- Reinforce the idea that mathematical functions can have minimum or maximum values.
- Check the use of strategies for solving the questions and algebraic writing of the objects requested, especially for the issue of minimization of material (researcher field notes).

Based on this decision, in the second stage of the lesson study, dedicated to planning, the task prepared by the group adopted as its context the rotational grazing of dairy cattle, because the institution, where the research lesson was held, IFC, is located in a region with strong agricultural activity and milk production, characterizing the interdisciplinary approach of the topic 'maximums and minimums'. The task was presented as seen in **Figure 2** (Richit et al., 2021b).

Decision about the duration of the research lesson mobilized reflections in the group that went beyond the specific issue of time, in that it considered the objectives for the lesson, the distribution of the hours of the calculus component established by the course curriculum and the care taken to avoid disengaging the students from solving the task.

> Estrela: I was thinking about the time [needed to do this]. Usually when we plan something, in practice it takes longer. So, thinking about this remote context in which the [lesson is to be held], should we propose something for one evening? Is it preferable to work on everything in one evening or [perhaps leave a part as] a hook for the next lesson? Considering that the lesson is all night long, from seven to ten, [we have to think about whether this is suitable] for remote teaching (6th session).

Considering the participants' observations, the group decided to organize the lesson around a task, so that all the moments for exploring the task would be completed in one evening (a four-hour lesson). The decision to conduct the research lesson in one evening considered the fact that the students were used to this schedule of four calculus classes one after the other, and the professor's need to continue with the other curriculum topics for the discipline in the following classes. In addition, the decision was made based on Amy's concern for the possibility that the students would forget or lose interest in the task if it was conducted on two different days.

After this decision, the group began to discuss *dynamics of the research lesson*, to deliberate on the different moments for exploring the task, the duration of each stage and how to systematize the aspects covered in the text.

> Amy: [In the moment when] I will do this familiarization about the context of the task as a way of presenting the task, rather than handing out the sheet, as it says in our plan: doing the reading and everything else. I think it's important to put the main aspects of the task on slides because our students have a more visual perspective. To try to look, to visualize a bit of what they're hearing [as I speak]. So, I [think it's better] to use some slides, because there are various aspects of veterinary science, animal welfare, planting, that I will not remember at the time. So, I'd like to suggest presenting some aspects of the task on slides (9th meeting).

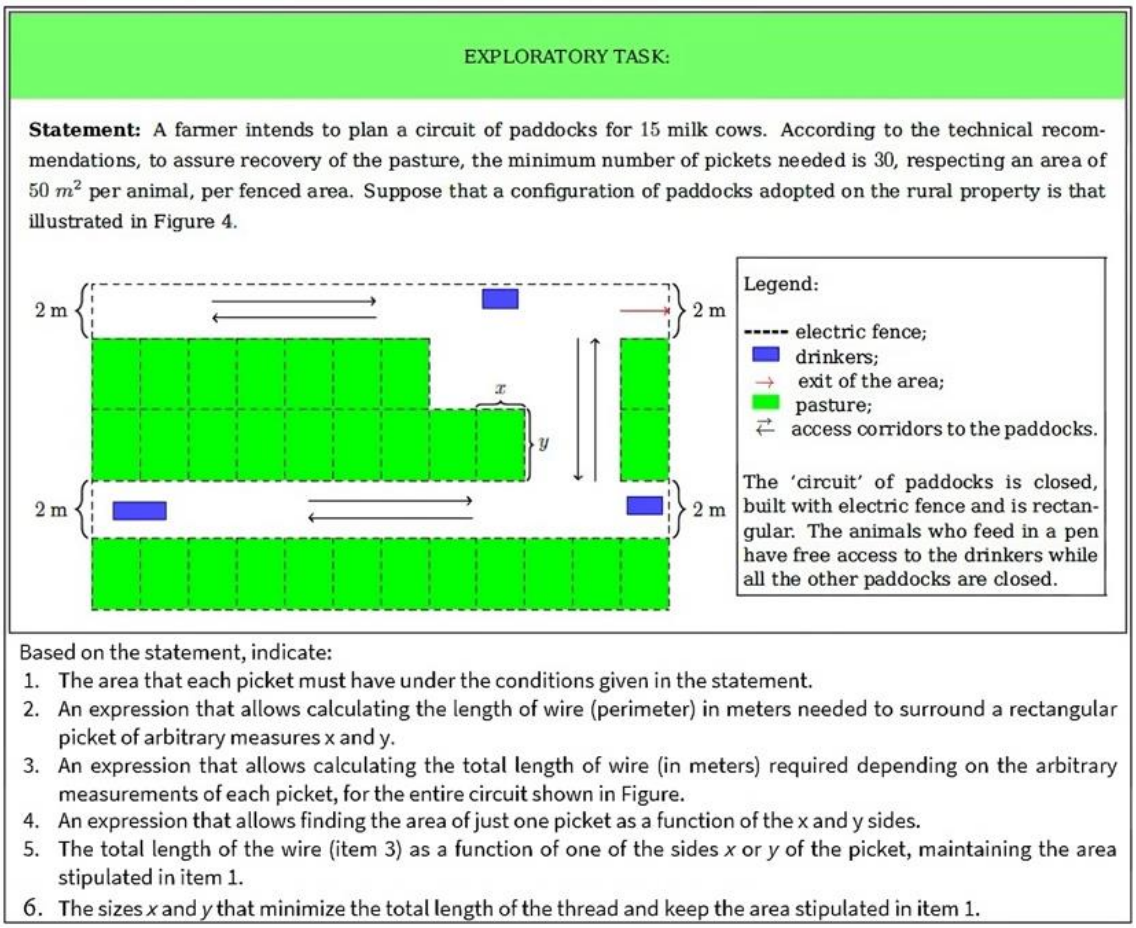

Figure 2. Task prepared for research lesson (Source: Archive of researchers)

Table 2. Moments of research lesson

Moments	Description	Duration
Problematization	Presentation of the task and problematization of aspects related to its context.	10 min.
Autonomous work	The students, organized into groups, dedicate themselves to solving the task.	60 min.
Collective discussion	Presentation of the groups' strategies, representations, solutions, and conclusions.	30 min.
Systematization	Systematization of the concepts, properties, and mathematical notation of 'maximums and minimums'.	20 min.

In the same way that the introduction of the task required careful negotiation in the group, the moment of collective discussion of the resolutions and the evaluation of learning was another important aspect in *decision-making* process among the participants.

> Amy: [And at] the end, at the stage of [the discussion] of the solutions, I wondered: are we going to have time to [analyze the solutions]? For example, if the group comes up with an eight-line solution, am I going to be able to immediately identify whether that solution is correct? I was thinking a bit about this aspect, that it might not be so easy to analyze and discuss the resolutions. [How can we] do this? Or [do we look] at the solution they've come up with? I realized this in the research task I did. If I already pointed out that the first group had not solved it correctly, when the second group went to talk, they ended up incorporating some of what I said, which went in the direction of what was correct. [In other words, the group] altered their presentation, which was no longer what they had discussed, no longer what they had constructed as a solution. So I do not know if I should make observations after each group has explained. But perhaps only at the end, presenting the solution to each of the questions in the task. Maybe do something more along those lines (9^{th} meeting).

According to Amy, the way the collective discussion is carried out influences the socialization of resolutions. Thus, after this discussion, the group defined the dynamics of the collective discussion and the duration of each moment of the lesson, observing the principles of the exploratory approach, as shown in **Table 2**.

Regarding the dynamics of the collective discussion, considering Amy's comments, the participants decided that each group of students would present the resolution of a question from the task, which would be commented on by the other teams. Only at the end would the teacher comment. This strategy helped to ensure that everyone took part in the collective discussion and prevented the solutions from being replicated by the groups.

The negotiation around this decision mobilized aspects about the role of the teacher of the lesson, as well as the strategies to be adopted to encourage exploration of the task, especially at the moment of collective discussion. This moment of the lesson (collective discussion) must be well orchestrated by the teacher to prevent a resolution that is shared first from influencing the

presentation other groups of students, leading them to rework their answers based on what their classmates have just presented or on the comments from the teacher of the lesson.

The interaction between teachers who shared experiences in the same field, from two perspectives (that of mathematics education and teaching calculus), was an important strategy for consolidating knowledge and establishing principles about observation in this lesson study. It also focused the observers' attention to different but complementary aspects of the students' actions on the task, revealing interesting strategies and conclusions for the teams' solutions.

At the end of the lesson study, *decision-making* was consolidated as a principle of collaboration through the group's desire to continue working in partnership. To this end, future actions were planned (research projects and the production of co-authored work) to consolidate the group as a professional learning community, because throughout this process, much more than planning a research lesson, a group was formed, with a sense of belonging and which understands collaboration as a key to improvements in education.

Therefore, professional and academic sharing permeated all the stages of the lesson study, favoring more informed *decision-making*. And both–*professional sharing* and *decision-making*–inseparably enhanced professional collaboration by providing a context for the participants to collaboratively carry out work that is usually conducted in isolation. In this way, the culture of collaboration contributes to the professional development of university professors because it provides them with a context for growth while challenging them to go further.

DISCUSSION & CONCLUSIONS

The analysis showed that the dynamization of the lesson study with university mathematics professors has collaboration as both a starting point and its horizon, highlighting it in light of *professional sharing* and *decision-making* inherent to university teaching.

Professional sharings promoted a context of listening and recognition of the participants' experience (Richit & da Ponte, 2019) because everyone's contributions were encouraged and valued, positively influencing the development of the lesson study, strengthening the relationships among the participants and consolidating a professional learning community sustained by collaboration (Day, 2001). The sharing encompassed the content of the subject of calculus, conceptual aspects, and properties of 'maximums and minimums', the resources and strategies for approaching this curricular topic, and influenced the elaboration of the task for the research lesson and how to develop it in the classroom.

Professional sharing involving teaching activities and experiences enabled participants to reflect on their own practice, hear and analyze the experiences (practice cases) of their peers, reflect on the challenges of teaching calculus (Hargreaves, 1995) and substantiate strategies to address them (Najikhah et al., 2023), which were tried out and analyzed in the research lesson. From this perspective, professional sharing supported the participants' professional learning because they had the opportunity to socialize and reflect on different teaching experiences, observe a teaching practice planned by the team, and reflect on this practice (Plauborg, 2009). In addition, professional sharing created opportunities to reflect on more than teaching calculus, which emphasized university teaching as a moral activity (Day, 2001).

Therefore, the dynamics of the lesson study, especially the atmosphere of welcoming and listening to individual and collective voices, fostered and enhanced different types of sharing, which were imbued with other principles of collaboration, such as dialog, cooperation, support and encouragement (Hargreaves, 1995), group work, overcoming hierarchies, and shared reflection (Richit et al., 2021a).

Decision-making was a key aspect in the dynamics of the lesson study, which was consolidated from the choice of curriculum topic at the start of the process to the administration of the research lesson, considering all the moments of this stage from the perspective of the exploratory approach. In addition, recent changes in Brazilian university teaching have brought challenges, requiring teachers' involvement and reflection about each dimension of their practice and compelling them to make decisions (Brodie, 2021), especially about how to teach in synergy with the course goals and needs of the students.

Just as professional actions at schools or higher education institutions are interdependent on the work of peers and other education professionals (Cavadas & Branco, 2023; Lima & Fialho, 2015; Richit, 2023), the analysis showed that decisions in the context of the lesson study were influenced by suggestions and the different roles (as observers, teachers, rapporteurs, critics, proponents of actions) assumed by the group members throughout the process. In this respect, we believe that although the participants worked in higher education, because they belonged to different institutions and had professional trajectories in areas with different emphases (mathematics and mathematics education), their experiences were somewhat different, which contributed positively to *decision-making* in the lesson study.

From this perspective, *professional sharing* and *decision-making*, conceived as principles of professional collaboration, enhanced the collaborative nature of teaching in higher education (Cavadas & Branco, 2023; Richit, 2023), insofar as they promoted interaction between the perspectives of teaching calculus and mathematics education and made it possible for professionals from different geographical and institutional contexts to work together. Collaboration therefore enabled participants to develop an affinity of moral purpose and complementarity of practice in university teaching (Day, 2001).

In this way, the lesson study promoted professional collaboration, to the extent that a context of listening and valuing everyone's voices and experiences was consolidated, the collaboration occurred at ever higher levels. As the lesson study progressed, with each meeting held, *decision-making* was permeated by longer discussions and more comprehensive reflections.

This indicates that collaborative *decision-making* presupposes a welcoming context in which everyone feels a sense of belonging, that they are listened to and valued.

In short, *professional sharing* and *decision-making* processes were catalysts that strengthened professional collaboration, enabling the participants to reflect on: aspects that interfere with student learning, different strategies for and alternative ways of teaching, and on the observation and analysis of student learning (Cerbin, 2011) in calculus. These reflections enabled the group to propose a task with the potential to deepen the study of the topic 'maximums and minimums' and help students overcome their difficulties in this subject.

The analysis shows that lesson study is based on collaboration and simultaneously enhances it as part of the professional culture, because it establishes a learning community that mobilizes different principles of teaching professionalism, such as the basic knowledge that supports teaching, values and ways of conducting university teaching, and the autonomy of professors. In this sense, the dynamics of lesson study trigger a process of cultural transposition by modifying professional dispositions (Lewis, 2012) and deconstructing established practices, reconsidering aspects related to the aims and intentions (Mellone et al., 2019) of university teaching.

A possible limitation of this study is that we only conducted and analyzed one lesson study with this group, which may have restricted our understanding of the collaboration that took place among the participants. We believe that conducting a second lesson study with these teachers could mobilize other collaborative principles and foster new understandings.

Another limitation concerns the categories covered. The analysis pointed to various principles of collaboration, such as dialog, trust, and mutual support; however, we chose to address *professional sharing* and *decision-making*, because both permeated each stage of the lesson study in a complementary way. However, future research could analyze the complementarity between other principles, such as dialogue and overcoming professional hierarchies.

Finally, considering that collaboration between university teachers in lesson study is approached from a specific theoretical and methodological perspective, we believe that new understandings can be developed by adopting other theoretical and methodological bases, which suggests the need for further research on this subject. For example, professional collaboration in lesson study has been a context for the development of principles that are fundamental to teaching mathematics in schools and universities, such as the principle of creativity (Seckel et al., 2019), modelization as a principle of fostering relevant mathematical processes (Ledezma et al., 2023), and general principles of mathematics didactics in lesson study according to the onto-semiotic approach (Breda et al., 2018; Font et al., 2023).

Author contributions: All authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: This study received funding from Conselho Nacional de Desenvolvimento Científico e Tecnológico [National Council for Scientific and Technological Development].

Acknowledgements: The authors would like to thank the eight teachers who participated in this lesson study for their intense engagement in the process and invaluable contribution to this research.

Ethical statement: The authors stated that the study is in line with the ethical criteria established by law and was approved by the Research Ethics Committee on 10 June 2021 (Process number 402748/2021-2). Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

REFERENCES

Alvine, A., Judson, T. W., Schein, M., & Yoshida, T. (2007). What graduate students (and the rest of us) can learn from lesson study. *College Teaching, 55*(3), 109-113. https://doi.org/10.3200/CTCH.55.3.109-113

Bødker, S., & Kyng, M. (2018). Participatory design that matters: Facing the big issues. *ACM Transactions on Computer-Human Interaction, 25*(1), 4. https://doi.org/10.1145/3152421

Borko, H., & Putnam, R. (1995). Expanding a teacher's knowledge base: A cognitive psychological perspective on professional development. In T. Guskey, & M. Hubermann (Eds.), *Professional development in education: New paradigms and practices* (pp. 35-66). Teacher College Press.

Breda, A., Font, V., & Pino-Fan, L. R. (2018). Criterios valorativos y normativos en la didáctica de las matemáticas: El caso del constructo idoneidad didáctica [Evaluative and normative criteria in mathematics teaching: The case of the didactic suitability construct]. *Boletim de Educação Matemática* [*Mathematics Education Bulletin*], *32*(60), 255-278. https://doi.org/10.1590/1980-4415v32n60a13

Brodie, K. (2021). Teacher agency in professional learning communities. *Professional Development in Education, 47*(4), 560-573. https://doi.org/10.1080/19415257.2019.1689523

Burroughs, E., & Luebeck, J. (2010). Pre-service teachers in mathematics lesson study. *The Montana Mathematics Enthusiast, 7*(2-3), 391-400. https://doi.org/10.54870/1551-3440.1196

Cavadas, B., & Branco, N. (2023). Science and mathematics teacher collaboration in higher education: A pedagogical experience using inquiry learning spaces. In B. Cavadas, & N. Branco (Eds.), *Handbook of research on interdisciplinarity between science and mathematics in education* (pp. 196-223). IGI Global. https://doi.org/10.4018/978-1-6684-5765-8.ch009

Cerbin, B. (2011). *Lesson study: Using classroom inquiry to improve teaching and learning in higher education*. Stylus Publishing.

da Ponte, J. P. (2003). Investigar, ensinar e aprender [Investigate, teach and learn]. In *Actas do ProfMat 2003* (pp. 25-39). APM.

da Ponte, J. P., Quaresma, M., Mata-Pereira, J., & Baptista, M. (2016). O estudo de aula como processo de desenvolvimento profissional de professores de matemática [Lesson study as a professional development process for mathematics teachers]. *Boletim de Educação Matemática* [*Mathematics Education Bulletin*], *30*(56), 868-891. https://doi.org/10.1590/1980-4415v30n56a01

Day, C. (2001). *Desenvolvimento profissional de professores: Os desafios da aprendizagem permanente* [*Teacher professional development: The challenges of lifelong learning*]. Porto Editora.

Denzin, N. K., & Lincoln, Y. S. (2006). *O planejamento da pesquisa qualitativa: Teorias e abordagens* [*Qualitative research planning: Theories and approaches*]. Artmed.

Font, V., Calle, E., & Breda, A. (2023). Uso de los criterios de idoneidad didáctica y la metodología lesson study en la formación del profesorado de matemáticas en España y Ecuador [Use of the didactic suitability criteria and the lesson study methodology in the training of mathematics teachers in Spain and Ecuador]. *Paradigma, 44*(2), 376-397. https://doi.org/10.37618/PARADIGMA.1011-2251.2023.p376-397.id1424

Fujii, T. (2016). Designing and adapting tasks in lesson planning: A critical process of lesson study. *ZDM Mathematics Education, 48*, 411-423. https://doi.org/10.1007/s11858-016-0770-3

Guskey, T. (1997). Research needs to link professional development and student learning. *Journal of Staff Development, 18*(2), 36-40.

Hargreaves, A. (1998). *Professores em tempo de mudança*: *O trabalho e a cultura dos professores na idade pós-moderna* [*Teachers in times of change: The work and culture of teachers in the postmodern age*]. McGraw-Hill.

Huang, R., Li, Y., Zhang, J., & Li, X. (2011). Developing teachers' expertise in teaching through exemplary lesson development and collaboration. *ZDM Mathematics Education, 43*(6-7), 805-817. https://doi.org/10.1007/s11858-011-0365-y

Ledezma, C., Breda, A., & Font, V. (2023). Prospective teachers' reflections on the inclusion of mathematical modelling during the transition period between the face-to-face and virtual teaching contexts. *International Journal of Sciences and Mathematics Education*. https://doi.org/10.1007/s10763-023-10412-8

Lewis, C. (2002). *Lesson study: A handbook of teacher-led instructional change*. Research for Better Schools.

Lewis, C. (2012). Schools where teachers learn from each other. *Child Research Net, 28*.

Lima, J. A., & Fialho, A. (2015). Colaboração entre professores e perceções da eficácia da escola e da dificuldade do trabalho docente [Collaboration between teachers and perceptions of school effectiveness and the difficulty of teaching work]. *Revista Portuguesa de Pedagogia* [*Portuguese Journal of Pedagogy*], *49*(2), 27-53. https://doi.org/10.14195/1647-8614_49-2_2

Mellone, M., Ramploud, A., Di Paola, B., & Martignone, F. (2019). Cultural transposition: Italian didactic experiences inspired by Chinese and Russian perspectives on whole number arithmetic. *ZDM Mathematics Education, 51(1),* 199-212. https://doi.org/10.1007/s11858-018-0992-7

Morris, J. C., & Miller-Stevens, K. (2016). *Advancing collaboration theory: Models, typologies, and evidence*. Routledge. https://doi.org/10.4324/9781315749242

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L. Hart, A. Alston, & A. Murata (Eds.), *Lesson study research and practice in mathematics education* (pp. 1-12). Springer. https://doi.org/10.1007/978-90-481-9941-9_1

Najikhah, F., Rondli, W. S., Haryati, H., Astuti, D., & Suningsih, S. (2023). Improving the quality of education and professionalism of teachers through lesson study. *Journal Scientia, 12(03)*, 2184-2190.

Nguyen, D. T., & Tran, D. (2022). High school mathematics teachers' changes in beliefs and knowledge during lesson study. *Journal of Mathematics Teacher Education, 26*, 809-834. https://doi.org/10.1007/s10857-022-09547-2

Pischetola, M., Møller, J. K., & Malmborg, L. (2023). Enhancing teacher collaboration in higher education: The potential of activity-oriented design for professional development. *Education and Informatics Technologies, 28*(6), 7571-7600. https://doi.org/10.1007/s10639-022-11490-x

Plauborg, H. (2009). Opportunities and limitations of learning within teachers' collaboration in teams: Perspectives from action learning. *Action Learning: Research and Practice, 6*(1), 25-34. https://doi.org/10.1080/14767330902731293

Quaresma, M., & da Ponte, J. P. (2021). Developing collaborative relationships in lessonstudy. *PNA, 15*(2), 93-107. https://doi.org/10.30827/pna.v15i2.16487

Richit, A. (2021). Desenvolvimento profissional de professores: um quadro teórico [Teacher professional development: A theoretical framework]. *Research, Society and Development*, *10*(14), 1-15. https://doi.org/10.33448/rsd-v10i14.22247

Richit, A. (2023). Professional development of professors in lesson study. *Educação Unisinos*, *27*, 1-20. https://doi.org/10.4013/edu.2023.271.20

Richit, A., & da Ponte, J. P. (2017). La colaboración docente en estudios de clase en la perspectiva de profesores participantes [Teaching collaboration in class studies from the perspective of participating teachers]. *Paradigma (Maracay)*, *38*(1), 330-352. https://dialnet.unirioja.es/servlet/articulo?codigo=7302923&orden=0&info=link

Richit, A., & da Ponte, J. P. (2019). A colaboração profissional em estudos de aula na perspectiva de professores participantes [Professional collaboration in lesson studies from the perspective of participating teachers]. *Bolema, 33(64)*, 937-962. https://doi.org/10.1590/1980-4415v33n64a24

Richit, A., da Ponte, J. P., & Tomasi, A. P. (2021a). Aspects of professional collaboration in a lesson study. *International Electronic Journal of Mathematics Education, 16*(2), em0637. https://doi.org/10.29333/iejme/10904

Richit, L. A., Richit, A., Richit, A., Teilor, B. A., Pedroso, C. A., Melo, M. V., Agranionih, N. T., Neves, R. B., & Zimer, T. T. B. (2021b). The paddock problem: Didactic material created in a Lesson Study to investigate the topic of maxima and minima in Calculus. *Zenodo* [Lesson, 2021, July 27]. https://doi.org/10.5281/zenodo.5140590

Richit, A., da Ponte, J. P., & Tomkelski, M. L. (2019). Estudos de aula na formação de professores de matemática do ensino médio [Class studies in the training of high school mathematics teachers]. *Revista Brasileira Estudos Pedagógicos* [*Brazilian Journal Pedagogical Studies*]*, 100*(254), 54-81. https://doi.org/10.24109/2176-6681.rbep.100i254.3961

Richit, A., Richit, L. A., & Teilor, B. A. (2023). Abordagem de máximos e mínimos em um curso universitário de cálculo [Approach of maxima and minima in a university course in calculus]. *Boletim de Educação Matemática*, *37*(77), 1036-1062. https://doi.org/10.1590/1980-4415v37n77a06

Richit, A. & Tomkelski, M. L. (2020). Secondary school mathematics teachers' professional learning in a lesson study. *Acta Scientiae - Revista de Ensino de Ciências e Matemática*, *22*(3), 2-27. https://doi.org/10.17648/acta.scientiae.5067

Richit, A., & Tomkelski, M. L. (2022). Meanings of mathematics teaching forged through reflection in a lesson study. *Eurasia Journal of Mathematics, Science and Technology Education*, *18*(9), em2151. https://doi.org/10.29333/ejmste/12325

Robutti, O., Cusi, A., Clark-Wilson, A., Jaworski, B., Chapman, O., Esteley, C., Goos, M., Isoda, M., & Joubert, M. (2016). ICME international survey on teachers working and learning through collaboration: June 2016. *ZDM Mathematics Education, 48*(4), 651-690. https://doi.org/10.1007/s11858-016-0797-5

Romeu, T., Guitert, M., & Sangrà, A. (2016). Teacher collaboration network in higher education: Reflective visions from praxis. *Innovations in Education and Teaching International, 53*(6),592-604. https://doi.org/10.1080/14703297.2015.1025807

Seckel, M. J., Breda, A., Sánchez, A., & Font, V. (2019). Criteria assumed by teachers when arguing about mathematical creativity. *Educação e Pesquisa* [*Education and Research*], *45*. https://doi.org/10.1590/S1678-4634201945211926

Stigler, J. W., & Hiebert, J. (2016). Lesson study, improvement, and the importing cultural routines. *ZDM Mathematics Education, 48*(4), 581-587. https://doi.org/10.1007/s11858-016-0787-7

Takahashi, A., & Mcdougal, T. (2016). Collaborative lesson research: Maximizing the impact of lesson study. *ZDM Mathematics Education, 48*(4), 513-526. https://doi.org/10.1007/s11858-015-0752-x