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Grade Twelve Students Establishing the Relationship Between Differentiation and Integration in Calculus Using graphs

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Abstract

Calculus is an important subject for science, engineering, and other fields of studies but phenomenally it is abstract and difficult to learn. Despite its importance, the teaching of introductory calculus always emphasizes manipulation of algebraic notations and rote learning. Most students learn the how instead of the why of calculus due to extensive use of algebraic symbols and notations. Therefore, graphing activities were developed based on the learning cycle approach and the lesson was taught for an hour in the classroom. This study investigates the effectiveness of the lesson in understanding the relationship between differentiation and integration calculus and to measure the students' attitude towards the learning unit. Sixty-five grade twelve students from higher secondary school were selected for the study. The test scores showed that there was significance improvement in the post-test scores compared to the pre-test scores. It was also found that the lesson was effective and enriching.

Keywords: calculus, differentiation, integration, graphing activity

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Introduction

The ideas of calculus are one of the greatest achievements of the human intellect (Hughes-Hallett et al., 2003) since calculus has demonstrated the power to illuminate the most fundamental problems in mathematics, physical sciences, biological sciences, and engineering. Calculus has reduced complicated problems to simple rules and procedures by using symbols and notations not only to represent a shorter way of writing but also to find the solutions easier. However, using those symbols and notations might lose the original pictures of the problems.

By contrast, teaching and learning of calculus begin with the symbols and notations and follow by several examples of its applications. Lecture-based teaching methods are the top choices in calculus instruction for decades. Students develop a skill at memorizing formulas and algebraic procedural steps instead of conceptual understanding. Besides, a vast number of calculus textbooks are available, covering every conceivable approach; these calculus textbooks contain a lot of abstract symbols and notations. A wide range of problems in the textbooks is solved using memorized formulas and procedural steps. According to Orton (1983a and 1983b) most students have learned calculus ‘how’ rather than ‘why’ due to extensive use of notations and symbols in teaching and learning. The real meanings of symbols and notations that students learned in the classrooms are not interpreted explicitly in the context of real world situations.

With these reasons, mathematics educators felt that calculus education needed reform focusing on conceptual understanding rather than the acquisition of procedural skills (Peterson, 1986; Steen, 1988). Since the calculus reform movement began, many researches were conducted on concept-based approaches to teach the fundamentals of calculus, e.g. a realistic approach (Kaput, 1994), a guided reinvention (Gravemeijer & Doorman, 1999), a computer-assisted approach (Lang, 1999), and a graphical approach (Tall, 1986). Most of these approaches employed contextual examples (e.g., distance and velocity of a moving car) without the real activities. As a result, students were required to imagine the context of the situations. If students cannot form clear mental pictures of contextual examples, they memorize the steps and follow rote learning. Graphing is a powerful tool in teaching as well as learning calculus. Graphical representations in calculus can help students to visualize underlying concepts. Graphs can translate and interpret algebraic formulas and data. Moreover, graphs often reveal mathematical results simply and clearly.

Students have a vague concept of algebraic notations in relation to geometric interpretations. Mundy and Graham (1994) also reported that the difference between the performances on the procedural items and the conceptual ones was due to the separate understanding of geometrical and algebraic context in calculus. From our pilot study, which was carried out in undergraduate students, they had difficulty in explaining the relationship between differentiation and integration. They recognized that “integration is the inverse process of differentiation” whereas they remained silent when asked to explain “how”. Since the concepts of calculus are originated from contextual applications, using contextual activities and interpreting them in the form of graphs would ultimately help students to relate the concepts of calculus to algebraic symbols and notations. In addition, the students will get the physical feel and visualization of the concepts.

This study focused on development of graphing activities based on the learning cycle approach to help students to establish the relationship between differentiation and integration in calculus. The main objectivities of this study were to find out the effectiveness of the developed learning unit on the students' understanding of the relationship between differentiation and integration in Calculus and to measure the students' attitude towards the learning unit. This study aimed to address the following research questions;

i). To what extent can the learning unit enhance the students' understanding the relationship between differentiation and integration in Calculus?
ii). What is the students' attitude towards the learning unit?

The Learning Cycle Approach

The learning cycle approach is an inquiry-based teaching model, which first emerged in the 1960s when Robert Karplus and his colleagues implemented in the Science Curriculum Improvement Study (SCIS) program (Lawson et al., 1989). It was developed by Robert Karplus based on the constructivist theory of intellectual development proposed by Jean Piaget (Karplus, 1980). In Piaget's intellectual development, the learning is not view as a transfer, but as an active construction of knowledge by the individual based on the knowledge already held (Piaget, 1952).

The learning cycle approach has an advantage in learning by ordering the instructional activities compatible with Paiget's cognitive development. In order to facilitate accommodation, the activities in the exploration phase expose the learner to a segment of the environment that demonstrates the information to be accommodated. In the second phase, the activities help the learner to accommodate the information. Finally, to organize the accommodated information, the activities are developed to help the learner to see the relationship between new information and other previously learned information (Abraham, 1997).

The learning cycle has been implemented in various studies even though the names of the phases have changed. Originally, Exploration, Invention, and Discovery phases were named by Karplus and his colleagues. Many authors have modified the names of these phases, eg. Barnes, Driver, Karplus, Erickson, Nussbaum and Novic, Renner, and Rowell and Dawson, but the learning format and sequence of the phases remain the same (Lawson et al., 1989; Sunal, 2007). However, the phases of exploration, concept introduction, and concept application described by Anton Lawson (1988) and Michael Abraham (1989) are the foundation phases most closely related to the pioneer in the learning cycle, Robert Karplus, and called Lawson-Abraham model of learning cycle.
The Lawson-Abraham's model of learning cycle consists of the Exploration, the Concept Introduction, and the Concept Application phases.

(i). Exploration phase

This is the most active phase for the students. They learn through their own actions and reactions with minimum guidance in an activity to expose them to the concepts. The students try out their knowledge by observation and investigation through the activity. The students are expected to encounter situations that they cannot explain with their present ideas or reasoning patterns. The teacher acts a facilitator by probing guiding questions and serving as a resource for the students.

(ii). Concept Introduction phase

In this phase, the concept is introduced and explained with help from the teacher. The concept is usually derived from the data or classroom discussions. This step should always follow exploration and relate directly to the pattern discovered during the exploration activity. The students should be encouraged to identify as many new patterns as possible before the concept is revealed to the class.

(iii). Concept Application phase

In this phase, the students explore the usefulness of the concept they have learned and apply it to new situations. This phase is necessary to extend the range of applicability of the new concept. Without a number and variety of applications, the concept's meaning may remain restricted to the examples used at the time it was initially defined and discussed. In addition, application activities aid students whose conceptual reorganization takes place more slowly than average or who do not adequately relate the teacher's original explanation to their experiences.

Methodology

1. Participants

Single group pre-test post-test research design was used in this study. Two sections of grade twelve science students were selected for the study. These students had already learned introductory calculus in grade eleven.

2. Procedures

A pre-test was conducted for sixty minutes prior to the intervention. The learning unit on the establishment of relationship between differentiation and integration in Calculus was taught for one hour. At the end of the instruction, a post-test and an attitude test were conducted.

A same set of five open-ended questions was used for both the pre-test and the post-test. The first question examined whether students could relate the graph of an anti-derivative to that of its derivative as well as the units in those graphs in the straight-line motion context. The second question examined whether students could relate the graph of a derivative to that of its anti-derivative in the same context. The third and fourth questions examined the students' conceptual understanding of integration and differentiation respectively. The fifth question examined how students relate the derivatives (slopes of the lines) to the integrals (areas under the lines) of the lines, and also to examined how students establish the relationship from differentiation to integration

The attitude questionnaire consisted of twelve Likert-type items and three open-ended questions. The purpose of the questionnaire was to find the students' attitude towards the relationship between differentiation and integration and towards the learning unit. The Likert scale in the questionnaire included "1 = Strongly disagree", "2 = Disagree", "3 = Neutral", "4 = Agree" and "5 = Strongly agree".

A paired sample *t*-test was used for data analysis to determine whether significant difference between the pre-test and post-test scores exists. The Cronbach's Alpha reliability coefficient of the post-test was 0.67. The frequencies of the responses to each questionnaire item were separately tabulated and interpreted. The Cronbach's Alpha reliability coefficient of the questionnaire was 0.82.

Graphing activities learning unit

The Lawson-Abraham model of learning cycle was used to frame the development of a learning unit on the relationship between differentiation and integration in calculus, which employed graphing activities. The students were divided into groups of 5 students. All the instructions for the group activities were provided on the worksheet given to the students. The details of the activity in each phase of the learning cycle are described below.

(i). Exploration phase

In this phase, students were asked to sketch the graph of the constant speed of a moving car for five hours, and to divide the area under the line into five equal parts as shown in Figure 1. Students were asked to find the area of each part under the line and to find the unit and the meaning of those areas, which should help them realize the graphical relationship between speed and distance.

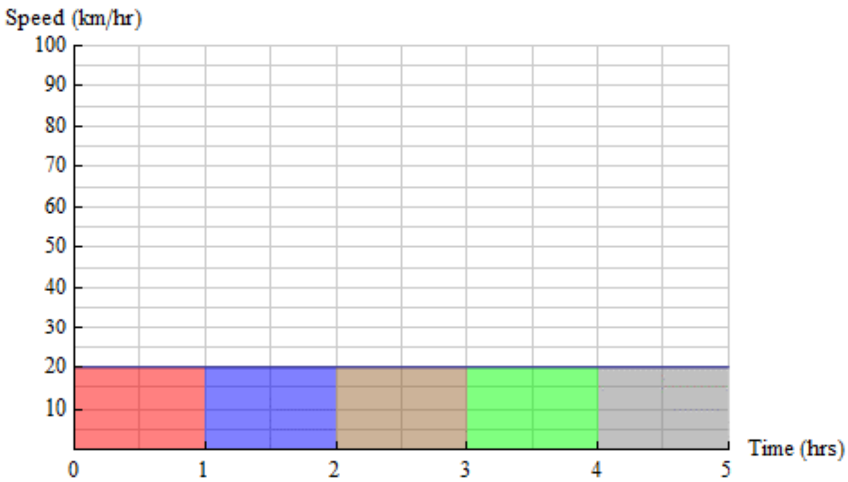
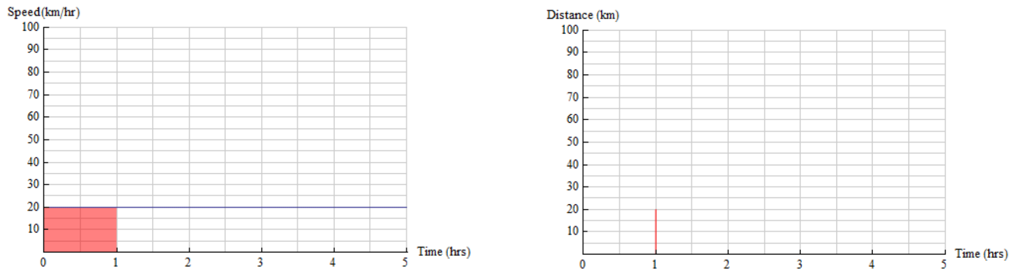


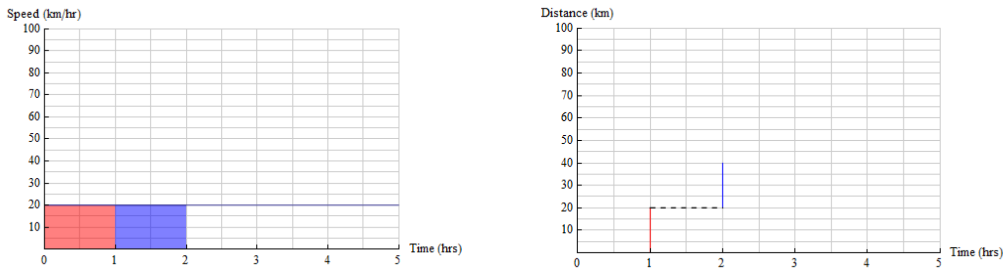
Figure 1 Graph showing constant speed of a car and area under the line.

Students were asked to plot each area (distance) on another graph paper and compare the two graphs, which were actually the graphs of a derivative and its anti-derivative as shown in Figures 2-6. Students found the area under the line from $t=0$ to 1 hour, 1 to 2 hours, 2 to 3 hours, 3 to 4 hours, and 4 to 5 hours (see Figures 2(a)-6(a)) and sketched the area on another graph as shown in Figures 2(b)-6(b), which eventually yielded the area under the speed-time graph (see Figure 7(a)) and the distance-time graph (see Figure 7(b)). They were further asked to find the equations of the two graphs in Figure 7. Being more familiar with algebraic notation, the equations should help them in confirming the relationship between the two graphs.



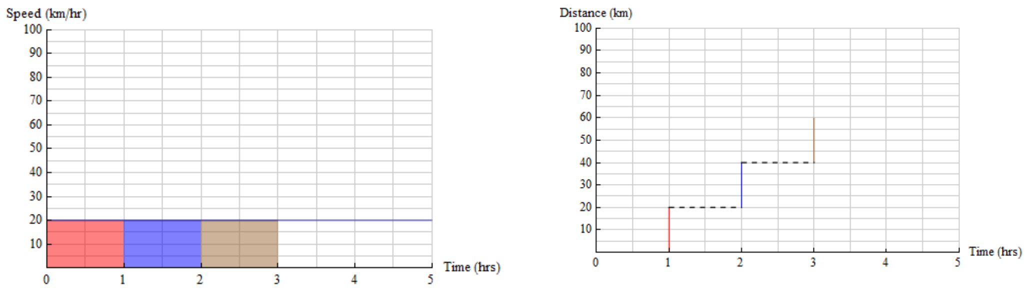
(a) (b)

Figure 2 Area under the line and distance during the first hour



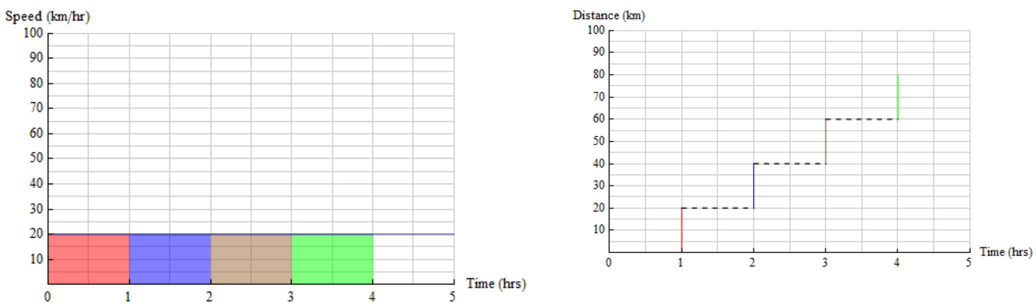
(a) (b)

Figure 3 Area under the line and distance during the second hour



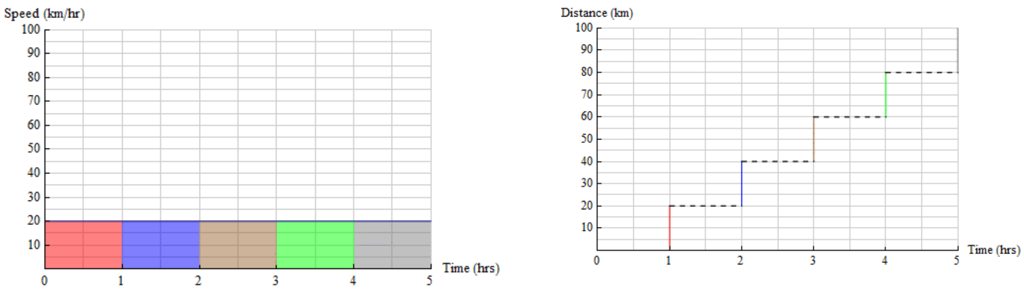
(a) (b)

Figure 4 Area under the line and distance during third hour



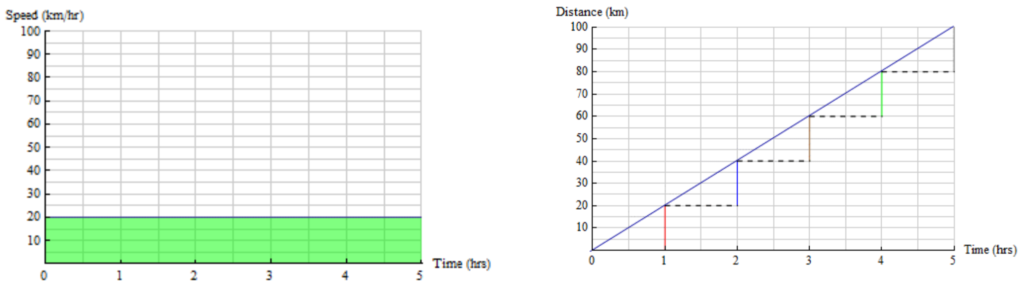
(a) (b)

Figure 5 Area under the line and distance during fourth hour



(a) (b)

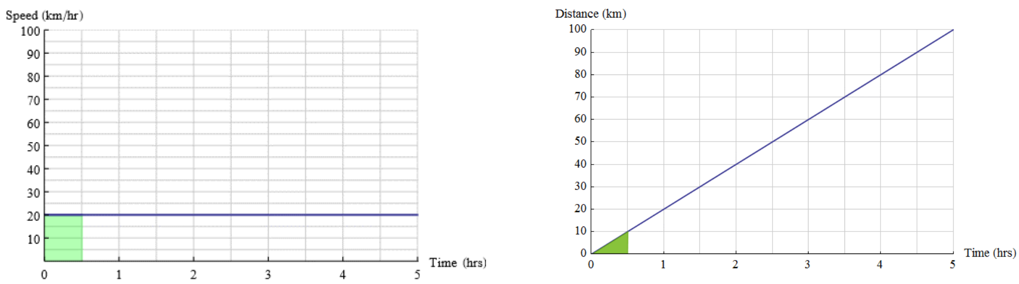
Figure 6 Area under the line and distance during fifth hour



(a) (b)

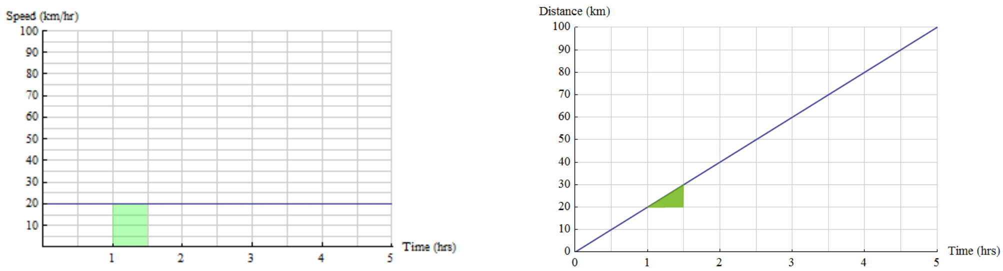
Figure 7 Area under the speed-time graph and slope of the distance-time graph

To help students see that the graphical relationship work for non-integers as well, they were asked to find the areas and the distances during the last half hour prior to t = 0.5, 1.5, 2.5, 3.5 and 4.5 hours as shown in Figure 8-12.



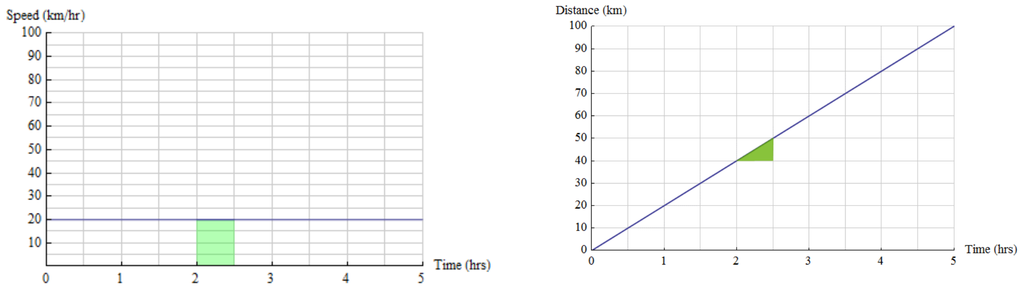
(a) (b)

Figure 8 Area under the line for $t = 0.5$ hours and slope of the line at $t = 0.5$ hours



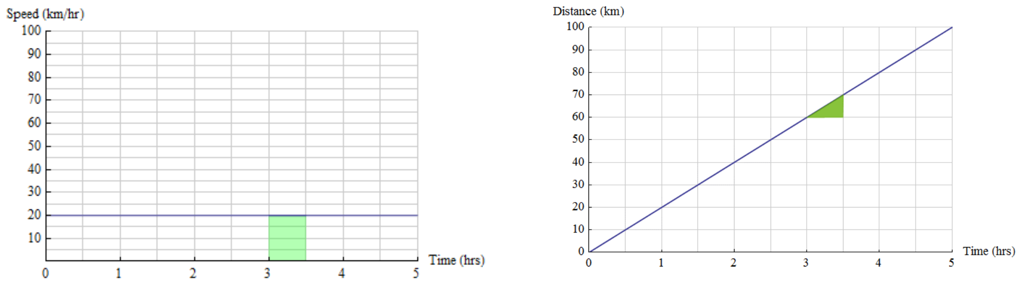
(a) (b)

Figure 9 Area under the line for $t = 1.5$ hours and slope of the line at $t = 1.5$ hours



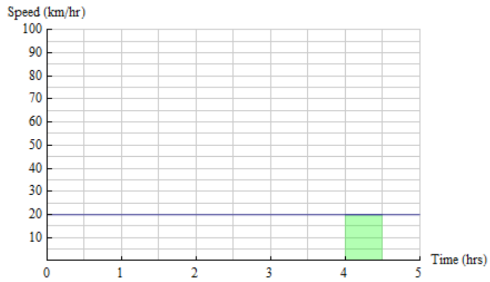
(a) (b)

Figure 10 Area under the line for $t = 2.5$ hours and slope of the line at $t = 2.5$ hours

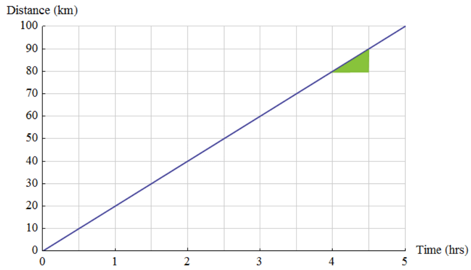


(a) (b)

Figure 11 Area under the line for $t = 3.5$ hours and slope of the line at $t = 3.5$ hours



(a)



(b)

Figure 12 Area under the line for $t = 4.5$ hours and slope of the line at $t = 4.5$ hours

Finally, they were asked to directly calculate the area under the line in Figure 7(a) from *t* = 0 to t = 5 hours by integration and to confirm that the result agreed with the area and the distance in the two graphs. They were also asked to find the slope of the line in Figure 7(b) at *t* = 0.5 hours, which gave the point on the line at *t* = 0.5 hours in Figure 7(a).

(ii) Concept introduction phase

From the exploration phase, students should begin to have an idea about the relationship between differentiation and integration. The concept introduction phase should help them formulate the idea more completely. The students were asked the following questions.

i). What do you get if you find the area under the graph in Figure 7(a) by integrating the equation of the line from *t* = 0 to t = 5 hours algebraically?

ii). What is the unit of the area? And what does the unit of the area tell you?

The students should be able to see that finding the area under the line and integrating the line of the equation would give 100 km, which would indicate the distance travelled by the car in 5 hours as shown in Figure 7(b). Then, the students were also asked the following questions.

i). What do you get if you find the slope of the line graphically and differentiate the equation of the line algebraically of the graph in Figure 7(b)?

ii). What is the unit of the slope?

The students should be able to see that the finding the slope of the line graphically and differentiating the equation of line algebraically would give 20 km/hr which would indicate the speed of the car on the graph as shown in Figure 7(a). Then, the students were asked; do you see any relationship between the graphs in Figures 7(a) and 7(b) in terms of differentiation and integration in calculus? Explain?

Now, the students should be able to see the relationship between differentiation and integration graphically, algebraically, and contextually from the activity and to conceptually understand that integration is the inverse process of differentiation.

(iii) Concept application phase

In this phase, the context was still a moving car but accelerating at 2 m/s^2 for 10 seconds instead of travelling at a constant velocity. From the exploration and concept introduction phases, students should have an idea how to figure out the equation of the area under the line from a given graph in general. The students should be able to figure out the equations of the lines from the graphs as shown in Figure 13.

Finding the area of the shaded region under the graph in Figure 13(a) would yield $2t_0$ which could be generalized to the equation of the line ($v(t)=2t$) in Figure 13(b), and finding the area under the shaded region under the graph in Figure 13(b) would give $\frac{1}{2}\times 2t_0 \times t_0 = t_0{}^2$ which could be generalized to the equation of the line in Figure 13(c)

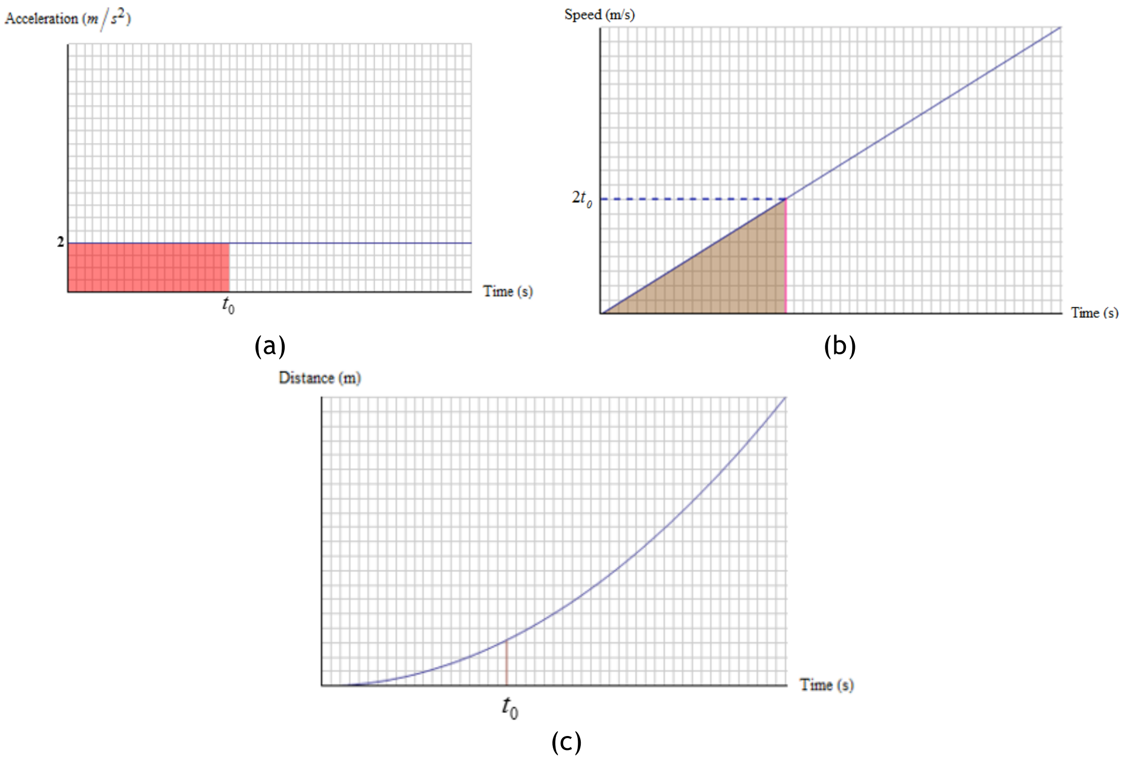


Figure 13 Graph showing how to derive the general equations of the lines

The students were asked to sketch the acceleration-time graph, to find the area under the graph as shown in Figure 14, and to plot the area, which was in fact the velocity, on another graph paper as shown in Figure 15. Then, finding the area under the graph in Figure 16 would give the distance, whose graph is shown in Figure 17.

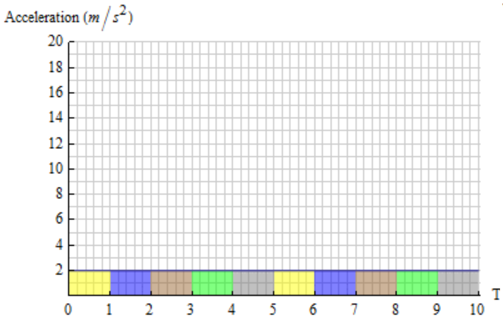


Figure 14 Acceleration-time graph.

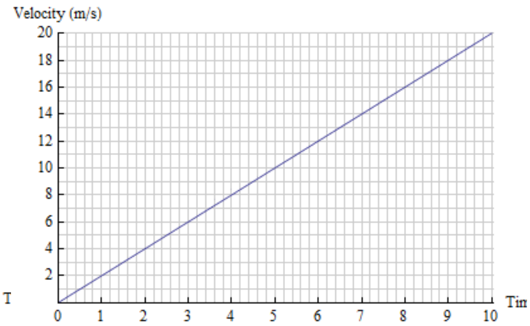


Figure 15 Velocity-time graph.

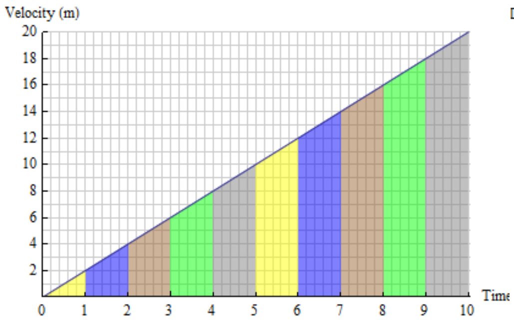


Figure 16 Velocity-time graph showing area under the line.

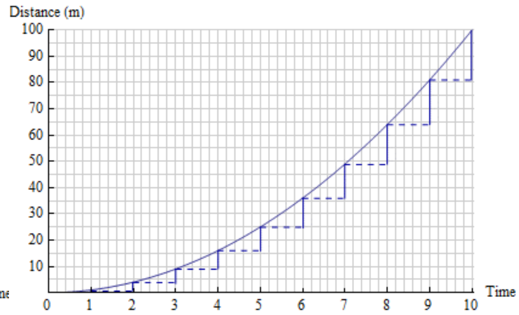


Figure 17 Distance-time graph.

Finding the derivatives of the distance-time graph in Figure 17 would give back the velocity-time graph in Figure 15 and finding the derivative of the velocity-time graph would give the acceleration-time graph as shown in Figure 16. Then, students could use the algebraic method of integration to confirm the findings of the velocity-time and distance-time graphs from the graphical method and the same was true for the reverse process of differentiation. It should be noted that the non-linearity of the distance-time graph could also be used to emphasize the instantaneous nature of a derivative, both graphically and algebraically.

Results

1. Students' performance

A paired sample *t*-test showed that the average post-test score was significantly greater than the average pre-test score (see table 1). After the intervention, there was significant improvement in students' performance. The mean score in the pre-test was extremely low as many students found the questions difficult, which indicated that students had no conceptual understanding in regard to differentiation and integration in general and to the relationship between differentiation and integration in particular. Some of the students scored much higher in the post-test despite the fact that the intervention lasted for only one hour.

Table 1 Paired sample *t*-test of pre-test and post-test result

Test	Mean	Standard deviation	t	Sig. (2-tailed)
Pre-test	4.36	3.09	16.83	0.00
Post-test	13.50	3.27		

2. Students' attitude towards the learning unit

The questionnaire was administered to 65 students for 30 minutes after the post-test to address the second research question: What is the students' attitude towards the learning unit? The questionnaire consisted of twelve closed-ended Likert-scale questionnaire and one open-ended (1) suggestions and comments regarding the activities in the learning units. (see Table 2)

The students' responses to each Likert-type item were analyzed by the frequency and mean of the students' responses to determine the students' attitudes towards the learning units as shown in Table.2

Table 2 Students' responses to the questionnaire.

	Items	1	2	3	4	5	Mean	SD
1	I like the activities in learning calculus.	1	5	8	28	23	4.03	0.97
2	I found the graphing activities in the calculus lessons interesting.	1	5	11	35	13	3.83	0.89
3	It was too difficult to learn calculus by doing graphing activities.	11	23	19	7	5	2.57	1.13
4	I find reading the textbook in detail is by itself sufficient for me to learn calculus.	31	16	12	3	3	1.93	1.13
5	I learn calculus better by reflecting on these activities instead of only by book and memorizing.	1	2	7	36	19	4.08	0.81
6	I look forward to solve more problems in calculus after the activities.	6	11	17	19	12	3.31	1.22
7	I will understand better if other topics in mathematics are taught using activities like the ones used in this calculus lesson.	2	7	14	26	16	3.72	1.05
8	The time was too short for the lesson.	4	4	13	26	18	3.77	1.11
9	The calculus lessons need more exercises until I understand and become fluent.	0	3	5	26	31	4.31	0.81
10	Mathematics teacher teaching calculus is enthusiastic in teaching calculus.	0	2	24	32	7	3.68	0.71

	Items	1	2	3	4	5	Mean	SD
11	Mathematics teacher teaching calculus is encouraging and approachable during the lessons.	0	6	17	34	8	3.68	0.81
12	Mathematics teacher teaching calculus taught the lessons too fast and I could not follow the instruction.	17	18	15	11	4	2.49	1.23

In the closed-ended Likert-scale questionnaire, 51 out of 65 students liked the activities in the calculus lessons and 48 students found the activities in the learning units interesting and enriching. Thirty-four students found that it was not very difficult to learn calculus by doing the activities and 47 students found that reading the textbook in detail was not sufficient for them to learn calculus. Fifty-five students responded that they learned calculus better by reflecting on the activities instead of only by reading books and memorizing, and 31 students looked forward to solve more problems in calculus after the activities. The majority of the students preferred to learn other topics in mathematics using activities like the ones used in the calculus lessons. Forty-four students found that the time was too short for the lessons in calculus and 57 students needed more exercises in the lessons.

The majority of the students felt that the time was too short for the lessons. The problem of time limitation was not under our control. We had to follow the mathematics curriculum issued by Department of Curriculum and Research Development, which specified in detail how long each topic should be taught. However, the majority of the students found that the instructor was enthusiastic in teaching calculus and encouraged the students in the learning process.

Discussion and Conclusion

In introductory calculus courses, a lot of attention is paid to how to do the calculations and manipulations of formulas instead of why and how they work. Students are usually taught calculus by means of what Ryan (1992) described as ‘the rush to rule’ where the meaning is ignored and students operate on a purely mechanical level, pushing symbols and notations on paper. This is the main problem that students face in conceptual understanding of calculus. Calculus originated from the study of motion, which is realistic in nature with rich history and experiences common to all human beings. Students are hardly taught calculus using graphs or realistic or experimentally real situations and visualization tools. Although, there have been numerous studies done to teach the concepts of the fundamentals of calculus but there has been no concrete study done to improve the teaching of the relationship between differentiation and integration. Differentiation and integration are taught separately mostly using an algebraic approach, which makes it difficult to visualize the relationship between differentiation and integration. Students just spell out the relationship by following what is written in textbooks—“Integration is the inverse process of differentiation”—and cannot explain how and why as it is difficult to visualize the relationship seeing only algebraic symbols and notations.

The pre-test result showed that the students often lacked certain conceptual understanding in differentiation and integration from traditional mathematics teaching. The students learnt calculus without actually understanding differentiation and integration as well as their relationship. The concepts of differentiation and integration were traditionally taught by focusing only on algebraic methods in an introductory calculus course at the higher secondary level. The students saw calculus as a series of process associated with algorithms and could not apply the concepts in the contextual situations. This agrees with Tall's (1992) findings that students instead of having conceptual view of the symbols and notations in differentiation and integration, they focus only on a process-oriented view. The students encountered difficulty in relating the functional notations of differentiation and integration to the context of motion.

The average score of the post-test was significantly higher than that of the pre-test, indicating that the developed learning unit could enhance the learning achievement of the students. The hands-on graphing activities helped the students think logically, develop their own reasoning skills, and ultimately invent their own concepts of the relationship between differentiation and integration. Sokolowski et al. (2011) and Orhun (2012) also employed graphing activities in contextual settings to enhance students' understanding of calculus. The reliability of the posttest was rather low due to the difficulty in learning calculus and to the open-endedness of the questions.

The students found the activities interesting and enriching probably because they took active roles in the lesson and felt motivated to learn calculus, hence leading to better performance. To really understand the relationship between differentiation and integration, students obviously need to understand both differentiation and integration, which are themselves based on more fundamental concepts like limit and continuity and discontinuity of a function. To understand these concepts, the lessons must focus on context-based activities rather than on algebraic methods (Tall, 1992). Students usually learn and retain better when they are actively involved in the lessons and the concepts can be visualized. The coverage of the fundamentals of calculus is necessary before the learning of the relationship between differentiation and integration can take place. Such a coverage will require a much longer intervention duration. It is hoped that a coherent lesson on calculus based on contextual activities will be developed.

Learning by doing is an unreplaceable approach consistent with the constructivist theory of learning. As said in Chinese proverb "Tell me and I will forget. Show me and I may remember. Involve me and I will understand." When students are physically and mentally involved in the activities, they get the real feel of what they are doing which helps them retain what they have learned. In this study, the activities are designed to help students understand the calculus concepts better by involving physically in the contextual graphing activities.

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