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Pre-service Secondary Teachers' Mathematical Pedagogical Content Knowledge Self-concept related to their Content Knowledge of Functions and Students

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ABSTRACT

Pre-service teachers' beliefs, attitudes, and values about mathematics have a significant influence on their self-concept about mathematics as a subject and determine how confident they are to teach it. The purpose of this study was to examine Zambian pre-service secondary mathematics teachers' pedagogical content knowledge self-concept about the function concept in relation to their knowledge of content and students. Data was collected from 150 pre-service teachers using a sequential approach in two phases. The first, quantitative phase, involved 150 pre-service teachers who responded to a functions survey and a mathematical pedagogical content knowledge survey. The second, qualitative phase, involved two pre-service teachers who were purposively selected from phase one to respond to vignettes and interviews for an in-depth understanding of their knowledge. Results of the study revealed that pre-service teachers' level of their pedagogical content knowledge self-concept was low. They would not be confident enough to teach the function concept in secondary school. Results further revealed that their knowledge of content and students was weak. A weak correlative relationship between pre-service teachers' KMLS and KM was revealed whereas a moderate correlative relationship of their KL and KC was revealed. It was further revealed that there was no significant correlative relationship between KTS and their knowledge of the function concept Thus, pre-service teachers needed to improve before leaving university for them to effectively teach secondary school concepts.

Keywords: pre-service teachers' self-concept, perceptions, mathematical pedagogical content knowledge, concept of a function

INTRODUCTION

Studies on pre-service teachers' pedagogical content knowledge (PCK) of mathematics have continued to occupy considerable space in teacher education research (Even, 1992; Hitt & Kontorovich, 2017). The concept of PCK was first propounded by Shulman (1986). In his conceptualization Shulman emphasized the need for teachers to have strong knowledge of subject matter and pedagogical knowledge for effective teaching to take place. He viewed an effective teacher as one who possessed strong subject matter knowledge with a solid knowledge base of teaching strategies.

In developing his initial ideas about PCK, Shulman (1987) listed and placed PCK as being equal to other knowledge domains for teaching namely content knowledge, curriculum knowledge, knowledge of students, general pedagogical knowledge, knowledge of educational history, values and philosophy, and knowledge of educational system (cited in Aksu & KUL, 2016). Shulman's ideas were then developed by mathematics

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education researchers in their quest to improve teacher education (Fennema & Franke,1992; Hill, Ball, & Schilling, 2008; Kulm & Wu, 2004). However, in the study of pre-service teachers' PCK little attention has been paid to PCK perceptions as they relate to the concept of a function.

The concept of a function is regarded as a key topic in the Zambian secondary school mathematics curriculum because knowledge of function concepts is essential in students' understanding of related topics in calculus and trigonometry (Marban & Sintema, 2020). Thus, pre-service teachers' knowledge of teaching strategies that would enable them teach function concepts effectively emerges as an important aspect of teacher education. Studying pre-service teachers' Mathematical pedagogical content knowledge (MPCK) in relation to the concepts of a function would inform teacher education providers about the awareness and confidence levels of pre-service teachers when confronted with mathematical concepts about the topic. It would also provide information about areas of MPCK that need more attention and can initiate revision of course materials in higher learning institutions aimed at improving pre-service teachers' MPCK skills.

Pre-service Teachers' MPCK Self-concept

Teachers' self-concept about their knowledge of subject-matter play a key role in how well or bad they will implement the mathematics curriculum. For mathematics which has often been a major cause of fear and anxiety among secondary school students (Jeffery et al., 2018) and pre-service teachers (Bates, Latham, & Kim, 2011), it is important to continue examining pre-service mathematics teachers' PCK perceptions in relation to their subject-matter knowledge if we are to achieve a quality and effective pool of teachers in Zambia. The Zambian educational system places great importance on the instructional practices that are used by teachers in classrooms because they are directly related to student outcomes (Ministry of Education, 2013). Good instructional practices that accommodate students' instructional and subject-matter needs are preferred because they usually have a positive impact on student achievement.

Newton et. al (2012) in their study to investigate relationship between mathematics subject matter and teacher efficacy among pre-service teachers found that their teaching of mathematics was largely influenced by low confidence. Gresham (2009) in his study about pre-service teachers' mathematics anxiety and teacher efficacy found a correlation between pre-service teachers' anxiety and their liking of mathematics. The pre-service teachers with a high degree of mathematics anxiety were found to have a low teacher efficacy and were negative about the subject. Thus, teachers' perceptions are important in determining the preparedness of mathematics pre-service teachers to embark on a teaching career (Elmahdi & Fawzi, 2019).

In their study of mathematics teachers content knowledge and PCK in problem posing, Lee et al. (2018) focused on teachers' knowledge of mathematics content, knowledge of content and students and knowledge of content and teaching. The study revealed good content knowledge of problem posing for the participating teachers. With regards to knowledge of content and students, and knowledge of content and teaching, it was revealed that teachers' level of awareness was good despite some observed obstacles that inhibit the effective use of problem posing in class. To increase PCK awareness among pre-service teachers, it is important to introduce pedagogical courses from first year when pedagogy is to be taught side-by-side with discipline specific content or to have a pedagogical certified program which students will enroll in after graduating with a degree in mathematics (Matos, 2020). This would equip pre-service teachers with necessary pedagogical skills needed for effective teaching of mathematics (Hine & Thai, 2019). In their study to investigate prospective mathematics teachers' self-perception of readiness to teach secondary school mathematics (Hine & Thai, 2019) argued that effective teaching of mathematics would require mathematics teachers to have significant level of mathematics subject matter knowledge and mathematical pedagogical knowledge. They concluded that prospective teachers' needed extra training to develop their subject matter and pedagogical knowledge required for them to adequately teach high school mathematics concepts.

Sanchez-Jimenez (2020) discussed PCK from the perspective of development of professional knowledge for mathematics teachers. They claimed that professional mathematics knowledge for teachers should be characterized as *mathematics knowledge to teach* and *mathematics knowledge for teaching*. The two are crucial in the professional training of a mathematics teacher. The former is concerned with the learning institution where the actual learning occurs whereas the latter refers to the tools the teacher uses in the action of teaching. This perspective is important for teacher training and development of PCK for mathematics teachers because it enhances PCK awareness among mathematics teachers. In fact, in acknowledging the importance of knowledge to teach and knowledge for teaching da Costa (2020) in his study that investigated knowledge to teach arithmetic concluded that not only was knowledge for teaching structured but also incorporated pedagogical opportunities expressed in the knowledge to teach.

In their study to explore pre- and in-service mathematics and science teachers' technological pedagogical content knowledge (TPACK) Airwaished et al. (2017) developed a framework that would be used to capture important qualities of the kind of knowledge necessary for teachers' effective pedagogical practice in an environment that supports use of technology in teaching. This framework was useful in improving in-service teachers' knowledge in some aspects of TPACK. Kim's (2017) study about the relationship between pre-service teachers' TPACK and their beliefs found that pre-service teachers who believed in student centered approaches to learning mathematics and technology use in teaching exhibited high levels of mathematics knowledge and TPACK than their counterparts whose belief about teaching was teacher centered. In a similar study Usak et al. (2013) found that pre-service teachers who enrolled in courses where instructors were learner centered had high achievement and developed positive attitudes toward learning than those whose instructors were teachers centered.

In a study to examine pre-service teachers' knowledge for teaching mathematics, Beswick and Goos (2012) found that teachers with low confidence levels exhibited low knowledge and were less aware of their mathematical knowledge. In another study aimed at assessing pre-service science teachers' PCK and subject matter knowledge Usak et al. (2011) found that pre-service teachers had gaps in subject matter knowledge and were teacher centered in their approach to teaching. They also exhibited teaching attitudes that were biased toward memorization of facts at the expense of developing process- oriented skills which are important for learning. In another study aimed at assessing the relationship between teaching experience and PCK, Duran and Usak (2015) revealed that teaching experience had a positive impact on the pedagogical content knowledge of a science teacher although teachers with several years of teaching experience had difficulties with their content knowledge. In a study to examine the interplay between teacher beliefs and teacher knowledge Blomeke et al. (2014) revealed that MPCK influenced teachers' beliefs in the sense that it made teachers to have more constructivist beliefs about teaching. The study concluded that teacher education which strengthened pre-service teachers' MPCK developed constructivist teacher beliefs.

Pre-service Teachers' Knowledge of the Concept of a Function

There is research evidence in literature on studies focused on the concept of a function in Zambia (Malambo, 2019; Marban & Sintema, 2020; Sintema, Phiri, & Marban, 2018) and abroad (Even, 1992; Kontorovich, 2017; Ozgen, 2010; Paoletti, 2020; Ubah & Bansilal, 2018; Wasserman, 2017). Studies that have been conducted in Zambia have investigated pre-service teachers' knowledge of the function concept on a wider basis which included but not limited to definition of a function, quadratic, composite, inverse, one-to-one and different representations of functions. Studies in contexts outside Zambia focused on specific aspects of the function concept where pre-service teachers had difficulties, with other studies extending to secondary school students' difficulties.

Pre-service teachers have been reported in previous studies of having difficulties with the concept of domain and range of a function (Anold, 2004; Aziz & Kurniasih, 2019; Dorko & Weber, 2014). They exhibited misconceptions and could hardly define domain and range. They often mixed up and confused the two concepts. This is in spite of research evidence showing that strong knowledge of domain and range would improve their comprehension of inverse functions as well as linear transformations. Pre-service teachers were found to have weak knowledge of inverse functions and could not make meaningful connections between inverse functions and other functions. They were also unable to sufficiently explain notation used to denote inverse functions (Even, 1992; Kontorovich, 2017; Paoletti, 2020; Wasserman, 2017).

Pre-service teachers have in prior research showed weaknesses in their knowledge of composite functions (Kontorovich, 2017; Ozgen, 2010). It was observed that they had misconceptions about composition of functions and could confuse composition of functions with ordinary multiplication of two algebraic terms. This was an indication that some of them were not ready to teach composite function concepts in secondary school. Their knowledge of quadratic functions was also reported to be weak (Aziz & Kurniasih, 2019; Huang & Kulm, 2012). As a result they were unable to select good representations involving quadratic functions for the learners. Their inadequate knowledge of this concept would affect their teaching in future if they did not improve.

One of the most researched aspects of the function concept in relation to pre-service teachers is their knowledge of different representations of the concept of a function (Aziz & Kurniasih, 2019; Dorko & Weber, 2014; Gagatsis & Shiakalli, 2004; Martinez-Planell, Gaisman, & McGee, 2015). Different representations of the function concept usually take the form of tables, algebraic symbols, ordered pairs and graphical representations. Pre-service teachers have had difficulties in translating from one representation to another and this inability to flexibly move between representations has been a major gap in their knowledge of functions. Their weak knowledge of flexibility to translate between different representations was not as one of the reasons they had weak mathematical reasoning. The inability to understand different representations of functions was also noted in secondary school students. Secondary school students reported difficulties completing tasks involving different representations (Elia et al., 2007; Hitt, 1998).

Summary of the Theoretical Background of the Study

This study used the theoretical lens of the mathematical knowledge for teaching (MKT) framework (Ball et al, 2008) in defining and understanding teacher knowledge of content and students. The MKT framework which was developed based on Shulman's (1986) PCK framework was basically designed to help mathematics education researchers understand knowledge required for mathematics teachers to effectively teach the subject.

The concept of PCK

Shulman (1986) expounded teacher knowledge by cataloging three types of knowledge namely (i) subject matter content knowledge, (ii) pedagogical content knowledge and (iii) curriculum knowledge. Subject matter content, in the case of mathematics, is concerned with the content structure of the subject which teachers must master and unpack for the learners in a way that enables learners to easily grasp mathematics concepts. This knowledge is discipline specific. He referred to PCK as knowledge of the teachability of content. In explaining the concept of PCK, Shulman (1986) reflected on the aspect of teacher's knowledge of instructional strategies. He posited that teacher's knowledge of instructional strategies referred to "the ways of representing and formulating the subject that make it comprehensible" … to students … by using "examples, illustrations, analogies, explanations and demonstrations" that satisfy the cognitive needs of learners.

The other aspect of PCK he alluded to, related to teachers' knowledge of students. This involved teacher's knowledge of "conceptions, misconceptions and pre-conceptions that students of different ages and backgrounds bring with them to the learning of *mathematics*" (p. 9). He postulated that content knowledge was as important to the preparation of a teacher as pedagogical knowledge and emphasized that the two constructs needed not to be studied as isolated components. He explained that PCK was a representation of the "blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organized, represented, and adapted to diverse interests of learners, and presented for instructions" (Shulman, 1987, p.8). Shulman's initial PCK ideas led many researchers to advance his ideas by coming up with different PCK models (Ball et al., 2008; Carlson & Daechler, 2019; Gess-Newsome, 2015; Grossman, 1990; Magnusson et al, 1999; Mark, 1990). The MKT by Ball and colleagues is of central importance to the current study.

The MKT framework

The MKT (**Figure 1**) is composed of two main knowledge domains, subject-matter knowledge and pedagogical content knowledge. Subject-matter knowledge is further divided into *common content knowledge (CCK)*, *specialized content knowledge (SCK)* and *horizon content knowledge (HCK)*. Pedagogical content knowledge, on the other hand was sub-divided into *knowledge of content and students (KCS)*, *knowledge of content and teaching (KCT)* and *Knowledge of content and curriculum (KCC)*. Some researchers built on the MKT framework to design their own frameworks (Steele et al., 2013). According to Ball et al. (2008)

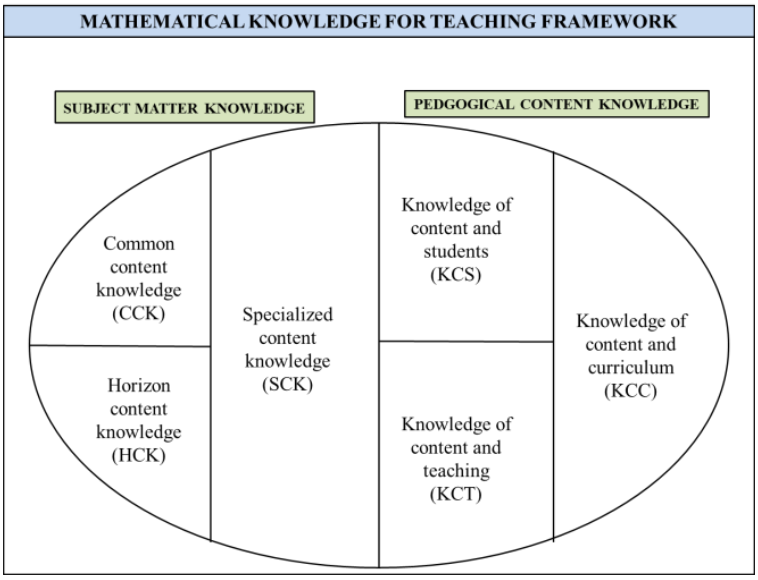


Figure 1. Mathematical knowledge for teaching framework (Ball et al., 2008)

- CCK is not unique to teachers. It is the kind of knowledge that other professionals possess and can be used by any educated person in a similar manner that a teacher uses it. For the purposes of teaching CCK is the commonly used knowledge to teachers during classroom teaching. In defining concepts, solving examples for students teachers mostly tap from CCK.
- SCK is a special kind of knowledge that is unique to teachers. This kind of knowledge equips teachers with skills that enable them to perform mathematical activities that other professionals cannot do. It enables them to identify patterns in student errors and devise ways of resolving such errors. SCK is specific to mathematics teaching and is not relevant to other fields.
- KCT refers to a blend of teachers' knowledge of mathematics subject matter and pedagogy. Teachers need to know a variety of teaching approaches and methods that would help in the effective teaching of mathematics concepts. KCT equips teachers with the ability to use appropriate teaching strategies in the delivery of concepts.
- KCS is a blend of teachers' knowledge of mathematics subject matter and that of learners' difficulties, errors and misconceptions. The teacher develops knowledge of understanding leaner mathematical characteristics that would help them learn effectively. KCS enables teachers anticipate, identify and resolve learners' misconceptions about mathematics.

This study exclusively focused on the knowledge of content and students (KCS) to study pre-service teachers' knowledge for teaching the concept of a function. In the context of this study KCS refers to pre-service teachers' knowledge of functions concept subject-matter and their knowledge of secondary students' misconceptions, difficulties and errors related to the function concept. It is hypothesized in this study that for a teacher to effectively teach the function concept he/she must have strong knowledge of the function concept as well as advanced knowledge of strategies that can best be used to teach it.

Recognizing that perception play a very important role learning mathematics, the purpose of this study was to examine Zambian pre-service secondary mathematics teachers' pedagogical content knowledge perceptions of the function concept in relation to their knowledge of content and students. Their PCK perceptions would correlate to their actual knowledge of subject-matter and their ability to identify and resolve students misconceptions and errors involving the concept of a function. This study is important to the improvement of mathematics teacher education in Zambia in the sense that it brings out some of the areas where pre-service teachers have difficulties with the function concept. In accomplishing its purpose, this study sought to answer questions about the level of PCK perceptions of pre-service teachers and their knowledge level about the ability to identify and resolve students' misconceptions and difficulties related to the function concept. The main questions this study sought to answer were:

1. What level of MPCK related to knowledge of functions and students is held by Zambian pre-service secondary mathematics teachers?

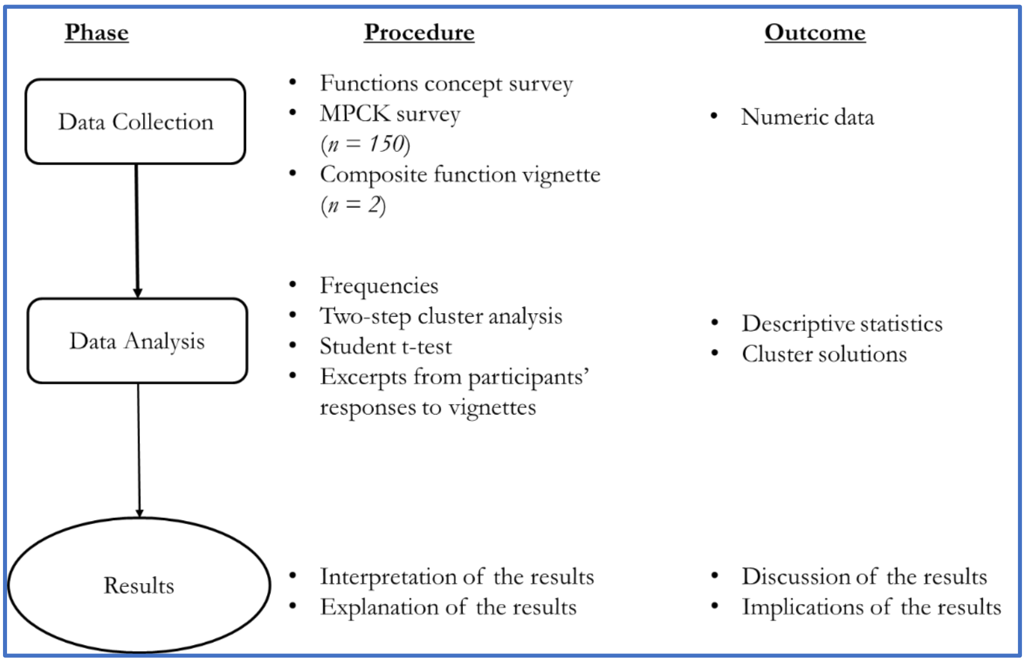


Figure 2. Visual representation of the research design procedure for this study (adapted from Creswell, 2014)

2. Is there a positive relationship between pre-service teachers' MPCK self-concept and knowledge of the concept of a function?

METHODOLOGY

This study is part of a large project that is currently ongoing. The project from which this study emanates focuses on Zambian pre-service mathematics teachers' subject-matter and pedagogical content knowledge which used a sequential explanatory mixed-method approach (Creswell, 2014) as its overall design. A mixed methods design was chosen because of its ability to provide an in-depth and complete understanding of the research problem which neither qualitative nor quantitative approaches can offer (Creswell, 2014).

Having already published two papers from the project (Marban & Sintema, 2020; Sintema et al., 2018), this study reports results of pre-service teachers' MPCK perceptions related to their content knowledge of functions and students. This study sought to answer a question concerned with pre-service mathematics secondary teachers' MPCK related to the concept of a function. This was revealed in their response to a vignette on composite functions which is used in this study. A visual representation (**Figure 2**) was developed to illustrate the research design procedures that were used in this study.

Participants

A total of 150 university students majoring in mathematics education were recruited from two public universities in Zambia's Copperbelt province. The participants were all third and fourth year students who had adequately covered the concept of a function in their university curricula, and having attended secondary school education in Zambia it was assumed that they also learned the topic at high school and were well oriented to the Zambian mathematics secondary school curriculum. All the 150 participants took part in responding to the MPCK and functions surveys while 2 of the 150 participants participated in responding to vignettes. In this study sample answers to one of the vignettes from two participants were presented.

Instruments

As data collection tools, the project on which this study is anchored used more than three instruments. This study presents results of the data collected using vignettes, MPCK self-concept survey and a pencil and paper test on functions. All the three instruments were subjected to a comprehensive validation process prior to being used to the data collection phase of this study. The validation process reported in this paper was performed by the authors because items that formed the instruments were adapted from priovious studies and

Table 1. Sample of pre-service teachers' MPCK self-concept survey items (Adapted from Bukova-Guzel et al., 2013)

MPCK factor	Sample items
KTS	I can design appropriate activities to present mathematical concepts
	I can use analogies to present mathematical concepts in instruction
KMLS	I can use mathematical language properly when presenting mathematical concepts
	I can use mathematical symbols properly
KM	I can anticipate students' possible difficulties about a topic
	I can design activities that will not cause students to develop misconceptions about the topic
KL	I know students' prior knowledge about the topic
	I can choose appropriate examples for students' developmental levels in my lesson
KC	I plan my lessons as to relate the purposes of the mathematics curriculum with students' needs
	I can evaluate the effectiveness of the activities I use in class for students' conceptual understanding

1. Given that $g(x) = \frac{3}{2x+1}$ and $z\colon\ x \to x^2 - 2x$,

   (a) Find the value of $g^{-1}(-5)$

   (b) State, with justification, two domains on which the function $z\colon\ x \to x^2 - 2x$ has an inverse.

   (c) Find an expression for $(z \circ g)(x)$ where $z \circ g$ denotes the composite function of z and g.

   (d) Evaluate $(g \circ g^{-1})(-5)$ where -5 belongs to the domain of g^{-1}.

Figure 3. Sample item of the MPCK functions survey (Adapted from Malambo, 2016)

2. A student is asked to give an example of a graph of a function that passes through the points A and B (See Figure 1).

   The student gives the following answer (See Figure 2).

Figure 1 Figure 2

   When asked if there is another answer the student says "No".

   (a) If you think the student is right, explain why.

   (b) If you think the student is wrong, how many functions which satisfy the condition can you find? Explain.

Figure 4. Sample item of the MPCK functions survey (Adapted from Even, 1993)

need to fit the context of the current study. The MPCK self-concept survey designed to measure pre-service mathematics teachers perceptions related to their PCK which was developed by Bukova-Guzel, Canturk-Gunhan and Kula (2013) was adapted to the Zambian context for this study. The survey was composed of five factors: Knowledge of Teaching Strategies (KTS), Knowledge of Mathematical Language and Symbols (KMLS), Knowledge of Misconceptions (KM), Knowledge of Learners (KL) and Knowledge of Curriculum (KC). A functions survey comprising 35 items across 9 questions used in previous studies (Even, 1990, 1993; Malambo, 2016; Watson, Ayalon, & Lerman, 2018; You, 2010) was developed for this study. Sample items of the MPCK self-concept survey are shown in **Table 1** while those for the MPCK functions survey are displayed in **Figures 3** and **4**.

A teacher gave the definition of the composite function and explained it on the board to his/her students. However, some of his/her students stated that they did not understand it completely. Then teacher gave the following example to the students.

In order to clean and dry our clothes in a laundry we use two machines, washing machine and dryer, respectively. Dry&Wash(clothes)

Dry[Wash(clothes)]=Dry[cleaned and wet clothes]=dried and cleaned clothes Combination of these machines works can be considered as a composition of functions

What do you think of this example?

Can this example cause students to misunderstand any points in the definition?

If exists, please explain these points.

If you were to explain the composite function by using a real life example, what will be your example? Explain how you will use it in class.

Figure 5. Vignette for assessing pre-service teachers' knowledge of composition of functions (Karahasan, 2010)

The study also adapted 8 vignettes on functions from previous studies (Ebert, 1994; Karahasan, 2010). The vignettes were used in previous studies (Ebert, 1994) to examine teachers' PCK of mathematics. In this study we are showing the results from one of the vignettes (**Figure 5**). The vignette was chosen because it was used previously (Karahasan, 2010) to examine pre-service teachers' PCK and it fits the Zambian context in the sense that definitions of concepts is part of the Zambian syllabus. This vignette was aimed at assessing pre-teachers' ability to relate composition of functions to real life. This is because when students fail to understand an abstract definition of a mathematics concept, teachers should be able to tap from the students' everyday life examples to enhance understanding.

Face and Content Validity of the Instruments

To ensure suitability for use in the Zambian context, the instruments were subjected to face and content validity. To accomplish this process we sought opinions of mathematics education experts from a public university and colleges of education who had at least three years secondary school teaching experience prior to their university teaching careers. This was because the experts needed to be familiar with the Zambian mathematics secondary school curriculum.

Validation of the vignettes

Evaluation of the face validity of the instrument where this vignette (**Figure 2**) came from was accomplished based on the level of agreement of 10 experts. The instrument was composed of 12 vignettes designed to measure teacher PCK of the function concept. Based on the comments of experts and 75% level of agreement of experts on the suitability of the vignettes to the Zambian contexts, 1 vignettes was removed from the instrument because they were below 75% level of agreement (Ohanian, 1990). The remaining 11 vignettes which met the agreement threshold formed the pre-final version of the instrument. The vignettes in the pre-final version of the instrument were not edited in any way because the experts recommended that they be used in the state in which they were. The pre-final version was then subjected to content validity evaluation.

The instrument was then evaluated for its content validity. This process involved assessment of each vignette for its clarity, relevance and coherence on a 4-point ordinal scale: *1 = does not meet the criteria, 2 = Low level, 3 = Moderate level and 4 = High level*. This was to ensure that the wording of the vignettes was clearly done without any ambiguities and each vignette was relevant to measuring MPCK. The entire instrument from which this vignette was picked was also evaluated for its sufficiency to thoroughly measure MPCK based on the same scale as other constructs. At this stage only vignettes that were rated 3 or 4 were retained in the final version. Following this criteria only 8 vignettes formed the final version of the instrument. The final version had a scale content validity index (S-CVI) of 0.90 and item content validity index (I-CVI) of 0.80. These validity indices implied that the instrument was valid for use in this study.

Table 2. Composite reliability and average variance extracted (AVE) values

Latent variable	Composite reliability	AVE
KTS	0.897	0.748
KMLS	0.886	0.796
KM	0.818	0.601
KL	0.941	0.889
KC	0.773	0.328

Validation of the MPCK functions survey

The pencil and paper test on functions followed a similar rigorous validation process as the vignettes. The development and validation procedure of the instrument was a two-stage process (Lynn, 1986) namely developmental stage and expert review/Judgment stage. The item pool consisted of test items from previous research on subject matter and pedagogical content knowledge of the function concept (Even, 1990, 1993; Leinhardt, Zaslavsky, & Stein, 1990; Malambo, 2016; Watson, Ayalon, & Lerman, 2018; You, 2010). The first version (pre-final version) comprised a total of twelve questions and 55 items. After going through the two validation stages the final version of the test consisting 35 items spread across 9 questions was established. Some questions in the test were open ended. This was to enable the researcher have in-depth understanding of prospective teachers' knowledge of the function concept.

To evaluate content validity index for the clarity, coherence and relevance of each item (I-CVI), inter-rater agreement with emphasis on the proportion of agreement by the raters was used. This method has the weakness of agreements sometimes being inflated merely by chance. To resolve this likely occurrence Lynn (1986) suggested that for five raters or less, the I-CVI of 1.00 would be accepted as a cut-off. This means that all the five needed to agree on the validity of the item. However, for six or more raters it was suggested that the I-CVI of at least 0.78 would be the accepted cut-off. This criterion was used in this study as a guide to delete, revise or substitute items that failed to meet the minimum threshold. For the purpose of this study only items that recorded ≥ 0.80 validity index were retained in the test. Comments from the raters were also considered when deciding which items to delete from the test. If an item had good item validity but was considered vague by the raters, it was deleted. The item content validity index was found to be 0.86. The S-CVI was calculated from all the items whose rating was 3 or 4 and found to be 0.90. In literature the acceptable threshold for scale content validity is 0.80 (Grant & Davis, 1997; Polit & Beck, 2004; Polit et al., 2007). Thus, the functions survey had acceptable S-CVI.

Pilot testing of vignettes and the MPCK functions survey

The final versions of the functions survey and the vignettes instrument were piloted using 10 pre-service mathematics teachers on separate days. The pre-service teachers who participated in this exercise were excluded from data collection for the main study. The purpose for piloting the two instruments was to check the time it would take to respond to each instrument so that the duration for the final data collection was fairly determined. The other reason was to obtain feedback from subjects whose characteristics were similar to those who would participate in the study. Thus, the pre-service teachers were asked to write down comments about which items they found vague and those which were difficult. They were also requested to state what difficulties they encountered and why they felt that some questions were vague. The results of the pilot test were good with positive feedback from all the participants. To this effect no further changes were introduced to the final versions of the two instruments.

Adaptation process of the MPCK self-concept survey

To adapt the MPCK survey a confirmatory factor analysis was performed to calculate the fit indices of the instrument. In this section we showcase part of the adaptation and validation process by discussing (i) Internal consistency reliability, (ii) Convergent validity and (iii) Discriminant validity of the instrument.

Internal consistency reliability: Internal consistency has commonly been measured using Cronbach's alpha. This has presented challenges such that some scholars suggested the use of composite reliability in PLS-SEM as a preferred measure of internal consistency (Bagozzi & Yi, 1988; Hair et al., 2012, 2014). A composite reliability of at least 0.7 is preferred but a minimum of 0.6 would be acceptable to achieve internal consistency. Examining **Table 2**, it can be seen that all the latent variables KTS, KMLS, KL, KM and KC had composite reliability greater than 0.7. thus, all the latent variables recorded high internal consistency.

Table 3. Fornell-Larcker criterion for checking discriminant validity

	KC	KL	KM	KMLS	KTS	MPCK
KC	**0.573**					
KL	0.060	**0.943**				
KM	0.158	0.314	**0.775**			
KMLS	0.302	0.126	0.322	**0.892**		
KTS	0.235	0.064	0.267	0.355	**0.865**	
MPCK	0.752	0.336	0.594	0.581	0.605	**1.000**

Table 4. Correlation of latent variables

	KC	KL	KM	KML	KTS	MPCK
KC	1.000					
KL	0.060	1.000				
KM	0.158	0.314	1.000			
KML	0.302	0.126	0.322	1.000		
KTS	0.235	0.064	0.267	0.355	1.000	
MPCK	0.752	0.336	0.594	0.581	0.605	1.000

Table 5. Heterotrait-Monotrait ratio (HTMT)

	KC	KL	KM	KML	KTS
KC					
KL	0.195				
KM	0.353	0.421			
KML	0.400	0.164	0.433		
KTS	0.321	0.075	0.358	0.451	
MPCK	0.916	0.355	0.726	0.660	0.671

Convergent validity: Convergent validity is established by considering the Average Variance Extracted (AVE) of each latent variable and an AVE of 0.5 and higher is acceptable (Bagozzi & Yi, 1988; Henseler, Ringle, & Sinkovics, 2009). The latent variables KTS, KMLS, KL and KM all had AVE higher than the acceptable minimum of 0.5. However, the variable KC had the AVE of 0.328 which was far below the minimum. This could have been caused be the poor indicator reliability of the indicators of KC.

Discriminant validity: Discriminant validity referred to how variance in the indicators is able to explain variance in the latent variables (De Sousa Magalhaes et al., 2012). **Table 3** shows indices of the Fornell-Larcker criterion for checking discriminant validity. Fornell and Larcker (1981) proposed that discriminant validity can be achieved by finding the square root of the AVE of each latent variable.

According to Fornell-Larcker (1981) and Chin (1998), if correlation values (**Table 4**) of other latent variables are less than the square root of the AVE then discriminant validity is achieved. Examining the table below it can be seen that all the latent variables except KC had the AVE values larger than the correlations in their columns. Discriminant validity was also checked using the HTMT to resolve the challenge presented by the latent variable KC.

The Heterotrait-Monotrait (HTMT) ratio for determining discriminant validity is said to be more efficient. Henseler, Ringle & Sarstedt (2015) proposed that the HTMT ratio of less than 1.0 meets the threshold for the establishment of discriminant validity. From **Table 5**, all the constructs had their HTMT ratios below 1.0. Thus, discriminant validity was achieved. However, other scholars have proposed even a lower threshold with Gold et al. (2001), and Teo et al. (2008) proposing a 0.90 threshold. A threshold of 0.85 was suggested by Kline (2011).

Table 6 shows the adjusted model fit indices. One of the most important indices for determining a good model fit is the standardized root mean residual (SRMR) which is responsible for measuring the approximate model fit by taking into consideration the difference between the observed correlation matrix and the model implied correlation matrix (Garson, 2016, p. 68). The SRMR less than 0.8 has been recommended to show good model fit (Hair et al., 2014; Hu & Bentler, 1998). However, it has been observed that "a lenient SRMR cut of point of less than 0.10" has been accepted in some instances (Garson, 2016, p. 68). Analyzing the table above it can be seen that the SRMR index for the estimated model was 0.094 which falls short of the "less than" 0.08 cutoff but is within the lenient 0.10 cutoff. Another index which is used to establish good model fit is the normed fit index (NFI) which must be above 0.9 for a good fit. In the case of the model under discussion the NFI was 0.605 which is lower than the cutoff.

Table 6. Adjusted model: summary of adjusted model indices

	Saturated model	Estimated model
SRMR	0.094	0.094
d_ULS	1.500	1.500
d_G	0.983	0.983
Chi-square	453.499	453.499
NFI	0.605	0.605

Table 7. Results of the auto clustering of the two-step cluster analysis of pre-service mathematics teachers

Number of Clusters	Schwarz's Bayesian Criterion (BIC)	BIC Change[a]	Ratio of BIC Changes[b]	Ratio of Distance Measures[c]
1	907.94			
2	788.97	-118.97	1.000	2.39
3	785.65	-3.32	.03	1.79
4	819.29	33.64	-.28	1.29
5	863.43	44.13	-.31	1.30
6	915.87	52.44	-.44	1.06
7	969.99	54.12	-.46	1.33
8	1030.59	60.60	-.51	1.26
9	1095.19	64.61	-.54	1.13
10	1161.63	66.44	-.56	1.01
11	1228.15	66.52	-.56	1.02
12	1294.87	66.72	-.56	1.14
13	1363.27	68.40	-.58	1.09
14	1432.63	69.36	-.58	1.03
15	1502.31	69.68	-.59	1.00

a. The changes are from the previous number of clusters in the table.
b. The ratios of changes are relative to the change for the two cluster solution.
c. The ratios of distance measures are based on the current number of clusters against the previous number of clusters.

Data Analysis

Using SPSS 23 a two-step cluster analysis was used as a major analysis technique for this study to derive clusters of pre-service mathematics secondary teachers based on the five components of the MPCK survey and their score in the pencil and paper test on functions. Descriptive statistics for the resultant clusters were calculated and a student t-test was performed to find out the mean difference of the pre-service teachers' MPCK domains. This was followed by a Pearson's principle moment correlation analysis for examining correlations between MPCK factors and performance in the achievement test. Excerpts from responses to a vignette on composite functions by participants from both clusters were also analysed in this study for the purpose of appreciating how pre-service teachers would address their learners' difficulties, misconceptions and errors when learning the function concept.

RESULTS

This section presents results of pre-service teachers' MPCK self-concept and their knowledge of functions. Pre-service teachers' MPCK self-concept was analyzed using two-step cluster analysis which resulted into two clusters.

Pre-service Teachers' MPCK Self-concept

As depicted in **Table 7**, the two-step auto clustering algorithm revealed that a two cluster solution was the best model for the data. This is because it minimized Schwarz's Bayesian Criterion (BIC) values and the change in them between adjacent numbers of clusters and the largest ratio of distances is for two clusters. Thus, two was the optimal number of clusters that best profiled the data.

The cluster distribution (**Table 8**) shows the composition of the clusters. Clusters 1 and 2 contained 72 and 79 subjects respectively which translate to 47.3 and 52.7 percent of the total number of subjects. The cluster quality was 0.4 which is fair and the ratio of the larger cluster to that of the smaller one is 1.11 which is a very good ratio.

Table 8. Cluster distribution of pre-service mathematics teachers based on their MPCK self-concept

		N	% of Total
Cluster	1	71	47.3
	2	79	52.7
	Total	150	100

Table 9. MPCK cluster profiles of pre-service mathematics teachers' self-concept

		KTS		KMLS		KM		KL		KC		MPCK Test Score	
		Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
Cluster	1	3.48	.89	3.82	.98	3.48	.92	3.44	.83	3.83	.72	41.23	10.56
	2	4.35	.60	4.70	.35	4.34	.49	4.50	.49	4.49	.39	55.80	14.73

Table 10. Results of the independent samples t-test of pre-service teachers' MPCK self-concept by cluster

Factor	Cluster	N	Mean	Std. Dev.	T	Df	Sig.
KTS	Cluster 1	71	3.48	.89	-7.09	148	.000
	Cluster 2	79	4.35	.60			
KMLS	Cluster 1	71	3.82	.98	-7.51	148	.000
	Cluster 2	79	4.71	.35			
KM	Cluster 1	71	3.48	.92	-7.20	148	.000
	Cluster 2	79	4.34	.49			
KL	Cluster 1	71	3.44	.83	-9.58	148	.000
	Cluster 2	79	4.50	.49			
KC	Cluster 1	71	3.83	.72	-7.10	148	.000
	Cluster 2	79	4.49	.39			
MPCK	Cluster 1	71	41.23	10.56	-6.90	148	.000
	Cluster 2	79	55.80	14.72			

Table 9 shows the cluster profiles of the Zambian mathematics pre-service teachers based on their MPCK factors and results of the pencil and paper test on the concept of a function. The MPCK factors that were used to form the clusters are KTS, KMLS, KM, KL and KC.

Cluster 1 is basically composed of pre-service teachers with low levels of self-concept of the MPCK factors when compared with their counterparts in cluster 2. Pre-service teachers in Cluster 1 also performed below average in the MPCK test on functions posting a mean performance of 41.23% with a standard deviation of 10.56. Those in Cluster 2 posted a mean performance of 55.80% with standard deviation of 14.73 which was statistically significant [$t(-6.90) = 148$, $p < 0.05$). A very interesting global picture is building from these initial results.

Pre-service teachers who performed below average in the MPCK test on functions also recorded lower scores in their MPCK factors. This implies that pre-service teacher participants with low performance in subject matter are more likely to have low confidence because they also exhibited low levels of MPCK self-concept. Pearson's principle moment correlation analysis (**Table 11**) revealed significant correlations between MPCK sub-factors and performance in the MPCK test on functions except KTS. The pre-service teachers who scored highly in the MPCK test on functions also posted high MPCK self-concept. The pre-service teachers with high level of MPCK self-concept and high scores in the MPCK test on functions represented positive pedagogical and subject matter knowledge development.

Table 10 shows the independence samples t-test results for the mean differences of pre-service teachers' knowledge based on clusters. The results revealed that there was a significant difference between clusters in pre-service teachers' MPCK factors and mean performance in the MPCK test scores on functions. The results further show that the two clusters were dissimilar based on the factors that were used to form them. The results indicate a higher MPCK self-concept mean score for pre-service teachers in cluster 2 than their counterparts in cluster 1. This implies that pre-service teachers in cluster 2 had better pedagogical skills required to teach mathematics that those in cluster 1 in secondary school.

Table 11 shows the results of Pearson's product-moment correlation analysis. This analysis was conducted to examine the relationship between MPCK self-concept sub-factors and the MPCK test scores of pre-service secondary mathematics teachers. As depicted from **Table 6**, there was a significant positive correlation between KMLS, KM, KL, KC and test scores obtained from the functions pencil and paper MPCK test. Although these correlations were positively significant, KMLS ($r = 0.291$, $p = 0.000$) and KM ($r = 0.196$, $p =$

Table 11. Correlations between MPCK factors of pre-service mathematics teachers' self-concept

		KTS	KMLS	KM	KL	KC	MPCK Self-concept	MPCK Test Score
KTS	R	1.00						
	P							
KMLS	R	.388**						
	P	.000						
KM	R	.450**	.386**					
	P	.000	.000					
KL	R	.311**	.298**	.341**				
	P	.000	.000	.000				
KC	R	.325**	.290**	.328**	.399**			
	P	.000	.000	.000	.000			
MPCK Self-concept	R	.726**	.690**	.731**	.681**	.637**		
	P	.000	.000	.000	.000	.000		
MPCK Test Score	R	.134	.291**	.196*	.380**	.318**	.375**	1.00
	P	.103	.000	.016	.000	.000	.000	

* $p \leq 0.05$ (2-tailed).
** $p \leq 0.01$ (2-tailed).

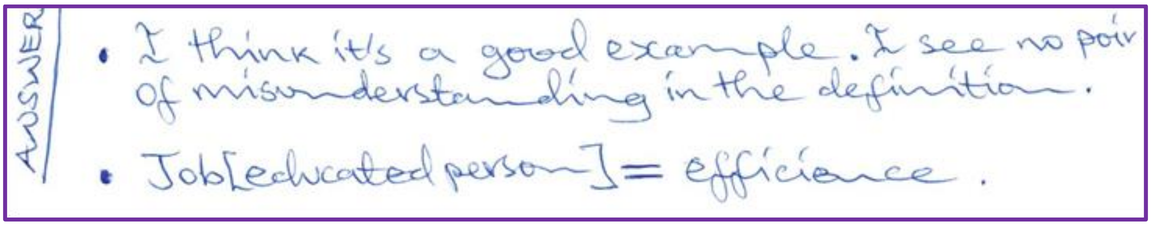


Figure 6. Excerpt of cluster 1 pre-service teacher's response to the vignette on composite functions

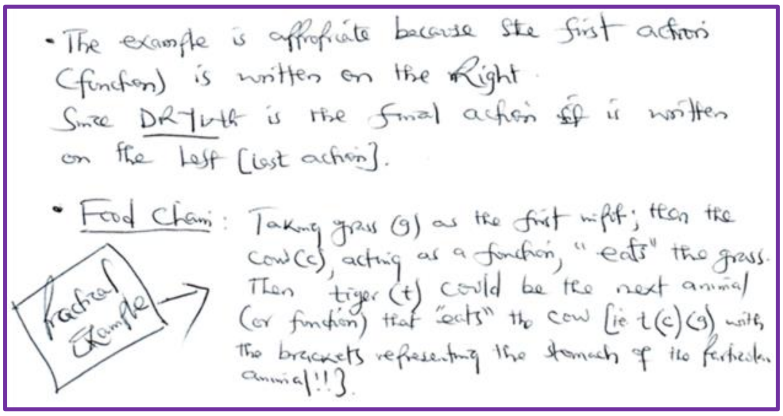
0.016) posted weak correlations with the test scores while KL (r = 0.380, p = 0.000) and KC (r = 0.318, p = 0.000) were moderately correlated with test scores. This implies that pre-service teachers with high MPCK self-concept performed better in the functions pencil and paper test that those with low MPCK self-concept. However, the correlation between KTS (r = 0.134, p = 0.103) and test score was not significant. This implies that the level of KTS was not related to one's performance in mathematics. Thus, if a pre-service teacher had high level of KTS it does not necessarily mean that their knowledge of the function concept would be high.

Pre-service Teachers' Responses to Vignettes

This section presents pre-service teachers' responses to the vignette (**Figure 5**). This vignette exemplified the definition of a composite function using a real-life situation. The vignette was calling on the teacher's understanding of the definition(s) and how a teacher can move between theoretical to practical or real-life examples in explaining a concept. **Figure 5** shows an excerpt of a response from a pre-service teacher in cluster 1. The teacher found the example in **Figure 2** suitable for teaching definition of composite functions. He also did not see anything about the given example that would cause misunderstanding of the concept related to the definition of composite functions.

However, the example the teacher provided was not good enough as it lacked detail. It was not a well thought example to fit the definition of composite functions. The equation given by the teacher in **Figure 4** was not elaborate enough to show which two functions were being considered. It was clear from this example that the pre-service teacher from cluster 1 had inadequate knowledge of real-life examples of composite functions. The pre-service teacher's inability to give a meaningful and valid real-life example of composite functions revealed his misconception about definition of composite function.

It can be argued that while the teacher was able to follow and understand the example presented in **Figure 5** by qualifying it to be a good example which cannot cause misunderstanding among students, his own knowledge of exemplifying definitions of composition of functions was undesirable. Among the key characteristics of pre-service teachers' MPCK is the ability to anticipate students' possible difficulties about a topic (functions in this case) and to design activities that would not cause students to develop misconceptions. The teacher whose answer is displayed in **Figure 6** would not be able to prepare a lesson on composite


• The example is appropriate because the first action (function) is written on the Right.
Since DR-Tith is the final action it is written on the left [last action].

• Food Chain: Taking grass (g) as the first input; then the cow (c), acting as a function, "eats" the grass. Then tiger (t) could be the next animal (or function) that "eats" the cow [ie t(c)(g) with the brackets representing the stomach of the particular animal!!]

Practical Example

Figure 7. Excerpt of cluster 2 pre-service teacher's response to the vignette on composite functions

functions that would resolve students' misconceptions. Thus, pre-service teachers in cluster one need to improve their knowledge of different concepts related to functions.

Figure 7 shows an excerpt from the answer presented by a cluster 2 pre-service teacher. The pre-service teacher demonstrated understanding of the definition of composite functions by stating that the given real-life example was appropriate in providing an alternative explanation regarding the definition. The teacher was also able to breakdown parts of the given real-life example in **Figure 5** and showed how they combined to fit the definition of composite functions. The real-life example given by the teacher was a true reflection of a composite function. It showed good understanding of using real life examples to explain mathematical concepts and epitomized the importance analogies play in teaching function concepts.

DISCUSSION

This section presents a discussion of the results obtained from pre-service mathematics teachers' MPCK self-concept and their knowledge of students via responses to a vignette (**Figure 5**). The study also examined the relationship between individual MPCK sub-factors and performance of pre-service teachers in a pencil and paper test on functions. While it has been established in this study that there is a significant difference in the MPCK self-concept between pre-service teachers in cluster 1 and those in cluster 2, this discussion section focused on the correlative relationship of MPCK sub-factors and knowledge of functions. Some factors showed no relationship with knowledge of the function concept whereas others showed weak to moderate relationships. No factor was strongly related to knowledge of the concept of a function. Cluster differences regarding pre-service teachers' KCS vis-à-vis their responses to vignettes will be discussed in the conclusion section of this paper. This was to ensure that the correlative relationship between MPCK self-concept and knowledge of the function concept is thoroughly discussed.

The findings of this study revealed a positive relationship between individual MPCK sub-factors and performance of pre-service teachers in the test on functions. This scenario suggests that high knowledge level of the concept of a function through improved performance in achievement tests is likely to improve pre-service teachers' MPCK self-concept and preparedness to teach upper secondary school mathematics concepts (Hine & Thai, 2019). It may also serve as an indication that pre-service teachers would develop their knowledge of the concept of a function with confidence. These findings are important for the improvement of teacher education in Zambia. The established relationship between MPCK and pre-service teachers' knowledge of functions entails that subject matter knowledge and MPCK require equal attention in teacher education (Shulman, 1986) high level of knowledge in one impacted on the other.

However, a closer look at the bivariate relationship between MPCK sub-factors and pre-service teachers' performance in the pencil and paper test on functions indicated that this relationship was not similar for all MPCK sub-factors. Pre-service teachers' MPCK self-concept about KTS ($r = .134$, $p = .103$) had no significant relationship their knowledge of the concept of a function. This result was surprising because knowledge of teaching strategies is crucial in the preparation and development of a mathematics teacher. This could be due to lack of teaching experience among the participating pre-service teachers. Maybe they did not gain a lot of KTS knowledge during their teaching practice period.

Pre-service teachers' MPCK self-concept about KMLS and KM showed a weak relationship with their knowledge of the concept of a function whereas their MPCK self-concept about KL and KC was moderately related with their knowledge of the function concept. These findings suggest that for this sample of pre-service teachers there wasn't a strong relationship between pre-service teachers' MPCK self-concept and their knowledge of the function concept. There is need for increased attention to pre-service teachers' MPCK during teacher training if institutions of higher learning in Zambia have to produce teachers that have high MPCK self-concept and confident to teach secondary school mathematics concepts. These results are consistent with the findings of Airwaished et al. (2017), and Kim's (2017) who found a relationship between knowledge of pedagogy and subject matter knowledge.

CONCLUSION

Results of this study have shown that pre-service teachers do not have the desirable level of MPCK self-concept and are likely to have low confidence when teaching mathematics. This was due to their low level of MPCK self-concept. It has also been revealed that pre-service teachers with low MPCK self-concept had weak knowledge of resolving students misconceptions. They also had insufficient knowledge of different representations of function. These results were consistent with previous research findings (Aziz & Kurniasih, 2019; Gaisman & McGee, 2015; Hine & Thai, 2019). Pre-service teachers with low MPCK self-concept in this study also showed inadequate knowledge of composite functions, similar to findings of prior research (Kontorovich, 2017; Ozgen, 2010).

Overall, interviews and vignettes showed that pre-service teachers were not fully prepared to teach secondary school concepts involving functions. There is need for universities in Zambia to structure teacher education programmes that will equip pre-service teachers in High school mathematics. This will help in the preparation of teachers that will effectively respond to the needs of schools because currently there is a gap between university mathematics and high school mathematics. Pre-service teachers should also orient themselves with secondary school mathematics because this is the content they are going to be teaching.

Implications for Future Studies

This study is a springboard for more research on function concepts. There is need for in-depth research on specifically different representations, inverse or composite functions. This will help have a detailed understanding of which particular aspect of functions is more difficult for pre-service teachers in Zambia. There is also need for more studies focusing on pre- and in-service teachers' perceptions of mathematics in Zambia. This study is one of the few studies focusing on perceptions. Therefore, more similar studies are needed to validate the current study.

More research focusing on teacher knowledge of content and students is needed in Zambia. This study established a correlative relationship between pre-service teachers' MPCK self-concept and their knowledge of functions. There is need for a study that would extend the focus to predictive relationship between pre-service teachers' MPCK self-concept and their knowledge of the function concept. Research on subject matter knowledge alone is not enough to improve mathematics teacher education. Zambia needs studies that combine subject matter knowledge and teacher knowledge of content and students because teachers don't teach in a vacuum, they teach students and thus they need to understand student needs at all times.

Disclosure statement

No potential conflict of interest was reported by the authors.

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