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Kindergartners' Use of Symbols in the Semiotic Representation of 3-Dimensional Changes

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ABSTRACT

Orientation skill's development is one of the topics studied in Mathematics Education because of its difficulty. In this article, we are concerned about the orientation skill of five-year-old children. For this end, we show a case study and a preliminary quantitative study of the symbolization used by children to depict graphically 3-dimensional changes in a plane. For this purpose, we have designed an activity based on Realistic Mathematics Education, where the children should find a treasure at the Childhood Education School and represent the itinerary between the classroom and the treasure in a map. We have also measured their spatial abilities through a specific test. The results show that, in one way or another, all the children understand the notion of 3-dimensionality and the changes in verticality, which they depict with specific symbols on the corresponding map. In any event, the semiotic representation depends on the orientation skill of the children. Thus, the types of symbols use vary with their orientation skills.

KEYWORDS
Childhood Education, map, orientation, Realistic Mathematics Education, semiotic representation, symbolization.

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Introduction

Nowadays we are surrounded by symbols and information. Because of that, one of the skills we should have is the ability to process all this information. In particular, we should know many geometrical concepts that we use intrinsically in our everyday life, even if sometimes we do not notice this. As an example, when we travel to another city, we must know how to orientate ourselves and how to interpret the city and transportation maps. All these

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human abilities can be grouped together inside the so-called visualization and spatial representation skills.

But, what happens when we are children? What should children learn in their early years? At this time, one of the most important things to learn is where we are, how to describe our surroundings so we can go from one point to another and how to describe to our classmates where there is something that we find relevant. All these skills should be developed in the classroom by taking everything around us into account. In particular, we need to understand some geometrical concepts.

But what do we understand by geometry? Using the words of Freudenthal in *Principles and Standards for School Mathematics*, "Geometry is grasping space…that space in which child lives, breathes and moves. The space that the child must learn to know, explore, conquer, in order to live, breath and move better in it" (referred in Clements, 1999, p. 1).

In this sense, previous research has contributed to casting light on the capability of young children to associate what they do to space, considered a fundamental part of geometry, and centred on visualization. In this connection, Yuzawa, Bart, Yuzawa & Junko (2005) describe which types of strategies are used by children from 3 to 6 years of age when they try to compare objects with different sizes, determining 4 different patterns of behaviour to resolve the question. Resnick, Verdine, Golinkoff & Hirsch-Pasek (2016) discuss the necessity to learn the concept of shape in Early Education with a good instructional tool and the way children can describe different shapes depending on their basic properties.

In the case of orientation, it is more difficult to find research on Childhood Education. For this reason, the purpose of this paper is to analyse how five-year-old children understand the space around them by focusing on their orientation skills. To this end, we have designed a teaching experiment based on Realistic Mathematics Education, where children have to follow some instructions to find a secret treasure at school and, after that, to successfully depict the path on a map.

Spatial Orientation

Before explaining what we understand by “spatial orientation”, we need to contextualize it. Spatial orientation can be considered one of the two major type of competencies of spatial thinking (spatial orientation and spatial visualization), which is defined as the human skill to imagine and reason about problems when space properties are involved.

For several years, there has been some controversy on the nature of this ability, whether it was a natural human ability or it could be developed. Some research was mainly centred on how blind children learned mathematical space properties (see, for example, Landau, Gleitman and Spelke (1981)). After several years of studies, the main conclusion is that in the case of spatial thinking “abilities have inborn beginnings, but are realized slowly over years of development” (Sarama and Clements, 2009: 171).

Returning to our main point, *orientation*, everybody has an intuitive way of describing it, but to explain its correct meaning, we need to use the definition provided by Sarama and Clements (2009: 161): "[…] spatial orientation involves understanding and operating on relationships between different positions in space, at first with respect to one's position and your movement through it, and eventually from more abstract perspective that includes maps and coordinates at various scales." It is clear that this description of spatial orientation has two definitions making allusion to different aspects of orientation; for this reason, in this paper, we will focus on the second one, which tries to describe in detail the crucial points involved when people need to orientate themselves in real spaces.

About this type of orientation, we should mention that there is a large amount of research on this topic focusing on representing the real space on maps together with the cognitive aspects involved in the process.

For example, Gaulin shows a collection of different representations made by students when trying to represent some specific physical objects constructed with multi-cubes concluding that students should familiarize themselves with different types of graphical representation of 3d-objects and with their relations in order to improve their spatial abilities related to visual processing and the interpretation of the figurative information involved (as cited in Fernández Blanco, 2011, p. 40).

In Hershkowitz, Parzysz and Van Dormolen (1996), the authors research the relationship between the space and the planar representations with secondary level students, coming to the conclusion that even if the models are isomorphic, the planar representations are not isomorphic with the 3-dimensional configurations represented.

Kotsopoulos, Cordy and Langemeyer (2015) study how 19 children, 8 to 10 years old, represent motion in large-scale mapping tasks. They analyse the student's drawings, oral description, and hand gestures while describing the tasks. Their results indicate that low achieving children produce fewer objects in their drawings, fewer gestures, and fewer verbal descriptions when engaging in a large-scale mapping task compared to high achieving children.

Carruthers and Worthington (2005) show the importance of children's own invented symbolism by analysing almost 700 examples of mathematical graphics produced by children between the ages of 3 and 8.

Orientation in Childhood Education and the relevance of landmarks

If we focus our interest on the treatment of orientation in early years of education, that is, in Childhood Education, first of all, it is important to keep in mind that the Organisation for Economic Cooperation and Development (2011) reports a positive correlation between this educational period and the mathematical competency some years later.

In this sense, if we examine international standards, we must mention the Principles and Standards of the National Council of Teachers of Mathematics (2000), which state on the issue of Geometry:

Learning geometry at all stages of education should enable to:

- Analyse characteristics and properties of geometric figures in two and three dimensions and develop mathematical arguments about on geometric relationships;

- Identify and describe spatial relationships using geometrical coordinates and other representational systems;

- Apply transformations and use symmetry to analyse mathematical situations;

- Use visualization, mathematical reasoning and geometric modelling to solve problems. (p. 42)

So, the relevance that orientation has with respect to the geometry block is clear. Furthermore, in the Pre K-2 period (from 4 to 7 years old), if we centre our interest on the ability "*Identify and describe spatial relationships using geometrical coordinates and other representational systems"*, this capability must enable child to:

- Describe, name and interpret relative positions in space and apply ideas about relative position;

- Describe, name and interpret the direction and distance when traveling in space and apply these notions;

- Find and name "places" with simple relations as "near" and coordinate systems such as maps. (p. 100)

In the Spanish case, the official curriculum for Childhood Education (form 0 to 6 years old), in the second cycle (from 3 to 6 years), makes explicit mention of orientation.

Some of the aspects that students should learn are "location of themselves and of the objects in space; relative positions, basic topological notions (open, closed, inside, outside, near, far...)" and "performing oriented displacements". The criterion to evaluate their development is: "The knowledge that children exhibit about spatial concepts (up, down, inside, outside, near, far...) will be considered " (Ministerio de Educación y Ciencia, 2008).

About research on Orientation in early years, Clements (1998) claims that

… it is unclear what kind of "mental maps" young children possess. Some researchers believe that people first learn to navigate only by noticing landmarks, then by routes, or connected series of landmarks, then by scaled routes, and finally by putting many mutes and locations into a kind of "mental map". (p.13)

…children must learn to deal with mapping processes of abstraction, generalization, and symbolization. Some map symbols are icons, such as an airplane for an airport, but others are more abstract, such as circles for cities. Children might first build with objects such as model buildings, then draw pictures of the objects' arrangements, then use maps that are "miniaturizations" and those that use abstract symbols. Some symbols may be beneficial even to young children. (p. 16)

To better understand how the previous guideline can be carried over to the classroom, Clements and Sarama (2009, p. 118) describe the learning trajectories for spatial thinking and spatial orientation depending on the age of the children as (we will only describe the period 0-6 years, corresponding to Spanish Early Childhood Education period):

• Children from 0 to 2 years of age are able to use a distance landmark to find an object or location near it, if they have not personally moved relative to the landmark.

• Children 2-3 years old use distant landmarks to find objects or location near them, even after they have moved themselves relative to the landmarks, if the target object is specified ahead of time.

• 4-year-old children locate objects after movement, even if target is not specified ahead of time. They search a small area comprehensively, often using a circular search pattern.

• For the 5-year-old case, a child, in general, "locates objects after movement (relates several locations separately from own position), maintaining the overall shape of the arrangement of objects. Represents objects' positions relative to landmarks (e.g., about halfway in between two landmarks) and keeps track of own location in open areas or mazes. Some use coordinate labels in simple situations".

• Children of 6-years of age locate objects using maps with pictorial cues.

Furthermore, they indicate some different instructional tasks that will help them to develop their spatial thought and orientation, emphasising the importance on doing different paths and discussing different routes, exploring outdoor spaces, etc. (as an example, "encourage children to mark a path from a table to the wastebasket with mask in tape. With the teacher's help, children could draw a map of this path…"(Clements and Sarama, p.119)).

In these activities, children should always be required to work with different symbolical descriptions of the paths by means of drawings, paintings, and verbal interaction. It is in these tasks where the words used by children are based on concepts that are problematic for them and, because of this, such tasks can be used as tools to evaluate and help children's development (Sarama and Clements, 2009, p.175).

Semiotic Representations and Symbols

In order to understand the relevance of semiotic representations, we need to start with what we know as "Mathematics Education". Taking into account the words of Elia, Gagatsis, Michael, Georgiu and Van den Heuvel-Panhuizen (2011), "Mathematics education includes a wealth of ideas and concepts and constitutes an area of human activity and thinking, which is characterized by the use of multiple representations" (p. 1842).

So, in this framework, a representation is any configuration of characters, images or concrete objects that stand for something else (Elia, Gagatsis, and Demetriou, 2007).

About semiotic representations, there are several studies that have tried to determine mathematical understanding through different representations used by the students. For example, in Childhood Education period, Elia et al. (2011) study how kindergartners use gestures to explain the meaning of some concepts related to spatial relationships between objects. They prove that five-year-old children, who took part in the activity, use gestures throughout the whole activity, concluding that gestures are essential in the learning process of early mathematics.

On the other hand, DeLoache (1991) believes that visual representations are important for mathematics. Also Carruthers and Worthington (2005) devote their efforts to analysing mathematical marks made by children in different teaching contexts. In our case, we want to see which types of visual representations are used by 5-year-old children (characterized by symbols) when trying to understand the 3-dimensionality property of space and translate it to a map; because, as far as the authors of this paper know, there is no previous research on what kind of comprehension Childhood Education children have of the 3-dimensionality of space.

Principles of the Realistic Mathematics Education (RME)

We remind the reader that, according to Freudenthal (1973), the RME has six principles:

• *Activity principle:* the students, instead of being receivers of ready-made mathematics, are treated as active participants in the educational process, in which they develop all sorts of mathematical tools and insights by themselves;

• *Reality principle:* the overall goal of mathematics education is to make the students capable of using their mathematical understanding and tools to solve problems. This implies that they must learn 'mathematics so as to be useful';

• *Level principle:* learning mathematics means that students pass through various levels of understanding: from the ability to invent informal context-related solutions, to the creation of various levels of shortcuts and schematizations, to the acquisition of an insight into the underlying principles and a discernment of even broader relationships;

• *Intertwinement principle:* it is also characteristic of RME that mathematics, as a school subject, is not split into distinctive learning strands. From a deeper mathematical perspective, the chapters within mathematics cannot be separated;

• *Interaction principle*: Education should offer students opportunities to share their strategies and inventions with each other. The students can get ideas for improving their strategies by listening to what others find out and by discussing these findings. Moreover, the interaction can evoke reflection, which enables the students to reach a higher level of understanding;

• *Guidance principle:* mathematics education should give the students a 'guided' opportunity to 're-invent' mathematics. [...]teachers have to provide their students with a learning environment in which the constructing process can emerge. (extracted from Van Den Heuvel-Panhuizen, 2000, p. 5-9)

Objectives of the Research

The three main objectives of this paper are:

(1) To analyse the ability of the five-year-olds to depict in a map a change of level produced in the space.

(2) To classify the symbols used in planar representations.

(3) To explore the possible relation between the use of symbols and spatial skills.

Methodology

The design of the teaching experiment is based in two pillars: the Realistic Mathematics Education and the use of landmarks to guide children in orientating them in real space, based on the research by Clements (1998) and the examples of instructional tasks of Clements & Sarama (2009).

Design of the Teaching Experiment

The design of the teaching experiment was divided into 4 phases, following Alsina (2012), as described completely in Author (XXX).

Context of the activity. The children receive a letter with information about a mysterious treasure of ancient pirates hidden somewhere in their school, and the way to find it is to follow the indications/landmarks around the building, which help children to follow the correct route to reach the treasure.

The itinerary is simple, they should go out the classroom, go up to the second floor, turn to the left, follow some signs up to the 6th door on the right, open the door, go into the classroom and follow other indications to find the treasure under a table, inside a chest. The most important thing in this itinerary is the *change of the floor*, where they should go up some stairs and, then, take some turns to the right, to the left and open some doors.

Description of the phases.

(1) First of all, we have examined the different aspects that could be worked with this activity in Educational Framework. In this case, our interest is centred on orientation skills, verbal and visual communication (representation systems) and learning about their surroundings.

(2) Secondly, we have done a small assembly with children to figure out which type of pre-ideas they have about pirates, space, maps…

(3) Thirdly, the students, divided into small groups, should find the treasure by following the symbols they can find and the indications described in the letter they have received. To find the treasure, the structure of the instructions given by the teacher was based on the use of landmarks (as said before).

(4) Finally, the children are required to represent the itinerary on a map and to describe it verbally explaining the meaning of their depictions and, if needed, describe the symbols used on it.

Landmarks used to help children on their search. To adapt the activity to the cognitive level of the children, we have used a collection of images in which some actions are represented. As in other activities in Childhood Education, where iconic pictures are used to describe to children which type of actions they should do, in our case, the pictograms help children to orientate and achieve the mission by themselves.

In Table 1 we show the symbols used, their meaning and their contextualization along the itinerary as landmarks.

Table 1 Symbols and contextualization as landmarks

Symbols	Meaning	Children's actions
	Go up (the stairs) /go straight the corridor	
	Go into (the classroom)	
 	Look under the table	

Participants

The participants have been five-year-old children of a public Early Childhood Education school of Galicia (Spain). The school is in Sigüerio, a small rural village, near Santiago de Compostela. The group consisted of 20 students (11 girls and 9 boys), and none of them required any special adaptation of the educational program.

Instruments

As we have said before, we have used several tools to calculate the impact of the teaching experiment on our research:

1. To analyse the interpretation of the children about the itinerary and the location of the treasure, we have used the handmade productions (maps) and the video recording archives of their verbal explanations;

2. To measure the spatial abilities, we have use the test "Pruebas de Diagnóstico Preescolar" (De la Cruz, 1988). This test defines 4 different factors/variables to determine some of the children's capabilities: relative locations/positions, orientation, perception and visual coordination. This author uses the sum of the first three variables to speak about the global spatial skill.

As the author describes on the test, it was designed to be resolved by educated 5-year-old children, and the reliability of the first three variables was calculated using the Spearman-Brown method. This gives the following reliabilities: 0.86, 0.89, 0.75 (respectively). So, we decided to use the same test because of the similarity in the conditions defining the sample used.

Results

The study has been divided into several subsections according to the objectives. First, we show the analysis of the productions carried out by children; secondly, the factors involved in the spatial skill of the children are analysed and, finally, we study the possible relationship between handmade productions and the children's spatial skills.

Maps Done by Children

We present a collection of different maps drawn by the children.

Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a vertical line, associating the climbing movement with the depiction used

Figure 1 Vertical line map

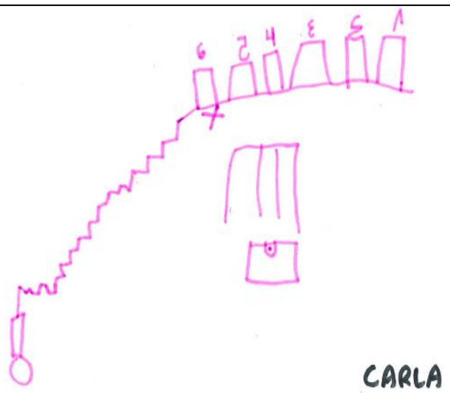
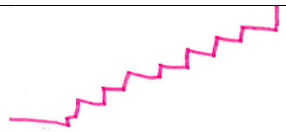
Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a polygonal line, associating the climbing movement with the horizontal projection of the stairs

Figure 2 Horizontal projection map

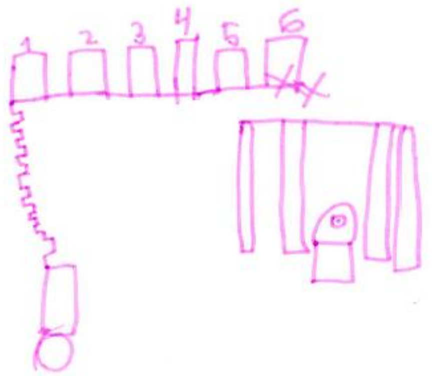
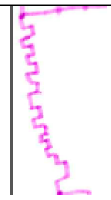
Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a polygonal line, associating the climbing movement with the vertical direction of the paper

Figure 3 Vertical stairs map

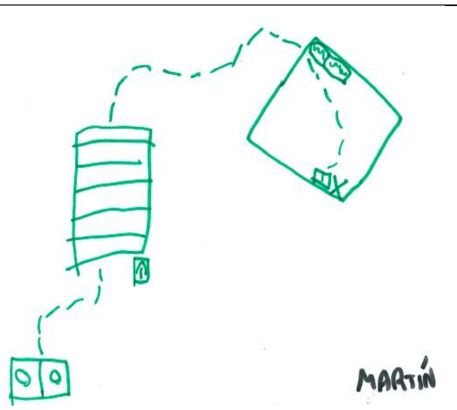
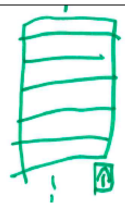
Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a different symbol, where all the steps of the stairs are depicted in the map.

Figure 4 Plane projection map

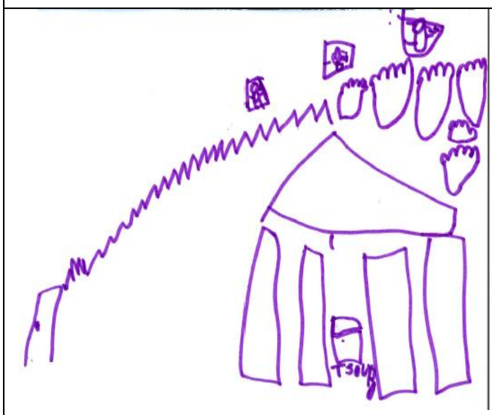
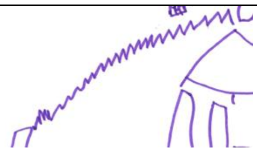
Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a polygonal line, associating the climbing movement with a diagonal line across of the paper.

Figure 5 Diagonal stairs map

Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a different symbol, where each step of the stairs is depicted as a separate object in the map.

Figure 6 Separate steps map

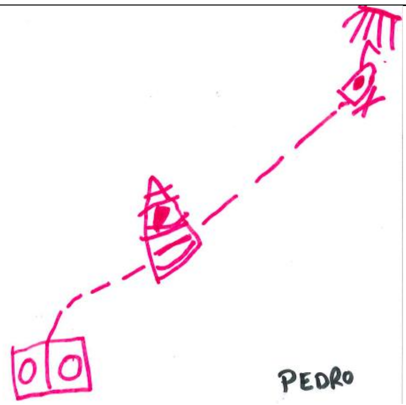
Produced map	Symbol used to depict the stairs
	 The child describes the action of going up the stairs as a different symbol, where the stairs are represented by a triangle with all the steps depicted in it.

Figure 7 Triangle map

The review of all the maps shows that the most common representation is a polygonal line when children try to represent a height change produced by the stairs. Sometimes it appears in a vertical way and sometimes diagonally, in 12 cases, children use this representation (see Figure 8 for some examples).

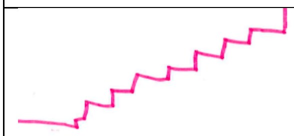
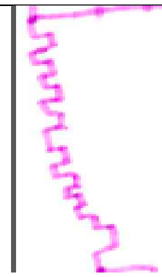
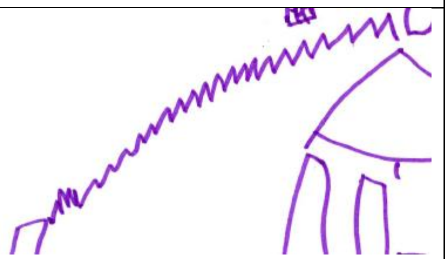
Different representations using polygonal lines		
		

Figure 8 Different representations of stairs using polygonal lines

In other 5 cases, a straight line represents the stairs. Furthermore, some of children use a vertical line and others use a horizontal line.

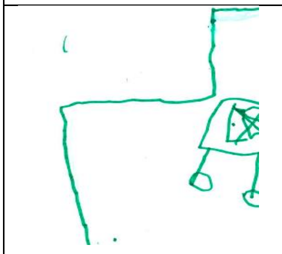
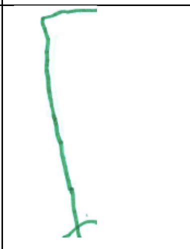
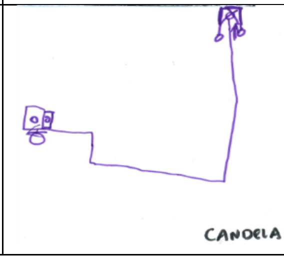
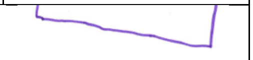
Produced map	Representation using vertical line	Produced map	Representation using horizontal line
			

Figure 9 Different representations of stairs using vertical/horizontal lines

Finally, we should point out that not all the children have used these previous representations; at Figure 10 some other interesting symbols used by kids are shown.

Figure 10 Other different symbols to represent the change of level

Some Parts of the Explanations of the Children about their Maps

In this section, we transcribe some parts of the conversation between the teacher and the children relative to the meaning of the map done, with the purpose of analysing the importance that children give to the change of vertical dimensionality through the representation used.

In short, we centre our interest on 4 cases to show in greater detail their interpretation (the names are fictitious), showing the transcription of the conversations extracted from the video record (translated from Galician into English), and after them, the conclusions that can be extracted:

First case: Ron.

-Teacher: Ron, describe me your map! (Ron ¡cóntame o teu plano!)

-Ron: This is the door of the classroom, (esta es la puerta del aula)

and we go up the stairs (y subimos las escaleras)

to the door, to the 6th door (hasta la puerta, hasta el aula 6)

and bend down, and we see the cards (y bajamos, miramos las cartas)

and after we find the treasure behind the table (y despues debajo de la mesa encontramos el tesoro.)

-Teacher: and What was? (¡ah! ¿e qué era?)

-Ron: Bimbo bread (Pan bimbo)

Figure 11 The door of the classroom, "go up the stairs", the other clasroom (from left to right respectively)

In this case, it is clear that child gives a big importance to the fact "go up the stairs" and the movements related to go up and down (behind the table). In Figure 2 we can see another example of this type of map.

Second case: Naroa.

-Teacher: Naroa, describe me your map!

-Naroa: this is our classroom (esto es nuestra clase),

here, you go up the stairs (aquí subes las escaleras)

and here we go straight (y aquí vamos pa'aquí)

and here we open the door (y aquí abrimos la puerta)

and here there is the treasure (y aquí está el Tesoro)

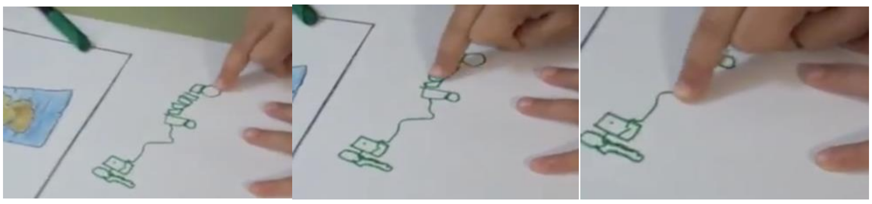


Figure 12 The sequence of the transcription is given: the first three sentences are shown

Figure 13 The sequence of the transcription is given: the last two sentences are shown

In this case, the child represents the movement of going up painting the steps of the stair, and she gives the 5 instructions people should know to arrive at the treasure, describing perfectly the main parts of the itinerary where the landmarks appear in the original place. Another example of the use of steps to describe the stair is given in Figure 6.

Third case: Paul.

-Teacher: Paul, describe me your map!

-Paul: we go out the door (salimos de la puerta)

we go up (the stairs) subimos (las escaleras)

and we find the treasure (y buscamos el Tesoro)

and it is under the table (y (está) debajo de la mesa)

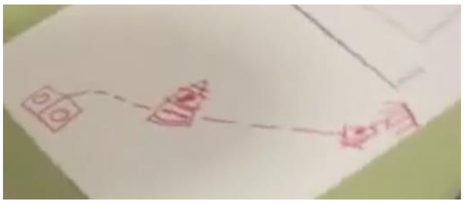


Figure 14 The map done by the child

In this case, it is clear that for the child the relevant parts of the itinerary are the main classroom, the stairs (painted with a different symbol, as a triangle) and the treasure. The rest of the parts of the itinerary are not relevant for him. This is a special case mentioned in Figure 10.

Forth case: Anne.

-Teacher: Anne, describe me your map!

Anne: you are here (aquí tás tí)

then go up the stairs (subes as escaleiras),

The door is the 6th (na porta seis)

Under the table there is the treasure (debaixo de unha mesa está o mapa)

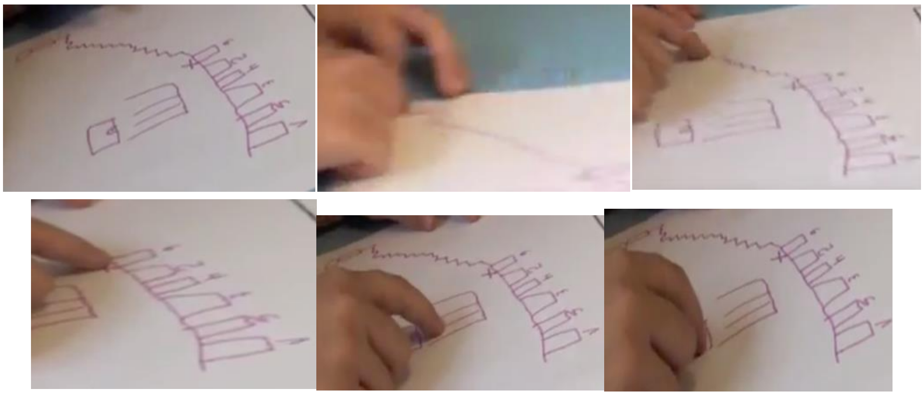


Figure 15 The map, where is the classroom, going up the stairs, the door they should find, the table inside the classroom and the position of the treasure (from left to right respectively)

In this case, the child is concerned about the way to describe the whole itinerary, the stairs, and the corridor where the classrooms of the first floor are located, and about determining the correct door that should be opened to find the treasure. In addition, she describes how the treasure is behind a table found at the 6th door. The representation of the stairs is mentioned in a previous section, in Figure 2.

Summarizing, as described before, in all the previous examples, it is obvious that the children need to describe the movement related to the fact that they have to go up the stairs in the middle of the itinerary, and this does not happen when they speak about the rest of the itinerary, depending on the case, they do not see the requirement to describe where the treasure was, or which was the number of the door on the second floor, or if they had to turn left or right after the climbing, etc.

In fact, some of the landmarks used to help children to locate themselves in the space have been really useful, guiding the children on spatial concepts as “go up, behind, go straight”. In all the cases, children feel they need to explain their itinerary by giving relevant details. In this way the teacher, who is asking about the treasure, will be able to understand the way to find it in a simple way. On the other hand, the last landmark, which is used to find the treasure behind the table, has not got the same weight as the rest of landmarks, since, after entering the classroom, children think it is simple to find the treasure with no instructions. This invites us to conclude that for some children the landmarks in the micro-meso-space are not required, because they have developed more the notions linked with space and they only need some relevant points when they are playing in the macro-space.

Spatial Skill of the Children

In this section, we focus our interest on the results shown on the test, which measures the spatial abilities of children. It is important to say that the purpose of this section is to investigate whether there is any possibility to establish any relationship between the symbols used by the children and their abilities. However, this section should be taken as an exploratory analysis; it does not intend to be a quantitative study, because of the reduced number of children involved in it. Anyway, in the following lines we present some percentages extracted from the test to describe the reality of the class and to specify to what extent the results vary, representing the diversity that can be found in this educational period.

Table 2 Numerical results obtained by the children in the spatial abilities test

Child’s code	Locations	Orientation	Visual coordination	Perception	Total sum
21	6	9	6	2	17
1	7	15	7	0	22
13	11	4	7	9	24
8	9	10	6	6	25
14	9	7	4	9	25
19	10	9	6	6	25
17	7	13	9	7	27
20	11	7	7	9	27
5	6	16	8	6	28
9	13	6	8	9	28
7	12	8	6	9	29
22	13	10	7	6	29

2	14	10	7	6	30
3	12	11	9	7	30
12	12	13	10	6	31
11	8	15	8	9	32
18	14	16	8	9	39
10	13	14	7	6	33
15	13	16	8	6	35
16	14	15	8	8	37

Table 3 Statistics for the different factors of spatial skill

	Locations	Orientation	Visual coord	Perception	Total sum
Percentile 1	8,75	8,75	6,75	6	25
Median	11,5	10,5	7	6,5	28,5
Percentile 3	13	15	8	9	31,25
Minimum value	6	4	4	0	17
Maximum value	14	16	10	9	39
Average	10,7	11,2	7,3	6,75	28,65
Deviation	2,75489994	3,736026597	1,34164078	2,40339671	5,15317990

Taking this into account, we can obtain the total results of the class depending on each variable. This part gives us a perspective about the ability of each child with respect to the different factors. In this sense, we centre our analysis on the variables: perception, location, orientation and sum of them.

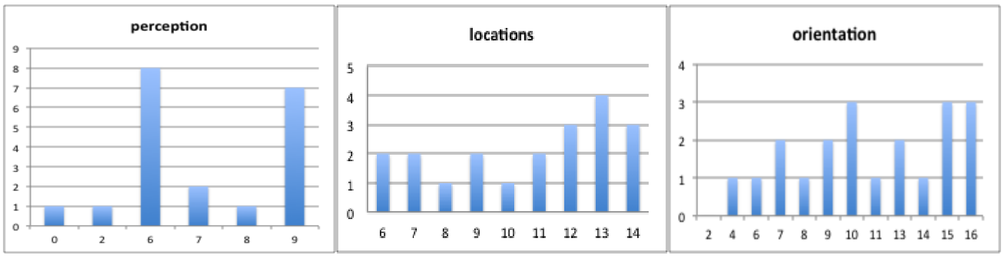


Figure 16 Number of children depending on correct answers on perception, location or orientation variables (left, middle and right respectively)

In order to better understand the aim of this test, we describe here what each variable measures, the maximum punctuation that can be obtained, and afterwards, we analyse the results yielded by our students.

In the case of perception variable, the maximum value that children can obtain is 9, and the minimum 0. The variable measures the ability of children to recognise specific objects from a picture. They should fix their vision on the picture and mark the objects asked by the teacher. As we can see above, a big percentage of the class (35%) has obtained the maximum score, that is, 7 children have a good perception; 11 children (55%) have a "medium" score in this factor obtaining 6-7-8 as total score.

With respect to "locations" factor, the score range goes from 0 to 14. This variable measures if the children can distinguish different positions of objects on the plane, if they can distinguish some objects from others that are similar but have been constructed by applying a rotation, a symmetry or a translation.

Again, the class can be divided into two groups, some children (35%) with good results in the test (13-14 punctuations) and the rest (65%) with worse results.

Finally, the orientation factor is used to determine the skill children have of copying an object drawn by means of a unique line, with some turns. The typical errors in this proof are related to the use of symmetries or turns different from the required ones. We should say that this is the factor that has obtained the worst results. The maximum score in this case is 16, and only a 30% of the class has done the test correctly (15-16 punctuations). Furthermore, there are some children (20%) with low results (6-7-8 points), and some others (45%) with intermediate results (9-10-11-13-14).

These first results invite us to analyse the skill defined by the sum of the previous factors. According to the instructions of the original test, the global ability of children is defined as the sum of the variables 'perception', 'locations' and 'orientation'; so we use this fact to group the children. This variable has a minimum score 0 and a maximum 39.

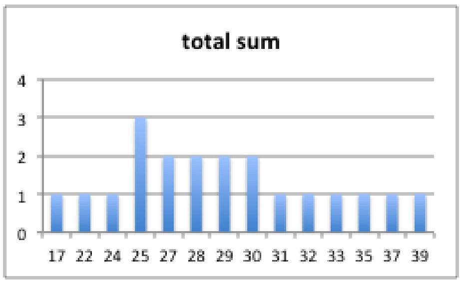


Figure 17 Frequencies of children depending on the punctuation of the "total sum" factor

As the previous graphic shows, the class is so diverse that we consider the quartiles of the variable to divide the class-group into the following 4 subgroups:

Group 1: [17,25], Group 2: [26, 28], Group 3: [29, 31], Group 4: [32, 39]

On the basis of this new grouping, we can describe some characteristics of our children depending on their scores (see Table 2):

First of all, children of Group 1 have obtained the worst results on the test. They show some difficulties to identify objects, they have not interiorized the notion of position correctly and they have some problems marking an asked object. This group will be regarded as low spatial skill group.

Second, children of Groups 2 and 3 have better results in general. They do better the exercises related with perceptions, but, in general, they have some problems with respect to location or orientation or both. We will refer to this union of groups as the group with medium spatial skill.

Third, children of Group 4 can do all the exercises correctly, with high scores, and in general they have no problem with the perception variable, and some errors, if any, are related with either orientation or location variables. Children of this group will be regarded as children with high spatial skill.

Analysis of the Possible Relation between the Spatial Abilities and the Representation Used

According to the total results and the type of representation used by children, as an exploratory study, we wondered if there could be any relation between them. For this purpose, we classify the symbols used depending on the spatial ability skill of the children, which results in the following table (Table 4):

Table 4 Skill and representations

Group	Polygonal line	Vertical or horizontal line	Other	
			quantity	Symbol
Low skill	3	1	1	
Medium skill	5	4	1	
High skill	4	0	1	

Even though the present research is not based on a huge quantitative study, it seems that, in general, when children have a bigger spatial ability, they stop using a simple line to try to depict the change of level (the stairs) by a more sophisticated symbol (in this case, the polygonal line). This pushes us to think that there is a relation between the spatial ability and the way of visual representation.

Conclusions

As we have seen before, at the age of 5, we can reach the following conclusions:

First, respect to the analysis of the ability of the 5 year-old-children to depict in a map a change of level produced in the space, it is clear they can realize the need to highlight a 3-dimensional change in a 2 dimensional representation (according to the learning trajectory described by (Clements and Sarama, 2009), but, furthermore, this research allows us to conclude that children have implicitly learned the notion of the space as a 3-dimensional mathematical object and they feel the need to describe any change on the vertical axis in the 2-dimensional representation.

So, even though it is difficult to speak about semiotic representation in Early Childhood Education, it is clear that children use an image to represent this spatial change, so it can be considered as a representation type, according to Elia, Gagatsis, & Demetriou (2007).

Furthermore, in the case studies we have analysed, children have used the landmarks in macro-space as a reference system to describe perfectly to another person the itinerary they should follow to find the treasure (Clements, 1998). In these dialogues children have explained correctly the directions involved in this process and the relevance of the symbol used for the 3d-change.

Second, we have seen that the way to represent this dimensional change is not completely a standard one; and, in fact, this depends on the development of children's mobility and the way they understand the space around them. Anyway, we have classified the symbols used in the planar representations, determining that the most common representation is a polygonal line, but there are many other representations, as vertical/horizontal lines, or others.

Third, we have used the test of De la Cruz (1988) as a tool for the description of the main characteristics of our students on spatial skills and to try to determine a possible relation between this spatial ability and the sophistication of the symbol used to depict the 3-dimensional change. After the analysis of test results, we have been able to describe which types of errors are more common depending on this skill, where low ability involves a huge number of errors related to the three variables. Children thus show some difficulties in identifying objects, they have not correctly interiorized the notion of position, and they have problems to point to an object they have been asked about; children with medium ability do better in exercises related to perceptions, but, in general, they have problems with respect to location or orientation or both; and children with high ability have no problem with variable perception, and some errors, if any, are related to orientation or location variables.

On the possible relation between this spatial ability and the sophistication of the symbol used to depict the 3-dimensional change, we conclude that it seems that when the spatial ability is bigger, the symbols used are more complex, thus is, they stop using a simple line to try to depict the change of level (the stairs) by a more sophisticated symbol (in this case, the polygonal line), and the reverse. This aspect gives us an opportunity to continue in the future with this research line, where we hope to be able to determine clearly if there is more to this possible relation than found so far or if it is only a matter of mere chance.

Finally, this study has helped us to understand how children "see" the space they have around them and how it is possible to work this complex notion, 3- dimensionality, in their early years. Anyway, an interesting research field is open in front of us, where we must investigate how complex mathematical notions (their intuitive meaning) are understood by kindergarteners.

Disclosure statement

No potential conflict of interest was reported by the authors.

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References

Alsina, A. (2012). Hacia un enfoque globalizado de la educación matemática en las primeras edades [Towards a holistic approach to mathematics education at the early ages]. *Números*, *80*, 7-24.

Carruthers, E. & Worthington, M. (2005). Making sense of mathematical graphics: the development of understanding abstract symbolism. *European Early Childhood Education Research Journal*, *13*(1), 57-79.

Clements, D. H. (1998). *Geometric and spatial thinking in young children*. Retrieved from ERIC database (ED436232).

Clements, D. H. (1999). Geometric and spatial thinking in young children. In J. V. Copley (Ed.), *Mathematics in the early years* (66-79). Reston, VA: National Council of Teachers of Mathematics.

Clements, D. H. & Sarama, J. (2009). *Learning and Teaching Early Math. The learning trajectories approach*. NY, New York: Routledge.

De la Cruz, M. V. (1988). *Pruebas de Diagnóstico preescolar*. Madrid: TEA Ediciones.

DeLoache, J. S. (1991). Symbolic functioning in very young children: understanding of picture and models. *Child Development*, *62*(4), 736-752.

Elia, I., Gagatsis, A. & Demetriou, A. (2007). The effects of different modes of representation on the solution of one-step additive problems. *Learning and Instruction*, *17*, 658-672.

Elia, I., Gagatsis, A., Michael, P., Georgiu, A. & Van den Heuvel-Panhuizen, M. (2011). Kindergartners' use of gestures in the generation and communication of spatial thinking. In M. Pytlak, T. Rowland, & E. Swoboda (Eds.), *Proceedings of the Seventh Congress of the European Society for Research in Mathematics Education* (1842-1851). Rzeszów, Poland: University od Rzeszów.

Fernández Blanco, Mª. T. (2011). Una aproximación ontosemiótica a la Visualización y el Razonamiento Espacial [An ontosemiotic approach to visualization and spatial reasoning] (Doctoral dissertation). Retrieved from TESEO (947559).

Freudenthal, H. (1973). *Mathematics as an Educational Task*. Dordrecht, The Netherlands: Reidel Publishing Company.

Hershkowitz, R., Parzysz, B. & Van Dormolen, J. (1996). Space and shape. In A. J. Bishop (Ed.), *International handbook of mathematics education 1* (161-204). Dordrecht, The Netherlands: Kluwer.

Kotsopoulos, D., Cordy, M. & Langemeyer, M. (2015). Children's understanding of large-scale mapping tasks: an analysis of talk, drawings, and gesture. *ZDM Mathematics Education*, *47*, 451-463.

Landau, B., Gleitman, H., & Spelke, E. (1981). Spatial knowledge and geometric representation in a child blind from birth. *Science*, 213, 1275-1277,

Ministerio de Educación y Ciencia. (2008). ORDEN ECI/3960/2007, *Currículo español de Educación Infantil* [Spanish Early Childhood Education Curriculum]. Retrieved from https://www.boe.es/boe/dias/2008/01/05/pdfs/A01016-01036.pdf

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Organisation for Economic Cooperation and Development. (2011). *Pisa in Focus 2011/1*. Retrieved from http://www.oecd.org/pisa/pisaproducts/pisainfocus/PiF1_esp_revised.pdf

Resnick, I., Verdine, B. N., Golinkoff, R. & Hirsch-Pasek, K. (2016). Geometric toys in the attic? A corpus analysis of early exposure to geometric shapes. *Early Childhood Research Quarterly 36*, 358-365.

Sarama, J. & Clements, D. H. (2009). *Early Childhood Mathematics Education Research. Learning Trajectories for Young Children*. New York, NY: Routledge.

Van den Heuvel-Panhuizen, M. (2000). *Mathematics education in the Netherlands: A guided tour*. Utrecht, The Netherlands: Utrecht University.

Yuzawa, M., Bart, W. M., Yuzawa, M., & Junko, I. (2005). Young children's knowledge and strategies for comparing sizes. *Early Childhood Research Quarterly 20*, 239-253.