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Euclidean Geometry's Problem Solving Based on Metacognitive in Aspect of Awareness

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ABSTRACT

Solving mathematical problems, as the main subject, is intended to improve one's ability in mathematics. The approach adopted in this present research was a qualitative one with the subject of the second semester students of mathematics in mathematics department. Six students consisting of two students under high, two middle, and two low ability categories were involved in this research. The data were obtained through four problems in the geometry subject test. The validity test employed was the item validity and the four exersices showed the coefficients of 0.79; 0.75; 0.70, and 0.82, respectively, meaning that the four exersices fulfilled the problem validity, meanwhile the test of reliability showed the coefficient of 0.78, namely the problems also met the reliability requirement. The results of the research showed that students were aware of what to plan and to do in the problem solving. The respondents realized them by writing the aspects they knew and the problems they intended to solve. In terms of the learning results, the two groups, high and middle, possessed some awareness in problem solving, but the students under the low category may be said to have less awareness of what to do in problem solving.

KEYWORDS
Awareness, metacognitive, problem solving

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Introduction

Any research on problem solving in term of the awareness aspect is the one dealing with any effort in making one aware of his activities made, including his thoughts, strategies adopted, need for planning, thinking process and his efforts to understand the problem (Heidari & Bahrami, 2012; In'am, 2015)

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The number of similar researches discussed is one of the meaningful aspects in doing this research and some researches relevant to the present research will be presented below.

Keichi (2000) in his research of metacognitive in mathematics education showed the following results: 1) metacognitive played important roles in problem solving; 2) students were more skillful in problem solving if they possessed metacognitive knowledge; 3) in the framework of problem solving, teachers often gave more emphasis on specific strategies and on some important characteristics in other problem solving activities, and 4) teachers showed impressive expressions in mathematical reasoning.

Meanwhile Saad (2004) in his study on algebra problem solving viewed for four metacognitive aspects showed that the awareness aspects were under the good enough categories. This finding is also confirmed by In'am's study (2012) on the development of the cognitive-based algebra learning model. The implementation of the model was made in State Junior High Schools in Indonesia, in each school six students consisting of two students under high ability, two under middle, and two students under low ability categories were involved. The research results showed that the awareness aspect was under good enough categories.

Another research dealing with metacognitive has been made by O'neil & Abedi (1996). Their research studied the influence of different question formats in mathematical evaluation on metacognition and attitudes. In the study, the respondents were 59 classes from eight Junior High Schools in California. The results of this research dealing with the cognitive strategy aspect showed that: there were significant influences of: 1) the sexes on the use of the cognitive strategy in answering open-ended and multiple choice questions; 2) the question formats on the cognitive strategies, and 3) the interaction of the sexes and the question formats on the use of the cognitive strategies, where female students employed far more cognitive strategies than male students, and both groups used more cognitive strategies on the open-ended questions than on the multiple choice question.

Heidari & Bahrami (2012) in their research showed that the awareness aspect, one of the metacognitive aspects, had a significant relation to the university students' thinking style in learning. This is in line with Nurdin's research (2007) on the development of the learning model using a metacognitive approach that may improve the students' mathematical capability.

Problem solving ability, critical thinking and analytical ability are really needed in the activities of learning mathematics, beside skills in working on the problems (Dochy, 2001; Hannula, et. al, 2004; Wahyudin, 2010). The students need them in order to possess the ability to obtain and to make use of information under this changing, uncertain and competitive condition.

As students of Mathematics Education department, they should have critical thinking ability and this capability should also be possessed by any graduates of higher education (Ennys, 2005; Perkins & Murphy, 2006).

The ability, however, cannot be obtained instantly. Therefore, the ability in problem solving should always be improved through mathematical learnings, and this as expected will give effects on the improvement of critical thinking ability (Kosiak, 2004; Yang, et. al, 2005; Perkins & Murphy, 2006).

It is also stated that to improve such a critical thinking may be made through the learning of geometry (Ruseffendi, 2006)

Dealing with learning materials, the implementation of the learning activities should adapt to the continuous changing conditions, the materials will not be textual anymore with the learning concept of solving materials in line with the preplanned program (Novotna, 2014). But the materials conveyed should be contextual, and the learning activities should always be adjusted to the existing condition, and the keys the students should know and understand should always be provided with (Komariah, 2011).

One of the materials that may provide the students with the logical thinking, besides the logic is the Euclidean Geometry. The aspect the students should study and learn dealing with the logical thinking may be seen in a series of activities in solving the Euclidean Geometry problems (In'am, 2003). The problem solving in the Euclidean Geometry is done by giving some statements in line with the logical orders and on the basis of reasons by basing oneself on the concerned theorems, postulates or definitions.

A theorem is any statement where their truth is obtained through a series of proofs, while a postulate is any statement of which the truth is obtained without any series of proofs, and a definition is any explanation on a unique characteristic, so that multiple interpretation will not happen (In'am, 2003). Concerning with postulate, some said that it is different from axiom. A postulate is used to give a support to a statement dealing with concrete material, while an axiom is needed to support an abstract proof.

Another activity to improve mathematical ability is made through mathematical problem solving, as the main discussion in learning mathematics (Saad, 2004; Haryani, 2012). There are some approaches employed when discussing mathematical problem solving. Polya (1971) proposes four steps in problem solving namely: 1) understanding the problem; 2) planning what to do; 3) implementing the solution in line with plan made, and 4) reviewing the result obtained through a series of activities in the problem solving. Moreover, the approach to problem solving may be made through cognitive aspects namely: 1) awareness; 2) cognitive strategies; 3) planning, and 4) reviewing (O'neil & Abedi, 1996).

A mathematical problem solving is an abstract and complicated process and involves human thinking and reasoning. Mathematical learning emphasizes the existence of symbols produced through a generalization process by making mathematical statements on various conditions (Lerch, 2004). Meanwhile the aspect to pay attention in the mathematical learning is the way the learners think when they are solving a problem (Memnun, et. al, 2012; Saad, 2004; Wahyudin, 2010).

The implementation of mathematical problem solving needs a creative thinking and systematic process (Blanco, et.al, 2013; In'am, 2015). This condition is an important learning facility to try to make a program of solving problems contextually, where a pattern is needed in order to be able to solve problems. A creative and logical thinking may be reached through the learning of Euclidean Geometry as an effort to train one's skill in problem solving (In'am, 2003). While, a mathematical creative thinking is an ability in knowing the solutions of any mathematical problems easily and flexibly (Novotna, 2014; Wahyudin, 2010).

Research question

Problem solving is a mental process requiring someone to think critically and creatively to look for alternative ideas and specific steps to face each hindrance (In'am, 2015; Mardzelah, 2007). Moreover, it is also an approach to solving a problem (Haryani, 2012; Saad, 2004; Wahyudin, 2010) and the activity has a specific characteristic in mathematics as a step in developing any mathematical knowledge.

On the basis of the problem dealing with a mathematical problem solving and one of the metacognitive aspects namely awareness, the problem in this present research is as follow: how is the Euclidean Geometry problem solving viewed from the metacognitive-based awareness aspect?

Methods

Research approach

The approach employed in this present research is qualitative in nature, intended to describe and to reveal the ability of the awareness aspect in metacognition (O'Neil & Abedi, 1996). It is a case study type of research (McMillan & Schumacher, 2001) which is in line with the characteristics to be revealed in this present study. Yin (1989) states that in a case study, a qualitative approach may be adopted. Merriam (1988) also supported a qualitative approach to a case study in the field of education.

Participant

The subject of this research was the second semester students of Mathematics Education Department in 2014/2015 academic year. The number of the students was six consisting of two students under high, two middle, and two low ability categories were involved in this research. As stated by Ruseffendi (2006) that the ability in solving Euclidean geometry problems is influenced by the level of ability one possesses.

The subject of this research was determined on the basis of the evaluation in solving Euclidean Geometry problems. They were grouped into three namely high, middle and low abilities. Those with high ability was coded as S-1 and S-2, medium as S-3 and S-4 and those with low ability as S-5 and S-6.

Data Collection

Data were collected through instrument, document and interview. The instrument used was the Euclidean Geometry ability test consisting of four items, and the instrument to be properly used in a research had fulfilled validity and reliability.

The instrument validity in this research is the item validity, namely the extent to which each item supports the whole scores, so that to understand the item validity, the item score and the whole score should be correlated. To understand the instrument reliability, the r correlation with the odd and even-item correlation method was analyzed by summing up the scores on the odd and even-items, the correlation between the two was counted (Arikunto, 2009), then it was analyzed using the SPSS 16.0 program. The validity test of the four items resulted in the coefficients of 0.79; 0.75; 0.70 and 0.82 respectively, meaning that the four test items had fulfilled validity, while for

their reliability, the coefficient obtained was 0.78, meaning the test items met the prerequisite of reliability.

The document used in this research is the result of the Euclidean Geometry problem solutions and interviews were made with those chosen as the respondents consisting of six students as mentioned above.

Data Analysis

This research started from the data subtraction made through the identification process of the results of the geometric problem solving by classifying the problems into high, middle and low categories. The categories are made by paying attention of the score results of the geometric test. Then the results of the data presentation supported by the interviews during the Euclidean geometric problem solving were explained.

In the data analysis of interviews as a step to observe the treatment of the metacognitive aspect of the awareness, a think aloud, as one of the ways to study one' s thinking manner, was employed. When one was solving a problem, what in one mind may be recorded and analyzed to determine his/her cognitive process dealing with the problem solved.

The process of the data analysis using the Think Aloud consists of eight stages: 1) transcribing the collected verbal data; 2) analyzing the available data; 3) reducing the data by making abstractions; 4) arranging units which are then grouped by coding them; 5) depicting the students' thinking structure; 6) analyzing the thinking process; 7) analyzing interesting matters, and 8) drawing conclusions (Subanji, 2007).

It is through these stages that students' ability in solving Euclidean Geometric problems from the awareness aspect in the metacognition would be understood.

Results

The research was committed through the test of the Euclidean Geometry subject and based on the data obtained through the test results of 180 second semester students in 2014/2015 academic year. The learning results were grouped into three categories namely those with high, middle and low achievement. From each group, two learning results were analyzed and the concerned respondents were interviews on the learning results as the step to analyze their awareness aspect from the metacognitive approach as stated in the following problem solving.

Problem Solving of the Respondents S-1and S-2

The answer to the problem as stated in Figure 2, shows that the respondent S-1 understood what to do in making a good plan. It is seen from the answer to the problem that meets the stages needed. Viewed from the stages of the proof of the triangle using the side angle side postulate, stating that *if there are two triangles of which the two enclosed sides and angles are congruent, the two triangles are congruent.* The first step shows the congruency of the two sides of the triangle $\overline{AB} \cong \overline{EF}$ obtained from the data known; this also applies for the two congruent angles $\angle 1 \cong \angle 2$ based on the known data. Then the proof of the congruency of the third side with a proper step is explained.

Pernyataan	Alasan
1. $\overline{AB} \cong \overline{EF}$	1. Diket
2. $\angle 1 \cong \angle 2$	2. Diket
3. $\overline{BD} \cong \overline{CE}$	3. Diket
4. BD = CE	4. Definisi kongruensi ruas garis
5. BC = BD - CD	5. Postulat pengurangan
6. DE = CE - CD	6. Postulat pengurangan
7. DE = BD - CD	7. Substitusi (4, 6)
8. DE = BC	8. Substitusi (5, 7)
9. $\overline{DE} \cong \overline{BC}$	9. Definisi kongruensi ruas garis
10. $\triangle ABC \cong \triangle FED$	10. Postulat si su si (1, 2, 9)

Figure 1. The Result of Respondent S-1 in Solving Problem No. 1

Figure 1 consists of two columns, where the first column shows the statement, and the second, the reason. The first to the second steps show the reason made, then the next is the definition of the congruence of the sides. The fifth and sixth postulates on the subtraction, while the seventh and eighth steps are substitution. Then the ninth step is the definition of the congruence of the line segment which is the same as the fourth side and the last is the congruence of the triangle in terms of the side angle side postulate.

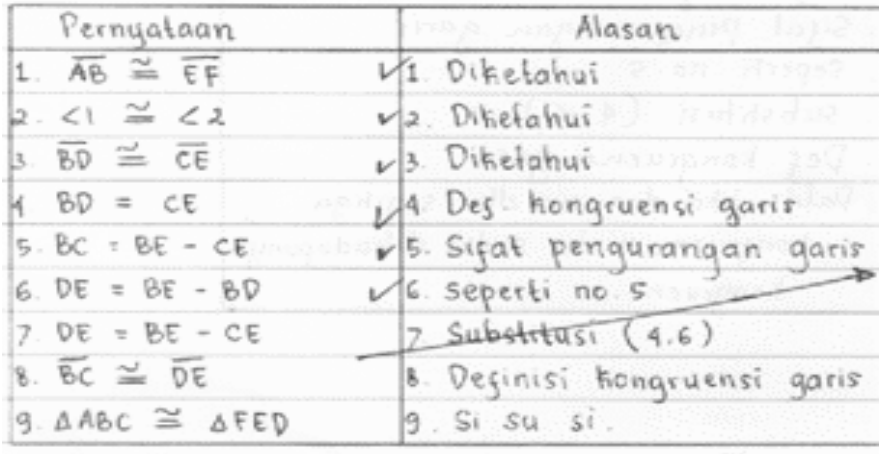
Based on the result, the researcher interviewed the respondent to support the finding of the document of the test result. The result of interview with the respondent S-1 was shown in the transcription below.

R: what did you do before doing the problem?

S-1: by paying attention to the problem, I really know what to do to solve it, I know what I should do and order to obtain the perfectness of the answer.

The condition as described shows that the respondent S-1 was really aware of what he should do to solve the Euclidean Geometric problem.

Figure 2 shows the depiction of the problem solving by high-ability respondent. From the stages made, it is shown that in solving a problem, he was aware of and understood about what to do. It is seen from steps 1 to step 7 showing that he really knew what he should write. But in the next step, there was a fault, where after the seventh step, it should be shown that the two-line segment has the same size, BC = DE, as a step to show that the two are congruent, $\overline{BC} \cong \overline{DE}$. A complete solution may be made by giving some information at the proof column, namely the last step using the reason of the side angle side theorem [1, 2, 8] meaning that the side angle side reason is located at the stages 1, 2 and 8.



Pernyataan	Alasan
1. $\overline{AB} \cong \overline{EF}$	1. Diketahui
2. $\angle 1 \cong \angle 2$	2. Diketahui
3. $\overline{BD} \cong \overline{CE}$	3. Diketahui
4. BD = CE	4. Def. kongruensi garis
5. BC = BE - CE	5. Sifat pengurangan garis
6. DE = BE - BD	6. seperti no 5
7. DE = BE - CE	7. ~~Substitusi~~ (4.6)
8. $\overline{BC} \cong \overline{DE}$	8. Definisi kongruensi garis
9. $\Delta ABC \cong \Delta FED$	9. Si su si.

Figure 2. Result of Problem Solving No. 2 of Respondent S-2

The description in Figure 2 is relatively the same as that in Figure 1 where it consists of two columns. The first shows the statement, and the second, the reason. The first to the third reasons are known, the fourth step is the definition of the congruence of the line segment where this step is similar to the eighth step. The fifth and sixth steps are the subtraction characteristics of the line segment, the seventh is substitution and the last step, namely the fifth step is the side angle side postulate.

To confirm the result, an interview with the student giving the answer as depicted in Figure 2 was made, where the result is as follow:

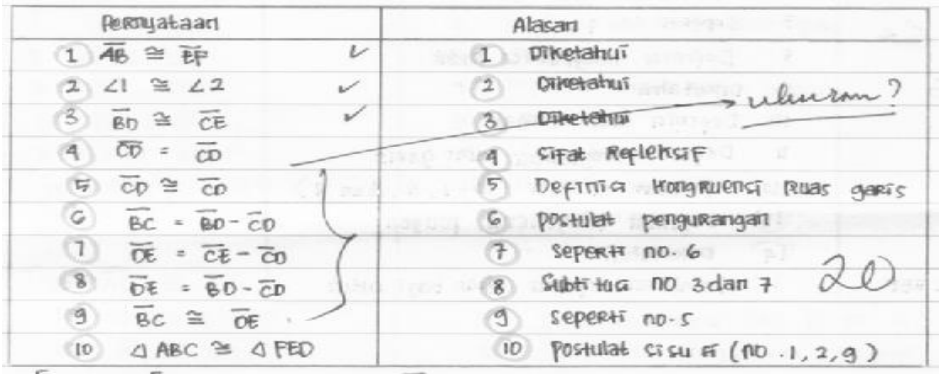
R: Do you know what you should do in solving the problem?

S-2: Yes sir... I understood what I should do to solve the problem … and I was sure that my work was right … and I did not recheck the result … but it seems that it is necessary to add a step before step no. 8… namely BC = BD…, …

The result of the interview shows that in the aspect of awareness in solving the problem, it seems that there no problem, the respondent understood and realized what he should plan and do to prove the congruency of the triangle.

Problem Solving of the Respondents S-3 and S-4

The results of the students' work under the middle category are shown in Figure 3.



Pernyataan		Alasan
1 $\overline{AB} \cong \overline{EF}$	✓	1 Diketahui
2 $\angle 1 \cong \angle 2$	✓	2 Diketahui
3 $\overline{BD} \cong \overline{CE}$	✓	3 Diketahui
4 $\overline{CD} = \overline{CD}$		4 Sifat Reflexif
5 $\overline{CD} \cong \overline{CD}$		5 Definisi kongruensi ruas garis
6 $\overline{BC} = \overline{BD} - \overline{CD}$		6 postulat pengurangan
7 $\overline{DE} = \overline{CE} - \overline{CD}$		7 seperti no. 6
8 $\overline{DE} = \overline{BD} - \overline{CD}$		8 Substitusi no 3 dan 7
9 $\overline{BC} \cong \overline{DE}$		9 seperti no. 5
10 $\triangle ABC \cong \triangle FED$		10 Postulat sisi sudut sisi (no. 1, 2, 9)

ulangan ?

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Figure 3. The Result of Respondent S-3 in Solving Problem No. 2

As stated in Figure 2, Figure 3 also consists of two columns, where the first column is the statement, and the second, the reason. The first to third reasons are known, the fourth is reflective in nature, the fifth step is the definition of the congruency of the line segment and this also applies for the ninth step. While the sixth and seventh steps is the subtraction postulate. The eight step is the same as the third one and the last step, the tenth, is the side angle side postulate.

Figures 3 shows that the student was aware of the planning and implementation steps in problem solving as shown at steps 1 to 3. Though at the step 4 a fault in writing, it should be CD = CD, happens, but it is not under the awareness aspect, but another aspect of the metacognitive approach as a step to show $\overline{CD} \cong \overline{CD}$. This fault also happens from steps 6 to 8 since the subtraction operation merely apples for the size of the line segment, meanwhile in the answer, a line segment was employed. However, it is seen that the student understood the steps he should make in solving the problem. It also happens at the step 10, as the last step where the student really understood the reason in proving the congruency of the triangle, namely using the side angle side postulate basing on the steps 1, 2 and 9. This condition is reinforced from the result of interview with one of the respondents as follow:

R: what did you know in solving the problems, did you understand what you should do before beginning to solve the problem?

S-3: E... I really understood what I should do in solving the problem … but I made a little bit mistakes in writing the symbol…. Since the time was up and I was sure that my work was right… so that I did not check it anymore.

Steps 6 to 8 should make use of the size symbol of BC = BD – CD; DE = CE –CD; then using substitution, DE = BD – CD is obtained and using the characteristic of a transitive equalization, BC = DE is gained and using the congruency line segment reason, the congruency of the two line is acquired. This result is also supported from the interview results, that the student was aware of what he should do, but there was a step lacking because he did not review the result.

Pernyataan	Alasan
1. $\overline{AB} \cong \overline{AC}$	1. Diketahui
2. $\angle B \cong \angle C$	2. Jika ada dua sisi yang saling berhadapan kongruen, Maka sudut yang menghadap sisi tersebut kongruen.
3. $\overline{ED} \perp \overline{AB}$	3. Diketahui
4. $\angle EDB$ siku-siku	4. Definisi tegak lurus
5. $\overline{EF} \perp \overline{AC}$	5. Diketahui
6. $\angle EFC$ siku-siku	6. Definisi tegak lurus
7. $\angle EDB \cong \angle EFC$	7. Kongruensi sudut siku-siku
8. $\angle CEF \cong \angle BED$	8. Dalil SD : Jika ada dua sudut kongruen dalam dua segitiga, maka sudut yang ketiga kongruen.

Figure 4. The Result of Respondent S-4 in Solving Problem No.3

Figure 4 consists of eight steps, where the first column shows the statement, and the second, the reason. The reasons of the first, the third and the fifth steps are known, the second is the postulate: in a triangle, if there are two congruent sides, the side facing the two sides is congruent. The fourth and the sixth steps are the definition of the perpendicular line, the seventh step is the congruence of the angle, and the last, the eight step, is the postulate: if there are two triangles with two congruent angles, the third angle is congruent.

In the respondent's answer as presented in figure 4 principally there is no problem, but there are some questions and reasons that are incomplete considered from the way of writing and the completeness of the reasons. The reason for the step 2 should be *If there are two congruent sides of a triangle, the angle before the two sides should be congruent.* This also applies for the step 4, and the writing of the right statement at the step is $\overline{ED} \perp \overline{AB}$ and $\overline{EF} \perp \overline{AC}$. While the reason at the step 7 should be written completely as a theorem: *If an angle is the right angle, then two angles should be congruent.* As a step to obtain a data accuracy, the following interview was made:

R: I see that what you wrote is incomplete. Why?

S-4: which one sir?

R: Look at the reason at steps fourth, sixth and the seventh steps!

S-4: what happens to my reason?

R: you write the definition of perpendicular, what is perpendicular?

S-4: Oh… yes sir, it should be the definition of perpendicular line, I was in a hurry sir.

The dialogue shows that the respondent was aware of his mistake in writing, and it can be stated that the awareness aspect in solving problems was fulfilled but due to another factor, namely as the respondent said, that he was in a hurry. It is often found out that in solving a problem, students often did not recheck because of various factors.

Problem Solving of the Respondents S-5 and S-6

The research results of the respondents under the low achievement category are presented at Figures 5 and 6 showing a series of answers they gave.

Pernyataan	Alasan
1. $\overline{AB} \cong \overline{AC}$	1. Diketahui
2. AB = AC	2. Definisi kongruensi ruas garis
3. $\overline{ED} \perp \overline{AB}$	3. Diketahui
4. $\angle EDB$ siku - siku	4. Definisi garis tegak lurus
5. $\overline{EF} \perp \overline{AC}$	5. Diketahui
6. $\angle EFC$ siku - siku	6. Seperti nomor 4
7. $\angle EDB \cong \angle EFC$	7. Definisi kongruensi sudut
8. $u\angle EDB = u\angle EFC$	8. Definisi poligon kongruensi
9. $\triangle EDB$, $\triangle EFC$ sebangun	9. Definisi segitiga sebangun.
10. $\angle CEF \cong \angle BED$	10. Postulat (si su si)

Figure 5. The Result of Respondent S-5 in Solving Problem No.3

Figure 5 contains 10 steps, and two columns where the first column is the statement and the second is the reason. The reasons of the first, third and fifth steps are known, the second step is the definition of the congruency of the line segment, the fourth and the sixth steps are definition of the perpendicular line. The seventh step shows the definition of the congruence of angle, the eight is the definition of the congruency of polygon, the ninth step is the definition of the triangle and the tenth step is the side angle side postulate.

On the basis of the document of the learning achievement of the respondents under S-5 category, it is seen that they can be said not to understand the problem they should solve. This condition may be seen from the second line where the respondent stated that AB = AC, where this is right for the implication of $\overline{AC} \cong \overline{BC}$, but dealing with the series of proof, the statement is not needed. This also happens at the line 7, where the statement is right, but the reason proposed is improper. The reason should be the theorem: *if there are two right angles, the two angles are congruent.* It is not different from statement at the line 8, where the statement $m\angle EDB = m\angle EFC$ is right since $\angle EDB \cong \angle EFC$, but dealing with the proof stages, the statement is not needed.

The statements given at steps 9 and 10 are not needed, since there is no relation to the previous statement and it seems that in the steps the respondents did not understand what to do in writing the stages in solving the problem. Any effort to support the finding as described in the result of solution, an interview with respondent S-5 was made and the following is the transcript of the interview:

R: I saw in your work, at stages 9 and 10 you wrote a statement on the congruency of a triangle and then congruency of angles with the side angle side theorem. Please explain it!

S-5: oh... I was a little bit confused sir ... I did not know what I should write in the statement, so that I merely wrote what in my mind, though actually I was not sure whether what I wrote is right.

The result of the interview shows that the respondent did not understand what solution to make. It can be stated that the respondent knew what to do namely solving the problem, but did not understand the steps he should write to string up the proof stages.

Figure 6. The Result of Respondent S-6 in Solving Problem No. 4

Figure 6 consists of 10 steps and the first column is the statement and the second is the reason. The first, third, fourth and seventh steps show that the reasons are known, the second step show the definition of the circle radius. The fifth and seventh steps show the definition of the perpendicular line, while the ninth step, the last step, shows the definition of the congruency of polygon.

Figure 6 shows that the respondent was aware of what he should plan and do in solving the problem. In steps 1 to 3 it may be said that no problem exists, but in step 5, after both congruent line segments $\overline{BC} \cong \overline{FD}$, it should be continued to prove that one side of the right angle is congruent, a step that should be made through the subtraction of the size of the line segment. Moreover, the steps 6 to 8 should not be stopped, but be continued using the congruency of the two angles, namely $\angle B \cong \angle F$. This condition is based on the fact that the two are right angles, and on the theorem: *if there are two right angles, the two angles are congruent.* Then it is shown that the two are right triangles and they prove the congruency, the theorem of the congruence of right triangles are employed: *if there are two right triangles and one of its right side is congruent, the two triangles are congruent.*

Discussion

Dealing with the research result, the awareness aspect in solving Euclidean Geometric problems supports previous researches that the metacognitive aspect in solving a problematic problem is the review aspect, while there is no problem for the awareness aspect (In'am, 2015). The students' ability in the awareness aspect is not free from the learning implementation leading the students always to show what they have not known and they have understood in the learning process. It is in line with the results of the previous researches (Larmar& Lodge, 2014) that for students at the beginning semester, some awareness of how they should learn should

be implanted through a metacognitive approach. Moreover, this research result also supports the research result (Saad, 2004) showing that the awareness aspect in solving problems in general is not problematic, but the review aspect in majority has raised some problems because there is no habit or time unavailability during problem solving. It is different from another research (Heidari & Bahrami, 2012) that the awareness aspect has a significant relation to the students' thinking style in learning. In addition, this research also supports Nurdin's research (2007) on the development of a learning model using a metacognitive approach to improving students' ability in mathematics.

The research results show that the category of respondents' ability becomes one of the factors in succeeding their problem solving for students under the categories of high and middle abilities because they have some awareness of the problem solving. But it does not apply for those in the low category because of their low awareness. This supports a research made by Khoiriyah (2012) that the university students' ability will influence their problem solving ability, this also applies for Murdanu's research (2004) stating that one of the difficulties in solving geometric problems is caused by the lack of their understanding of the problem.

Conclusion

The results of this present research shows that in solving Euclidean Geometric problems it seems that the respondents are aware of what to plan and to do in solving the problems among those under high and middle categories. Respondents may be aware of them by writing any aspects they know and the problem to solve. The three groups under the study, high, middle and low, possess some awareness in their problem solving, except those under the low category who have less or even do not have any awareness of what to do in solving their problems. This is supported by the analysis of the respondents' work reinforced by interviews to reveal the real condition in terms of the problem solution.

Therefore, it can be stated that the awareness aspect which is a part of the aspect of the metacognitive approach to problem solving shows that the respondents are really be aware of the plan and should plan in solving Euclidean Geometric problems, except those under the low category, who show less awareness of what they should do in problem solving.

Disclosure statement

No potential conflict of interest was reported by the authors.

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