

Integrating History of Mathematics into Curriculum: What are the Chances and Constraints?

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There exists a gap between what is espoused in the professional and scholarly arena regarding possible benefits of students' learning of the history of mathematics and teachers' perceptions of the use of the history of mathematics in curriculum. The purpose of this large scale research study was to gain understanding of the high school mathematics teachers' perceptions related to the integration of the history of mathematics into their instruction. In particular, we sought to identify those factors that encourage or discourage high school teachers' choice to include the history of mathematics as a systematic part of their mathematics courses. A total of 367 teachers, 12% of all high school mathematics teachers from one of the USA states, participated in an on-line comprehensive survey, which was designed by the researchers. Several factors, such as teacher's perception of the history of mathematics, teacher's knowledge of the history of mathematics, time, high stakes testing, resources and other were identified and are discussed in this paper. We also offer recommendations for teacher professional development and suggestions for further research.

Keywords: Teacher perceptions; history of mathematics; history of mathematics into curriculum

The overarching purpose of our large scale research study was to gain understanding of the high school mathematics teachers' perceptions related to the integration of the History of Mathematics (HOM) into their instruction. One of the goals of the study, which we address in this paper, was to identify those factors that encourage or discourage high school teachers' choice to include the HOM as a systematic part of their mathematics courses.

Several key assumptions, well known to the community of mathematicians and mathematics educators, served as a basis of our research. First, HOM provides a background for gaining a rich and deep understanding of the evolution of mathematical concepts. Second, understanding how and why basic mathematics concepts were developed by individuals through years of hard work, sacrifice, trials and tribulations one has an opportunity to trace the intellectual development of humankind. In this manner, mathematics is placed within a clear and practical human context demonstrating the utility of mathematics concepts. Third, learning HOM may increase students' interest and enhance positive attitude toward mathematics.

While there is not enough explicit empirical proof for such assumptions, it is apparent that the teaching of historical development of basic concepts has become central to the teaching of all the basic sciences and virtually all of the social sciences. Indeed, since the work of Thomas Kuhn (1996), the history of science has been a rapidly growing field of research and publication. The popularity of the science histories of Stephen Jay Gould in biology and Stephen Hawking in physics, to name only a few, suggest how history can capture a very broad attention even to very abstract and complex subjects. If history of the

discipline is proving to be important in the basic sciences, why should it not also be true of mathematics? In the words of Glaishier (1848 - 1928), "I am sure that no subject loses more than mathematics by any attempt to dissociate it from its history." It seems commonsense to believe that it is beneficial to mathematics learners, particularly those who have difficulty understanding the significance and relational value of mathematical concepts, to be aware of the purposes and intellectual struggles of those who created mathematics and to appreciate the process of invention. In addition, teachers are often encouraged to teach mathematics as a social construction, an activity that makes sense through its usefulness, and it is fair to say that most teachers strive to inspire their students' interest in mathematics.

To value mathematics the students should have varied experiences related to the cultural and historical aspects (Krathwohl, Bloom, & Masia, 1973) of the evolution of mathematics so they can appreciate the role of mathematics in the development of our society. The idea that HOM should be an integral element of mathematics teaching was strongly argued by Siu and Tzanakis (2004), who questioned whether the notion of 'using history of mathematics' should be replaced with 'integrating history of mathematics', thereby encouraging the view that the HOM is inseparable from the subject itself.

The importance of the HOM in the school curriculum has been emphasized by professional councils such as the National Council of Teachers of Mathematics (NCTM), National Research Council (NRC) and National Council for Accreditation of Teacher Education (NCATE), and has been supported by research studies (e.g., Siu, 2004; Weng Kin, 2008).

The National Council of Teachers of Mathematics (NCTM, 2000), stated, "Mathematics is one of the greatest cultural and intellectual achievements of humankind, and citizens should develop an appreciation and understanding of that achievement, including its aesthetic and even recreational aspects" (p. 4). Interesting enough, while The Principles and Standards for School Mathematics (NCTM, 2000) outlined new visions and principles, which were designed to give students enriched and functional mathematics education manifested in conceptual understanding, there was no mentioning of HOM as a means to facilitate students' appreciation of and to value mathematics.

Swetz (1994) suggested that "The history of mathematics supplies human roots to the subject. It associates mathematics with people and their needs. It humanizes the subject and, in doing so, removes some of its mystique" (p.1). We sustain the position that mathematics is a "cultural phenomenon" (Wilder, 1968, p.xi), and that meaningful learning of school mathematics must be facilitated by studying the cultural significance of mathematics, the role of the evolution of mathematical concepts and scientific thought. Barbin et al. (2000) posited:

The conviction that the use of history improves the learning of mathematics rests on two assumptions about the process of learning. The more a student is interested in mathematics, the more work will be done; and, the more work that is done the greater will be the resulting learning and understanding. (p. 69)

Swetz, Fauvel, Bekken, Johansson, and Katz (1995) suggested that exposure to the HOM at the high school level can have "...a profound effect! For it is at the secondary or high school level that students first experience the power of mathematics and begin to realize the wide scope of its application and possibilities" (p.1). Unfortunately, the historical dimension

of mathematics is deficient, ignored or viewed as an "exotic luxury" as Whitrow (1932) suggested.

Research into teachers' perspectives of the inclusion of HOM in the classroom is scarce. Studies (e.g., Philippou & Christou, 1998; Schram, Wilcox, Lapan & Lanier, 1988; Siu, 2004; Smestad, 2009; Stander, 1989) indicated that teacher interests in and value of mathematics increased when introduced to the HOM. At the same time, these studies emphasized that teachers found no interest in *using* the HOM within the mandated curriculum.

Clearly, there is a gap between what is espoused in the professional and scholarly arena regarding possible benefits of students' learning of the HOM, the curriculum standards, which have no trace of the HOM, and teachers' views on the integration of the HOM in curriculum. Fasanelli *et al.* (2000) asserted, "These decisions are ultimately political, albeit influenced by a number of factors including the experience of teachers, the expectations of parents and employers, and the social context of debates about the content and style of the curriculum" (p.1).

Seemingly, the primary influence on a teacher's decision of whether or not to include the HOM in the classroom is the set of government created learning objectives, provided in the national or state curriculum requirements in USA. Fasanelli *et al.* (2000), report that many countries have a Magistrate of Education which outlines the educational goals for the entire country. The countries such as Austria, Brazil, China, Denmark, France, Greece, Italy, New Zealand, Norway, Russia, have a national set of frameworks standards that explicitly incorporate HOM into the learning standards.

As of the writing of this paper, 46 states of the United States of America have adopted the Common Core State Standards (www.corestandards.org). The goal in the adoption of the common standards is to insure that students across the United States are provided with a curriculum that is unified in rigor and content.

An interesting dichotomy is that, according to Fasanelli *et al.* (2000), "The present ICMI study is posited on the experience of many mathematics teachers across the world that the history of mathematics makes a difference: that having history of mathematics as a resource for the teacher is beneficial" (p. 1), yet nowhere in the United States Common Core State Standards is there any mention of the learning of the HOM. Is this not sending a political message to the teachers that learning the HOM plays no role in learning the subject? If on a state level and a national level there exists the implicit message that the learning the HOM is not important, why would a teacher consider the contrary? Yet, as our study shows, some teachers do include HOM in their instruction. And, since the integration of the HOM is not explicitly supported by major current state and national policies, the teachers' judgment is most likely a function of their perceptions, attitudes and probably education and training.

Method

We designed a comprehensive survey scale instrument to address the overarching purpose of the study and its goals. Having weighed the advantages and the disadvantages of both on-line and postal mail, we chose to disseminate a web based survey via SurveyMonkey™, a

web-based software program (www.surveymonkey.com) and have the participant responses sent to the company's server for convenient yet confidential access.

Participants

This study took place within the confines of one of the USA states, which had 372 operating public high schools including charter schools with around 2,909 mathematics teachers (State Department of Education, 2010). To encourage participation, we first contacted via email all public and 7 private high school principals (total 379) requesting forward to their mathematics teachers an invitation to participate in an anonymous, on-line survey. The number of the private high school mathematics teachers remains unknown. Only 6 principals declined the request. We also utilized our extended personal contacts and sent a bulk of e-mail messages to high school mathematics teachers requesting their participation. The invitation letters to the teachers included a description of the study and detailed instructions on how to access the survey through the link to the web site SurveyMonkey™ and contact information should they have any questions. All were encouraged to access the survey within two weeks. However, due to slow response and school break, we sent follow up e-mail messages informing the teachers about the extension of access to the survey. A total of 367 teachers participated in the on-line survey, which is about 12% of all high school mathematics teachers in the state.

Instrument

The survey's six parts included 110 items. Some items were designed by the researchers. Other items were adopted with modification from surveys of previous studies. Part I, Attitudes Towards Mathematics as a Discipline, consisted of 39 items. The items were adopted from the research of Tapia and Marsh (2004). The 17 items of Part II, Attitudes Towards Mathematics as a Discipline, were based upon Dutton (1962) as well as Shulman (1986) research and modified accordingly to serve the purpose of the research. Some of the ideas for Part III items (11 total) were based upon and modified from Alken (1974). Sixteen items in Part IV were prompted from the writings of Ernest (1993) and were modifications of research from Charalambous, Panaoura and Philippou (2009). Part V items (22 total) were prompted by the research of Tzanakis et al. (2000). Part VI items (7 total) were extracted from the NAEP Mathematics Teacher Background Questionnaire (2009).

A Likert scale consisted of 5 declarative sentences with choice responses varying in degree from strongly disagree with value of 1, disagree, neutral, agree, to strongly agree with value of 5. Among other declarative statements concerning teachers' philosophical orientations and beliefs about the nature of mathematics¹, participants were asked to select the responses that most closely related to their perceptions about mathematics, perceptions about HOM, the factors that encourage them or make it difficult to include HOM into their classroom instruction. We also collected extensive demographic data that included teacher education background, experience, the routes by which they obtained teaching license, etc. *Skip logic* (or branching) method was utilized to direct the teachers to a custom path of the

¹ Analysis of this portion of the survey is addressed in other publications.

survey that, depending upon their response of 'Yes' or 'No', led them to rate the statements pertaining to reason 'For' or 'Against' the inclusion of HOM.

Reliability analysis and scales formation. Prior to the collection of the data and in order to assure ease in delivery, quality of construction of the survey, and face validity a small pilot study with 12 contacts was conducted. The pilot study group and the participants of this study were mutually exclusive.

The analysis of the data was focused on teachers' beliefs about the nature of mathematics, perceptions about the HOM and the factors that affect their decision to integrate HOM in their curriculum. Of the 110 questions, 74 Likert (ordinal) questions were used to run a reliability analysis, which yielded a Cronbach's alpha value of .94.

To uncover the underlying structure of a relatively large set of variables (i.e., 74 Likert ordinal data items), an exploratory factor analysis with variance explained criteria were used, which resulted in formation of seven constructs: *Math Benefits*, *Joy/Interest in Math*, *Absolutist*, *Fallibilist*, *Teachers' Perceptions of History of Mathematics*, *Support the Inclusion of History of Mathematics*, *Does not support Inclusion of History of Mathematics*.

In this paper, we chose to highlight three constructs, *Perceptions of the HOM* (see Table 1), *Reasons for including HOM* (see Figure 1), *Reasons for not including HOM* (see Figure 2), and their relationship (see Tables 3, 4, and 5). The tables and the figures contain the statements with sufficient factor loading under each selected constructs. Analyses of the four remaining constructs have been addressed in other publications.

Data Analysis

Given the selected three constructs, our primary objective was to examine whether teachers' perceptions of the utility and importance of HOM influenced their decision to incorporate HOM in their instruction.

As a result of factor analysis, five statements had sufficient loading to form the construct *Teacher Perception of HOM*. These five statements and modal responses for the entire sample are shown in Table 1.

It is obvious that the majority of teachers responded favourably to the relevance of the HOM to mathematics learning, and believed that HOM has its values and place in the school curriculum. Apparently, many believed that learning of school mathematics may be facilitated by studying the cultural significance of mathematics, and that historical context may enhance motivation and encourage interest in learning mathematics.

There is little disagreement that *all students of mathematics should be taught some history of mathematics* (see Table 1, first statement). The fifth statement, on the other hand, suggests a possible constraint on the use of HOM. There are obviously many mathematics teachers who believe HOM is "necessary", "valuable", and an "assistance to learning", yet they think it does not make "understanding math easier". HOM may be viewed as an additional, time consuming burden, rather than a useful instrument for capturing students' attention and focusing their learning efforts. More discussion on this issue is provided below.

Nonetheless, the data for the entire sample suggest that if teachers' perceptions of the utility and importance of HOM influence their decision to incorporate HOM in their

instruction, the vast majority of our sample should be employing HOM. Unfortunately, as shown in Table 2 this is not true. A majority is using HOM but is only 55% of the total sample.

Table 1
Teachers' perceptions of HOM

	All students of mathematics should be taught some history of mathematics.		The history of mathematics is worthwhile and necessary to the understanding mathematics.		Knowledge of the history of mathematics is valuable to non-scientists or non-mathematicians.		Knowing the history of mathematics may assist students in learning mathematical concepts.		Understanding mathematics would be easier if the history of mathematics was taught.	
	Frq.	%	Frq.	%	Frq.	%	Frq.	%	Frq.	%
.00	20	5.4	20	5.4	20	5.4	21	5.7	21	5.7
Strongly Disagree	2	.5	5	1.4	8	2.2	2	0.5	10	2.7
Disagree	22	6.0	39	10.6	39	10.6	27	7.4	52	14.2
Neutral	52	14.2	128	34.9	111	30.2	59	16.1	162	44.1
Agree	205	55.9	140	38.1	156	42.5	206	56.1	98	26.7
Strongly Agree	66	18.0	35	9.5	33	9.0	52	14.2	24	6.5
Total	367	100.0	367	100.0	367	100.0	367	100.0	367	100.0

Table 2
Teachers' use of HOM

I use the history of mathematics in my classroom		
	Frequency	Percentage (%)
.00	30	8.2
No	133	36.2
Yes	204	55.6
Total	367	100.0

This point forward, we will refer to the group of 204 teachers who do include HOM as the 'Yes' group and the group of 133 teachers who do not include HOM as the 'No' group. It is clear when comparing Table 1 with Table 2 that there are many teachers in the 'No' group with a positive perception of HOM. By the same token, many teachers in 'Yes' group think that HOM may not necessarily make 'understanding mathematics easier'. Are there meaningful differences between the 'Yes' and 'No' groups relative to 'perceptions'? Table 3 and Table 4 show the responses of the 'Yes' and 'No' groups respectively. The comparison between two groups is shown on the Table 5.

Teachers in group 'No' (see Table 4) scored the items below the modal response of the group of all participants (see Table 1), which is consistent with their decision to not include the HOM into their instruction. Markedly, among many factors that affect teachers' decision, there is noteworthy evidence that teachers' beliefs about the importance of the HOM, their views on the benefits and values of the HOM play important role.

Table 3

Perceptions of HOM for those teachers who do include HOM

	All students of mathematics should be taught some history of mathematics		The history of mathematics is worthwhile and necessary to the understanding mathematics.		Knowledge of the history of mathematics is valuable to non-scientists or non-mathematicians		Knowing the history of mathematics may assist students in learning mathematical concepts		Understanding mathematics would be easier if the history of mathematics was taught	
	Frq.	%	Frq.	%	Frq.	%	Frq.	%	Frq.	%
.00	1	.5	1	.5	1	.5	2	1.0	2	1.0
Strongly Disagree	1	.5			2	1.0	1	.5	2	1.0
Disagree	3	1.5	11	5.4	7	3.4	7	3.4	18	8.8
Neutral	20	9.8	61	29.9	56	27.5	18	8.8	94	46.1
Agree	122	59.8	100	49.0	107	52.5	135	66.2	67	32.8
Strongly Agree	57	27.9	31	15.2	31	15.2	41	20.1	21	10.3
Total	204	100.0	204	100.0	204	100.0	204	100.0	204	100.0

Table 4

Perceptions of HOM for those teachers who do not include HOM

	All students of mathematics should be taught some history of mathematics		The history of mathematics is worthwhile and necessary to the understanding mathematics		Knowledge of the history of mathematics is valuable to non-scientists or non-mathematicians		Knowing the history of mathematics may assist students in learning mathematical concepts		Understanding mathematics would be easier if the history of mathematics was taught	
	Frq.	%	Frq.	%	Frq.	%	Frq.	%	Frq.	%
.00	1	.8	1	.8	1	.8	1	.8	1	.8
Strongly Disagree	5	3.8	5	3.8	6	4.5	1	.8	8	6.0
Disagree	25	18.8	25	18.8	29	21.8	19	14.3	31	23.3
Neutral	62	46.6	62	46.6	51	38.3	37	27.8	62	46.6
Agree	37	27.8	37	27.8	44	33.1	66	49.6	28	21.1
Strongly Agree	3	2.3	3	2.3	2	1.5	9	6.8	3	2.3
Total	133	100.0	133	100.0	133	100.0	133	100.0	133	100.0

We observe a significant difference in the two groups (see Table 3 & Table 4). Teachers in 'Yes' group appeared to be more likely to agree that all high school mathematic students

should be taught some HOM, that the HOM is worthwhile and necessary to the understanding mathematics, that the HOM is valuable to non-scientists or non-mathematicians, that knowing the HOM may assist students in learning mathematical concepts, and that understanding mathematics would be easier if the HOM was taught. That was anticipated and expected. It is just as important to note that the ‘No’ group, while not as strongly in support of these statements, is nonetheless quite favourably disposed to HOM except for the last statement regarding *‘Understanding math would be easier if the history of mathematics was taught’*.

Table 5

Independent sample test

	I include the History of Mathematics in my lessons	N	Mean	Std. Deviation	Std. Error Mean
Perceptions of History of Mathematics	No	132	3.206	.684	.060
	Yes	201	3.824	.573	0.40

<u>t-test for Equality of Means</u>							95% Confidence Interval of the Difference	
		t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Perceptions of History of Mathematics	Equal variances assumed	-8.906	331	.000	-.618	.069	-.754	-.481
	Equal variances not assumed	-8.585	245	.000	-.618	.072	-.760	-.476

The data suggest that teachers who tend to agree with the *Perceptions of History of Mathematics* statements will likely include the HOM in their classroom instruction. Possibly, this would indicate that teachers who view HOM as valuable, necessary, or worthwhile would tend to include HOM in their classroom lessons.

An interesting fact somehow related to inconsistency in ranking a few closely related statements can be observed. The teachers in ‘No’ group do not include HOM because they do not see that *‘Understanding mathematics would be easier if the history of mathematics was taught’*. However, the very similar statement, *‘Knowing the history of mathematics may assist students in learning mathematics concepts’* was rated at a much higher degree. One can raise a question why this group had such a varied rating for the seemingly similar ideas of *assisting*

in learning and making it easier. We speculate that the teachers in this group did not associate these statements, or didn't think that the whole purpose of teaching history of mathematics is indeed *assisting the students to learn, i.e., making it easier.*

But even more strikingly fact is that the teachers in 'Yes' group ranked the statement, '*Understanding mathematics would be easier if the history of mathematics was taught*', and the statement, '*Knowing the history of mathematics may assist students in learning mathematics concepts*' with a remarkable difference. It is also interesting to note that the 'Yes' group rated the statement, '*All students of mathematics should be taught some history of mathematics*' very high. However, seemingly one of the major reasons of inclusion of HOM (i.e., to make 'understanding mathematics easier') was ranked lower than expected. This would imply that 'Yes' group, also, did not view the learning of HOM as an aide to understanding mathematics.

We suggest that teachers would benefit from more guidance and training about the values and use of the HOM as a mean to assist understanding of mathematics.

Factors that Affect Teachers' Decision

The precipitating factors that affected 'Yes' group teachers' decision to include HOM are shown in the Figure 1 in descending order of modal response.

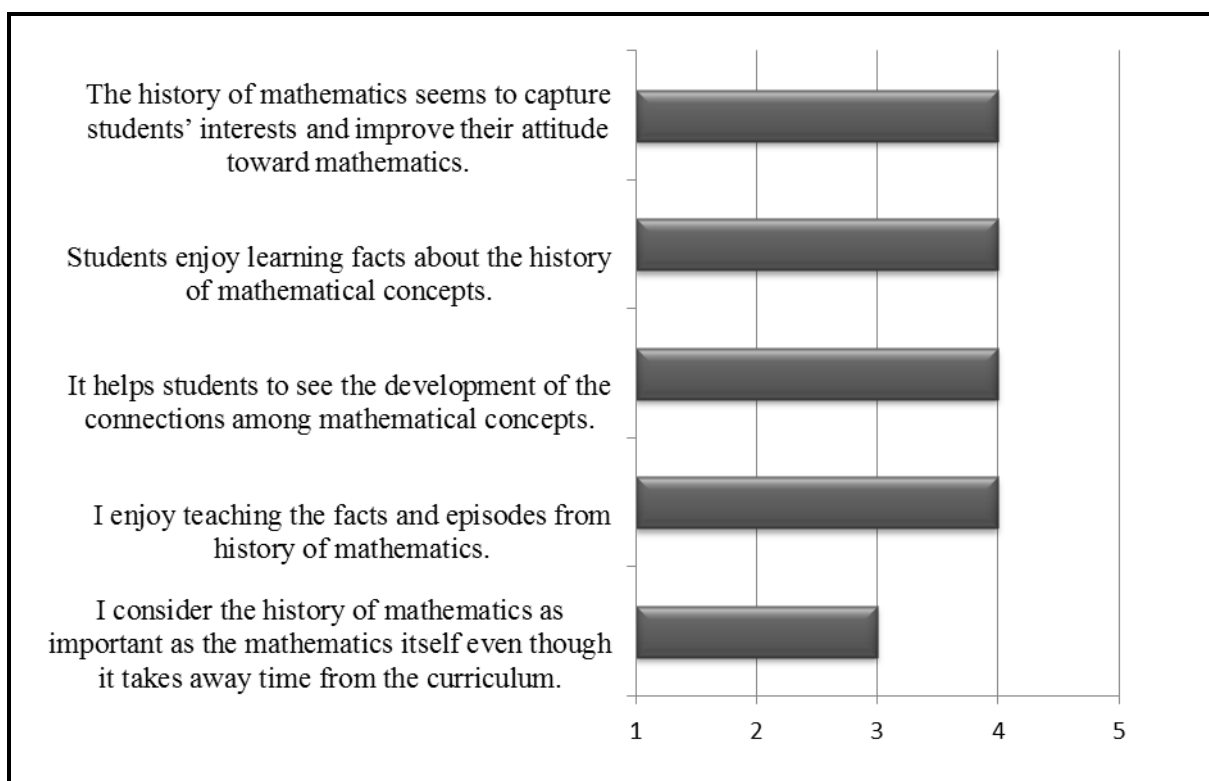


Figure 1. Factors that affect teachers' decision to include HOM

The top two responses indicate that 'Yes' group enjoyed teaching HOM, and the teachers believed their students enjoy learning HOM. The *enjoyment* seems a very important factor to this group. The third and fourth responses appear to center on the role that learning the HOM plays in the construction of knowledge.

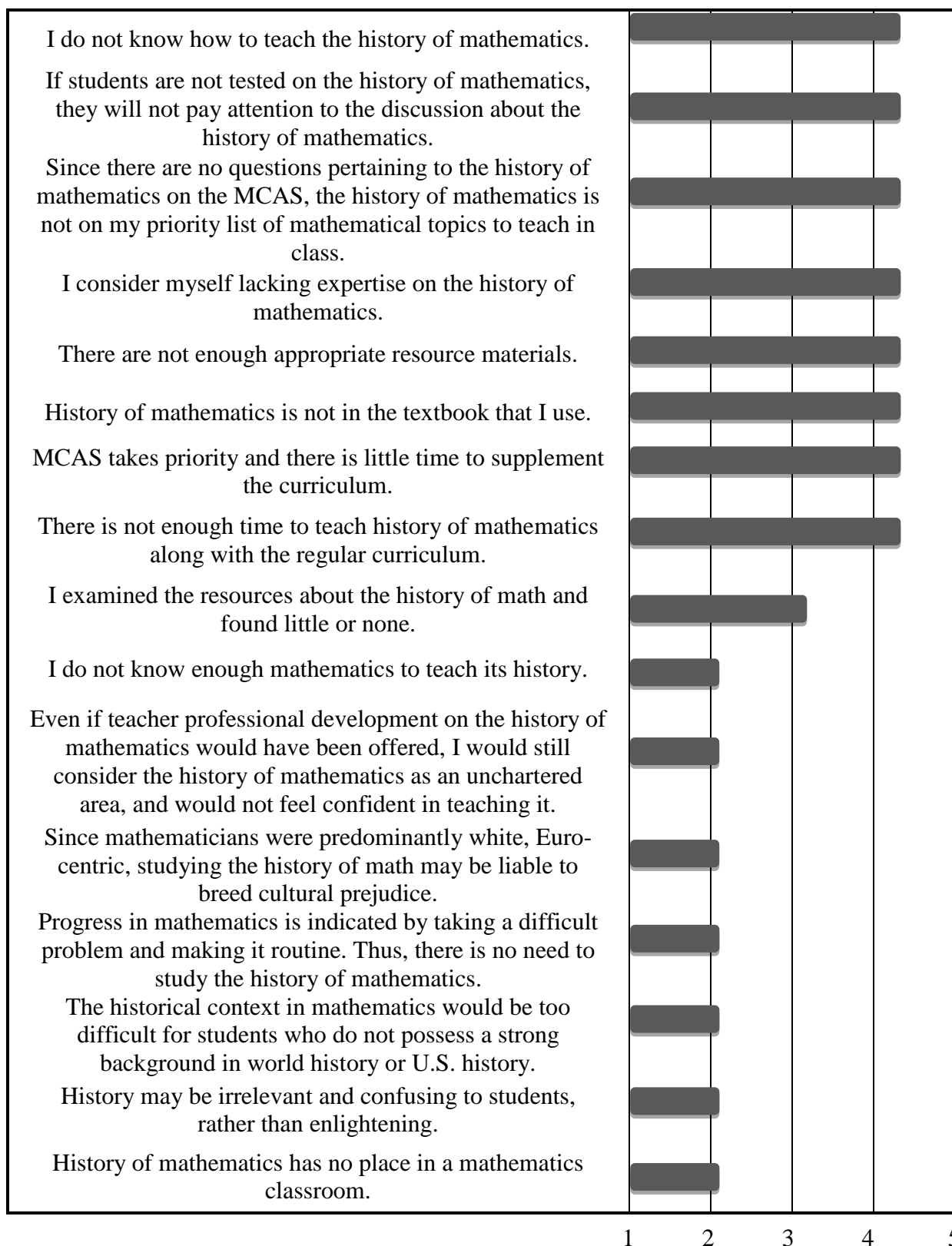


Figure 2. Factors that affect teachers' decision to not include HOM

Similar to the 'No' group, it seems the teachers in this group did not relate the fact that helping the students to see the development of and the connections among mathematical concepts, enhancing students' interests and improving their attitude toward mathematics are

closely linked to and are major contributors to making easier to understand mathematics. The factors that affect 'No' group are shown in the Figure 2 in descending order.

The ranking of each statement indicates the extent to which each mitigating factor would deter teachers from including HOM. All top eight factors were noteworthy and are addressed in the next section. Two of the factors (i.e., *I consider myself lacking experience in the history of mathematics*, and *I do not know how to teach its history of mathematics*) suggest a lack of confidence in the teaching of HOM. This prompted a closer examination of a relationship between teachers' decision to include HOM and the number of courses on the HOM taken by the teacher (see Table 6).

Table 6

The number of HOM courses taken by the teachers during their course study

		How many courses, on the history of mathematics, have you taken?				
		0 courses on the history of mathematics	1 course on the history of mathematics	2+ courses on the history of mathematics	Total	
I include the History of Mathematics in my lessons:	No	Count	80	43	7	130
		Expected Count	66.8	47.9	15.3	130.0
		% of Total	24.2%	13.0%	2.1%	39.3%
		Residual	13.2	-4.9	-8.3	
	Yes	Count	90	79	32	201
		Expected Count	103.2	74.1	23.7	201.0
		% of Total	27.2%	23.9%	9.7%	60.7%
		Residual	-13.2	4.9	8.3	
Total	Count	170	122	39	331	
	Expected Count	170.0	122.0	39.0	331.0	
	% of Total	51.4%	36.9%	11.8%	100.0%	

The results of the chi-square test of independence analysis indicate significance between the two variables ($p < .005$) (see Table 7).

To summarize this finding, in the group 'No' the majority of teachers never took courses on HOM and fewer teachers took 2+ courses on HOM. In the group 'Yes' fewer teachers never took courses on HOM and more teachers took 2+ courses on HOM. We can speculate that the more courses teachers take on the HOM, the more likely the teachers will include HOM in their instruction.

Table 7

Chi-square test results

	Value	df	Asymp. Sig. (2-sided)
Pearson Chi-Square	12.586 ^a	2	.002
Likelihood Ratio	13.380	2	.001
Linear-by-Linear Association	12.353	1	.000
N of Valid Cases	331		

a. 0 cells (.0%) have expected count less than 5. The minimum expected count is 15.32.

These data seem resonate with the data obtained on the *Perceptions of the History of Mathematics* construct. Analyzing all constructs and factors (Figure 1 and Figure 2), we offer several speculations and suggestions.

Discussion

There are myriad factors that affect a teacher's decision whether to integrate HOM into the fabric of their curriculum. In this paper we narrowed our consideration to the following important factors.

Time and High Stakes Testing

The shortage of class *time* is certainly a valid concern when it comes to consider supplementing the curriculum. This concern is amplified by the advent of the new Common Core State Standards required for classroom instruction (www.corestandards.org). It is highly likely that most teachers, across the United States, will soon be spending much of their planning and teaching time adjusting the curriculum to accommodate the new mandate. Unfortunately, nowhere in the Common Core State Standards there is a mention of students learning the HOM. Until federal and state education policy makers understand value in learning HOM at all levels of K-12 and supplement the Common Core Standards with their own requirement that students become familiar with HOM, it is dubious that HOM will be widely and consistently incorporated into an already prescribed curriculum.

State's high stakes testing was also indicated as a primary reason that has affected teachers' decision to not include HOM. There was a significantly high rank assigned by the participating teachers to the survey items clustered around issues related to testing (a modal ranking of 4 - 'Agree'). Obviously, *high stakes testing* is a priority, and unless HOM is included as an essential practice standard in State frameworks, most teachers would either not include HOM or would be forced to consider sacrifices of the curriculum to include HOM. However, it seems a pedagogical misconception to think that if students are not tested on the HOM, they will not pay attention to the classroom discussion about the HOM (see Figure 2). The benefit of the integration of HOM is not necessarily immediate and directly measurable. One cannot expect that there is a straightforward relationship between learning HOM and performance in mathematics. Nevertheless, the factors such as motivation, appreciation,

enjoyment, etc. are critical to facilitate engagement and interest toward mathematics, which are in turn prerequisites for better performance.

Teacher Knowledge of History of Mathematics

'*Confidence in knowledge and teaching history of mathematics*' is another highly ranked reason why teachers chose to not include HOM. The statement, '*I consider myself lacking expertise on the history of mathematics*' received a modal rating of 4 – 'Agree', and contains an important line of thought worth addressing. Research (Ball, 1988; Cooney, Shealy & Arvold, 1998; Dutton, 1962; Furinghetti, 2007; Philippou & Christou, 1998) suggested that teachers teach in the manner in which they were taught. They have subliminally built in the images of the pedagogy from their instructors. Thus, the teachers' presentation and discussion of mathematical concepts are largely affected by their past experience as students. This implies that the teachers who have had relatively little or no exposure to HOM being both high school and higher education students, may perceive themselves lacking expertise and consequently not expose their students to HOM.

We support the requirement of at least one course on the HOM during mathematics teacher preparation study. It would serve two important purposes to the teacher. Presumably, a course on the HOM would contribute to the expansion of the repertoire of mathematical content knowledge and build confidence in the teachers' ability to teach HOM. A teacher who is aware at least of a portion of the vast mathematical historical facts, events, developments, etc. and the role they play in connecting mathematical concepts to each other, will likely have inclinations to present mathematics as a story with centuries of twists, turns, trials and tribulations. Students will probably be excited to learn stories about "Pythagoras' irrational number scandal" and/or why some often use $22/7$ (which is a rational number) for π (which is irrational number). Thousands of years of the HOM provide a massive number of facts and stories which can be discussed in class to enrich mathematics curriculum. It is also feasible to assume that with increased repertoire of mathematical content knowledge and knowledge of HOM, teachers will be able to, confidently, present mathematics as a continuum of concepts, rather than a collection of disjointed concepts.

Evidence in Table 6 indicates that over half (51.4%) of the 331 teachers who responded to the survey statement, '*How many courses on the history of mathematics have you taken?*' have never taken a course on the HOM. This is perplexing. Victor Katz in his book, *A History of Mathematics: An Introduction* (1998) indicated that the majority of the United States certification requirements for teachers at secondary schools require a course in the HOM. This begs the question, "How and why were the majority of teachers responding to the survey certified with no courses on the HOM?" An answer to this question may reveal an ultimate constrain to the incorporation of HOM. In an open response, a participating teacher commented, "...even though one of the requirements for my Graduate Degree included the history of Math. [sic] That was conveniently wived [sic] for expediency." An option of "opting out" of a course on HOM, for the sake of expediency, is communicating to the teacher that HOM is not important and is not going to be used in the classroom. The ramification of such a decision needs to be questioned and examined. No one can ascertain the extent of benefit for teachers and students of learning HOM. However, to date, there

exists no empirical research that indicates HOM is *not* beneficial to teachers' and students' construction of mathematical content knowledge. This returns us to the point made in the introduction section that history of sciences has become increasingly important to the teaching of the basic sciences. Do the sciences have it wrong and only mathematics rightly appreciates the lack of benefit in understanding its history?

It also seems useful to notice that 90 teachers (27% of the total $N = 331$ responded) who indicated they had never taken a course on HOM, included HOM in their classroom instruction. This group of teachers is representative of a point discussed previously. We speculate that teachers who are intrinsically motivated and believe that HOM is an important and beneficial support to students' construction of mathematical knowledge will likely include HOM regardless their formal education.

Resources

The *availability of resources* is another deciding factor influencing the teachers' decision. Two of the most highly ranked statements of reasons for not including HOM were, '*History of mathematics is not in the textbook that I use*' and '*There are not enough appropriate resource materials.*' Time is a precious commodity for a teacher and having resources readily available and grade appropriate may be an explanatory factor for teachers' who do not include HOM. However, the current technological advances provide an amazingly easy access to an abundance of information related to practically any historical overview on any mathematics topic studied in the K-12 curriculum. We suggest that professional development projects, workshops, lectures, and study groups could serve as an excellent medium by which mathematics teachers at all levels can be introduced to the teaching of HOM. The survey asked the teachers if they would participate in the professional development focused on the History of Mathematics. Seventy-seven percent of teachers responded positively. This is a significant and key piece of the puzzle. There was not an overwhelming majority of teachers who felt HOM does not belong in the curriculum and there is a large majority who would be interested in knowing how to include HOM. It can be suggested that professional development provides a willing and advantageous avenue for the inclusion of HOM. Offering to teachers practical information on methods of integrating HOM may help to address the question of whether '*All students of mathematics should be taught some history of mathematics.*'

Enjoyment of Teaching and Learning of History of Mathematics

The humanizing benefit of the HOM strikes a chord with the affective domain of learning. As our data analysis shows, those teachers who *enjoyed* teaching HOM, believed their students *enjoyed* learning HOM. They believed that HOM helps students to see the development of the connections among mathematical concepts and provides the conditions to capture students' interests and improve their attitude toward mathematics, which in turn plays a critical role in the construction of knowledge and contributes to the development of "affective domain" (Krathwohl, Bloom, & Masia, 1973) of learning. Many researchers (Fauvel, 1992; Furinghetti, 1997; Siu, 2004; Smestad, 2009) believe the HOM provides students with the opportunity to construct a personal, visual, and emotional connection to the

development of concepts. Teachers who do include HOM are intrinsically motivated to integrate some topics from the HOM and agree that it will benefit the students by capturing their interest, and perhaps assist, in the learning of mathematics. It is unfortunate that HOM is not a part of the states' frameworks required curriculum. Being supported by the states' mandates or recommendations, teachers would not be alone with such a theory.

Conclusion

The HOM may not only be a benefit to the student but also to the teacher. The impact of the factors mentioned above is far reaching. Of particular importance is the fact that teachers indicated a lack of confidence in teaching HOM. The HOM may serve as the foundation upon which the teacher can construct strong mathematical connections which would, in turn, help to strengthen the teacher's mathematical content knowledge and confidence. Following are recommendations that may lead to greater integration of HOM in the classroom.

Teaching mathematics with no knowledge of the HOM is tantamount to a lawyer being allowed to practice law having no knowledge of the history of the judicial system, or scientists immersed in science without knowing its history. Higher education cannot waiver on the importance of knowing the HOM. To do otherwise, suggests there to be little or no importance in the value of knowing the HOM. It suggests that awareness of the interconnectedness and necessity of the development of mathematics concepts are not essential to the understanding of mathematics itself, and to teaching and learning of mathematics in particular.

This is a significant reason for a requirement, and adherence to teachers learning the history and *nature* of mathematics. There is strong need for students to view mathematics as a human creation that began thousands of years ago and is ever changing. Students who view mathematics as a set of discrete, unrelated topics may have difficulty in understanding the relational worth of each mathematical concept, its attachment and value to human life.

In addition to undergraduate and graduate education in the HOM, we suggest a systematic professional development pertaining to HOM. Professional development sessions are an excellent way to introduce teachers, at all levels, to the resources for teaching HOM, and it is a great opportunity for teachers to acquire the necessary pedagogical content knowledge as well as build upon mathematics content knowledge. The National Council of Teachers of Mathematics (NCTM) initiated a professional development scholarship emphasizing the History of Mathematics (NCTM, 2011). The goal of NCTM is to:

provide financial support for (1) completing credited course work in the history of mathematics, (2) creating and field-testing appropriate classroom activities incorporating the history of mathematics, and (3) preparing and delivering a professional development presentation to colleagues. (NCTM, 2011)

In summary, including HOM as part of classroom instruction requires a cadre of participants. It requires the teachers to include HOM in classroom instruction, not because it is mandated, but because there is an intrinsic desire to understand the importance of students making connections across the mathematical discipline. Teachers need to accept that HOM is one of the avenues that will accomplish that goal. It requires higher education, administrators,

and parents to expect a standard that teachers are entering the classroom with a strong mathematical background and confidence in their content knowledge. A teacher whose mathematical content knowledge is supported by knowledge of HOM will likely be aware of the use of HOM as an aide to students in their construction of mathematical concepts. For the HOM to be included in the classroom, publishers will need to provide teachers with the necessary and abundant resources that allow for the inclusion of HOM. Finally, it will require the inclusion of HOM as a Standard of Mathematical Practice for State frameworks. It was Aristotle who said, “If you would understand anything, observe its beginning and its development” (384 BC – 322 BC).

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