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ENHANCING STUDENTS' UNDERSTANDING IN CALCULUS TROUGH WRITING

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ABSTRACT. The purpose of this study was to investigate the effects of using writing activities on students' understanding and achievement in Calculus. The design of this study was quasi-experimental. The subjects of this study consisted of two secondary schools in one of the states in Malaysia. Each school was assigned one intact class of Form Four to be the experimental group and another one intact class as the control. The experimental group learned mathematics by using the writing activities for five weeks, while the control group learned mathematics by using traditional whole-class instruction. A 20-item Calculus Achievement test was designed with reliability .87. The findings showed that the experimental group exhibited significantly greater improvement on calculus achievement. The students showed positive reaction towards the use of writing. Findings of this study provide information to schools to take advantage of writing activities to promote understanding.

KEYWORDS. Calculus, Writing, Understanding, Achievement, Student.

INTRODUCTION

Mathematics learning is a complex and dynamic process. Most of us want our students to understand information that is presented to them or information that they discover for themselves. But, what, exactly, do we mean when we use the term understand? Understanding is not simply remembering mathematical concepts or being able to follow procedure. Understanding in mathematics learning requires more than simple recall of facts. Perkins and Blythe (1994) define understanding as being able to explain, finding evidence and examples, generalizing, applying, analogizing, and representing the topic in a new way. Learning currently no longer emphasizes correctness of the final answer but has shifted to emphasizing process, context, and understanding. Each topic in mathematics has its own conceptual understanding and skill mastery

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to be learned by the student. Teaching mathematics for understanding is to help students develop how to think and how to make decision (Allen, 1992; Borasi & Rose, 1989; Burton & Morfa, 2000; Countryman, 1992; Noraini, 2007). In the teaching process the mathematics teachers need to give more attention on steps to ensure their students not only grasp the mathematical concepts but can communicate and explain to others what they have understood (NorainiIdris, 2000, 2006, 2007).

Mathematics Educators always hope that students can understand what is being taught and not just regurgitate facts or merely apply procedures for solutions (Kazemi, 1998). Hence, recent emphasis in mathematics teaching has focused on not the final answer of the process. It has shifted to the context and understanding shown by the student. Writing is seen as one way to encourage critical and in-depth thinking, reflection and evaluation of understanding in students. Writing is one activity that can be a mechanism for assessing the level of mathematical understanding in students (Artzt & Armour-Thomas, 1992; Brown, 1997; Countryman, 1992; Noraini, 2006; Pugalee, 1997, 2001).

In this endeavor, the student not only learns to solve problems but thinks more deeply about why the method gives the solution. Besides that the writing activity encourages the student to connect a new concept with an existing one. This leads to mathematical understanding not at the instrumental level but relational and logical understanding.

Skemp (1976) states that to understand a concept, group of concepts or a symbol is to assimilate it into a suitable schema, that is, to form a connection between ideas, facts or procedure that is generally accepted. This is a dynamic and not static process. A concept is built from collected data, and then is related with other concepts to create another and more complex concept. Skemp (1979) differentiated understanding into three categories namely instrumental, relational and logical understanding as discuss below. According to Skemp (1979) there are three kinds of understanding.

Instrumental Understanding

Instrumental Understanding is the ability to apply an appropriate remembered rule to the solution of a problem without knowing why the rule works. In other words we know “how” but do not know “why”. Instrumental Understanding applied to the concept of average consist of knowing only the computational rule for calculating the simple average of a set of numbers. In this study the terms instrumental understanding, computational knowledge, computational ability, computational skill, procedural skill and procedural knowledge are used interchangeably when referring to instrumental understanding. The schemas formed by instrumental learning are short-term, the least and most quickly acquirable by which correct answers can be given; in other words rules, which we may regard as degenerate schemas. Students learn a set of these, each appropriate to a limited class of task.

Relational Understanding

Relational Understanding is the ability to deduce specific rules or procedures from more general mathematical relationships. In short, one knows both "how" and "why". Thus, in this study, the terms relational understanding, conceptual understanding, and conceptual knowledge were used interchangeably when referring to relational understanding. The construction of these relational schemas are the goals of relational learning. The operands may be newly encountered concepts, and the goal may be connecting these with an appropriate (relational) schema. Achieving this goal is equivalent to relational understanding, and in the process the schema itself has undergone further development. Another kind of goal may be to deduce specific methods for particular problems, or specific rules for classes of tasks. The ability to do this is evidence of relational understanding. Yet another kind of goal is to improve existing schemas, by reflecting on them to make them more cohesive and better organized, and hence more effective for the first and second kind of goal.

Logical Understanding

Logical understanding is closely related to the difference between being convinced oneself, for which relational understanding is sufficient, and convincing other people.Most of us subject our ideas to self criticism before making them public, formally or informally and constructing a proof which satisfies ourselves gives us confidence to write the explanation. In some cases, all we want is to satisfy ourselves. But the satisfaction of criticisms and self-criticism is secondary to the main goal, and indeed aids its achievement, the construction of ever more extensive and powerful mathematical knowledge, coherent, and without weaknesses or internal inconsistencies. This is quite an activity; writing formal proofs of this sort is learnt by reflecting.

To develop and attain understanding as suggested by Skemp, writing activities that encourage the students to build their own ideas and concepts are appropriate. This may make mathematics learning more meaningful.

Unfortunately, of late the mathematics teaching and learning process has encountered a worrying trend. Many students have used the wrong methods in the mathematics learning process. They think that learning mathematics by memorizing rules and substituting numerals into the chosen formula is the right way. Besides that, mathematics is said to be a subject where one is required to give a wrong or right answer (Miller, 1992).

Based on these reasons, it is no longer unusual to find students who use procedures without understanding the concepts behind them or those who understand very little of the concept behind each procedure used (Hiebert & Lefevre, 1986). According to Borasi and Rose (1989) only a few students hope for meaningful mathematics learning and only a small number of students see mathematics as requiring creative thinking. As a result, many students are often satisfied with symbol manipulation and routine solving of problems without achieving deep understanding of the concept or solution procedure for a topic.

Strangely, even with this attitude there are students who achieve high scores in mathematics. But they must know that these high scores are only for the short term because such scores do not help in developing mathematical concepts or problem solving ability necessary for long term success in mathematics (Borasi & Rose, 1989).

Of greater concern to teachers is that this attitude is carried over by the students to college or university level. Gordon (1997) found that many college students have a weak grasp of mathematical concepts, and that students weak in algebra in upper secondary school are not only weak in mathematics but also have a negative attitude toward the subject. These students only memorize algorithms or procedures and do not understand the meaning behind the procedures used (Gordon, ibid). In fact some students do not realize there is meaning in mathematics and that concepts exist for every procedure applied. They believe that solving mathematics problems means to run many operations using symbols without meaning (Oaks, 1992).

STATEMENT OF PROBLEM

Mathematics learning for understanding is not easy. Many students fail to understand the concepts taught to them. They solve problems by memorizing formulae and procedures teachers have taught them. The students merely put the required figures into the formula to arrive at the answer (Miller, 1992). In schools, many teachers emphasize how to write equations, draw graphs and solve problems until the final answer is obtained. Mathematics teachers seldom ask students to write down explanations because the teachers themselves are unsure how to mark the written tasks. Furthermore, the teachers do not know how to relate writing ability with numeracy and how writing can improve critical thinking and problem solving capability (Robinson, 1996).

Since mathematics is seen as placing importance on the final product (Miller, 1992), it is not surprising that students place importance on algorithms and procedures and do not try to understand the meaning behind the procedure applied. Students who use this approach may be competent in solving by applying algorithms but do not think deeply about the meaning behind a concept or procedure. The learning through memorization process will prevent them from solving more complex and abstract problems correctly.

Lack of understanding in mathematics often can make students lose interest in the subject and affect their mathematics achievement. The ability to use procedures and understand concepts in mathematics are two things necessary in learning mathematics (Hiebert & Carpenter, 1992). Understanding in mathematics learning generally involves actions to know concepts and principles related to the procedures and relating or creating meaningful relationships between existing concepts and newly-learnt concepts (Boroody & Ginsburg, 1990).

According to Skemp (1979), instrumental understanding is seen as knowing the rules without knowing why the rules can be applied, while relational understanding is known as knowing what should be done and why they should do it. Another type of understanding according to Skemp (ibid.) is logical understanding. In logical understanding, not only do students know what to do and why, they can also explain what they are doing to others.

Hence, to develop understanding in mathematics learning, changing the students' overall approach is necessary. The students' mathematical understanding should not just be at the instrumental understanding level but they should also achieve relational and logical understanding. Many efforts have been tried and writing across the curriculum has been introduced to develop relational and logical understanding (Robinson, 1996). Mathematics learning does not just stress the final answer but has shifted to process, context and understanding. Hence, exposing students to writing activities is timely. The writing activities used in this study involve students in explaining with clarity their mathematical ideas to help them develop deep understanding of a particular topic.

The focus of the writing activity in this study is on expository writing and log entries because the researcher would like to explore the students' understanding of concepts and procedures in differentiation through written responses and not to explore the affective aspects of the students. Because of that, the journal writing involves the students in explaining their views.

In this study, writing activities will be given to students in the experimental class asking them to think deeply about the topic taught, to write in sentences the steps in solving the problem, to explain the concept taught and to relate the concept taught with existing concepts. The process students undergo will be active learning; they need to define the basic meaning of the concepts or topics. Students also need to perceive the tasks they are engaged in as being more than just a procedure. Students must be able to explore and investigate their subject too. For example, they might be required to draw inferences from facts, apply existing knowledge to new problems, and give explanation for why certain strategies or procedures were chosen to solve the problems.

The topic of differentiation was used in this study to ascertain students' understanding as a result of writing activities. The topic of differentiation was chosen because, according to the mathematics syllabus for additional mathematics (Malaysian Ministry of Education, 2000), differentiation is a core topic for Form Four students. They are first exposed to differentiation topics such as limits, simple differentiation and differentiation applications at this stage. Differentiation also is an important component of calculus in mathematics especially for those who will proceed to higher mathematics or engineering. Nevertheless, many students find that differentiation is a difficult topic and they do not really understand the concepts and procedures in differentiation (Orton, 1983; White & Mitchelmore, 1996). In Malaysia, the Ministry of Education reported that students are weak in differentiation because they do not grasp the basic concepts and important skills in differentiation (Ministry of Education, 1996). Hence it is important for teachers to help students better understand the concepts and procedures.

Helping the students to understand concepts and procedures in differentiation involves efforts to help them visualize mental connections (Prus-Wisniowska, 1996). This process of interrelating can be achieved through building of connections between existing basic structures of understanding with newly encountered facts. Writing activities that involve active learning of mathematics help to develop these mental constructs.

Calculus has traditionally one of the most difficult concepts for students to understand and master. Although most students learnt the specific algorithm and procedure that they are taught, their general conceptual understanding often remains remarkably deficient. Recent years have seen increasing interest in developing the use of writing in the mathematics classroom (Allen, 1992; Burton & Morgan, 2000; Borasi & Rose, 1989; LeGere, 1991; Oaks & Rose, 1992). This is largely based on the premise that writing is an activity that is in itself conducive to learning.

The rules and procedures of school mathematics make little sense to many students. They memorize examples, they follow instructions, they do their homework, and they take tests, but they cannot say what their answers mean. Most of us want our students to feel that mathematics is an enjoyable and rewarding study, but do we succeed? Some of our students are successful whereas others are anxious and fearful. Mathematics achievement has been generally been poor. Several students are unable to explain what they have done in solving mathematics problems and how they obtained the answers. The rules and procedures of school mathematics make little sense to many students. The question now is, "How do we know if we are teaching for understanding?"

Understanding and knowing mathematics is doing mathematics (Allen, 1992; Borasi & Rose, 1989; Countryman, 1992; Dougherty, 1996; Noraini, 2007) We need to create situations where students can be active, creative, and responsive to the physical world. The researcher believes that to learn mathematics students must construct it for themselves. They can only do that by exploring, justifying, representing, discussing, using, describing, investigating, and predicting. Writing is an ideal activity for such processes. Writing can motivate and enhance the learning that takes place when students confront the concepts and procedures of mathematics.

In an investigation of the benefit of using expository writing, Bell and Bell (1985) concluded that "expository writing is an effective and practical tool for teaching math problem solving" (p. 214). Writing helps build thinking skills for mathematics students as they become accustomed to reflecting and synthesizing as parts of a normal sequence involved in communicating about mathematics (Pugalee, 1997). Writing should be encouraged as an integral part of the mathematics curriculum designed to help students in understanding mathematical concepts.

In the teaching and learning of mathematics (Noraini, 2007):

1. Writing helps students become aware of what they know and do not know, can and cannot do;
2. When students write they connect their prior knowledge with what they are studying;
3. Writing helps students summarize their knowledge and allows teachers to gain insights into students' understanding;
4. Writing helps students raise questions about new ideas;
5. Writing helps students reflect on what they know;
6. Writing helps students construct mathematics for themselves.

Many teachers agree that successful learning requires reinforcement, feedback, synthesis, and action. Students get immediate feedback from words that they produce. Writing mathematics can thus free students of the assumption that math is just a collection of right answers to questions posed by someone else (Borasi & Rose, 1989; Countryman, 1992).

CONCEPTUAL FRAMEWORK

Skemp's Theory of Understanding

In his definition of understanding Skemp (1979) stated that to understand a concept means to create connections between ideas, facts and procedures that are generally accepted. The ideas, facts or procedures are collected to form a basic concept, and these concepts are connected with other concepts to form new and more complex concepts. In this process what is expected is the development of relational and logical understanding and not just instrumental understanding (Skemp, ibid.).

Skemp (ibid.) described instrumental understanding as the ability to apply a rule or procedure without knowing why. The rule or procedure used can be applied only for certain tasks and the mental structures built through instrumental understanding cannot be easily modified. It requires memorization of the method or formula and can be used only for one type of problem.

By relational understanding Skemp (ibid.) meant knowing what should be done and why we should do it. It involves effort to know the concept and procedure and relating the new concept to an existing one. Relational understanding requires the student to choose, change and apply data, formulae and principles in new situations. The student who has attained relational understanding is capable of solving problems with minimal direction and minimal reliance on memory.

For example, once the teacher has taught the method for differentiating a polynomial function, the teacher gives the following problem: differentiate $y = \frac{1-x^2}{x^2}$ with respect to x. Those students with relational understanding would relate the new knowledge with the existing one and use the new equation $y = \frac{1}{x^2} - 1$ before beginning to differentiate. The student thus can resolve the problem by choosing a faster and easier solution.

Logical understanding involves the ability to connect a symbol and signifiers in mathematics to relevant mathematical ideas and connecting the ideas into a schema. Students with logical understanding can use their understanding to influence other students or prove mathematical statements. Logical understanding also involves efforts to demonstrate what is stated according to logic or proving a statement is true.

To expand and acquire understanding as suggested by Skemp (1976), writing activities that encourage students to build their own ideas and concepts about what they have learnt are appropriate. This active learning involves deep thinking and helps students to create mental structures. This mental development is conducive to mathematical understanding. Hence the writing activities in this study are aimed at creating a situation where students undertake tasks by searching and experiencing for themselves and reflecting on what they are doing so that mathematics becomes more meaningful. The learning process never stops at mere memorization but leads to proper mathematical understanding. Besides that, the teaching process using writing activities also gives teachers the opportunity to identify weaknesses and misconceptions in students.

In this study, writing activities are developed to encourage mathematical understanding in the context of the three types of understanding as defined by Skemp.

The above theoretical framework will be applied in this study for use on the Form Four students who have just begun to learn differentiation. If the writing activities can improve students' understanding of differentiation, this understanding can help students follow other topics in calculus with more efficiency.

COMMUNICATION THROUGH WRITING

Mathematics learning is a complex and dynamic process involving interactions between previously acquired levels of understanding and the conceptualization and incorporation of new material. Writing encourages a level of cognitive activities which maximizes the potential of the learner to modify and restructure mathematical knowledge.

As suggested by the National Council of Teachers of Mathematics (1989, p. 214) the ability to communicate mathematics involves being able to:

1. Express mathematical ideas by speaking, writing, demonstrating, and depicting them visually;
2. Understand, interpret, and evaluate mathematical ideas that are presented in written, oral, or visual forms;
3. Use mathematical vocabulary, notation and structure to present ideas, describe relationships, and model situations.

If we believe that students learn best by constructing and evaluating the knowledge that we wish them to acquire, we are likely to view students as research apprentices who gain ownership of knowledge by asking their own questions about existing knowledge.

Writing definitely plays the key role in the process of student knowledge-construction (Guckin, 1992; Luitel, 2002; Mayer & Hillman, 1996). Writing can help teachers answer specific questions about students;

1. Do students use math to make sense of complex situations?
2. Can they formulate hypotheses?
3. Can they organize information?
4. Are they able to explain concepts?
5. Can they use computation skills in context?
6. Do they use mathematical language appropriately?
7. Are they confident about using mathematical procedures?

Objectives of the Study

The purpose of this study was to investigate the effects of using writing activities onstudent understanding and achievement in Calculus.

Specifically this research project seeks to:

1. Find out the effects of writing activities on student achievement in calculus;
2. Find out the effects of writing activities on student attitude toward learning calculus and toward the subject; and
3. Find out the students' beliefs and attitudes about writing activities as a learning tool.

Specifically, it will seek answers to the following questions:

1. Do writing activities help improve student achievement in calculus?
2. Does writing help to improve the attitude of students toward learning concepts in calculus?
3. What are the perceptions of students towards writing activities in calculus?
4. What are the students' beliefs and attitudes about writing activities as a learning tool?

METHODOLOGY

Research Design

This study involves a quasi-experimental nonequivalent control group design. In this study the researcher did not assign subjects randomly to treatment. The subjects were chosen from students in existing classes in order not to disrupt the school routine. Nevertheless, the subjects in this study comprise students of similar grades in internal tests and had almost similar backgrounds. The research design is given in Figure 1.

Experimental Group	O1	X1	O2
Control Group	O1	X2	O2

Figure 1. Research design

O1 -represents pretest used to ascertain level of student understanding before treatment for the Experimental group

O2 -represents posttest given to test the level of understanding of the students after the treatment given to the experimental group

X1 -represents learning with writing activities

X2 -represents learning under the traditional whole-class instruction

The research design appropriate for a situation where random sampling is not possible is the non-equivalent pretest-posttest control group design. The researcher has chosen this design because it was not possible to select the students for the experimental and control groups. The students were from "intact" classes, having been selected by the school administration and it was not possible for the researcher to change the composition of the existing classes. Since intact classes were chosen it was possible to carry out the study without students realizing they were involved in the study.

Sample. The study took place in two of the public secondary schools in Selangor, Malaysia. Each school serves a diverse academic, social, economic, and cultural population. The participants in this study were students (male and female) in Form Four class. The students in this school were from the middle social-economic status. The average ages of the students were between 16 to 17 years. There were eight classes in Form Four in each school. After discussion with Principals and teachers from the two schools, two intact classes were identified from each school. Each school was assigned one intact class to be the experimental group and another one as the control group. Both groups had comparable socio-economic and ethnic background as well as comparable mathematics grades according to the teachers' grade book. The sample consisted of 42 students in the experimental class (EC) and 43 students in the control class(CC) in the first school and 43 students in EC and 43 students in CC in the second school.

Instruments. Three instruments were used for data collection: Calculus achievement test, Attitude inventory items, and Students perception questionnaire. The Calculus achievement test consist of twelve questions testing conceptual understanding, procedural knowledge, and problem solving. Attitudes were measured by a six item questionnaire. This questionnaire was designed to know the reactions when solving calculus problems, to assess feeling towards the use of writing activities, and perceived importance of calculus. The student perception questionnaire is a four item questionnaire to assess experience of writing in calculus classes; students in the experimental class were asked to answer the questionnaire.

Validity and Reliability. A 20-item Calculus Achievement Test was designed for this study. Its test-retest reliability in the pilot test was .87. After the test had been prepared, three experienced mathematics teachers with more than fifteen years of teaching experience were requested to check the test questions for content validity. Attitude Inventory items and questionnaire for students' perceptions were validated by the same teachers.

Instructional Activities. The goal of the writing activities in this study was to improve understanding and achievement in calculus. The researcher designed the writing activities to use during the 5-week treatment in the experimental class. The writing activities involve all students to define meaning of concepts, allow students the opportunity to express, explore, explain, criticize, and justify. Examples of the questions are as follows:

1. Differentiate $5x^5 - 12x^4$ with respect to x. Explain how you differentiate the problem in detail.
2. Given $y = (4x^3 + 6)/x$. Describe each step you use to differentiate the given problem and give reason for your choice of steps.
3. In words, write what you would do to solve this problem. Where appropriate, tell why you are doing that step for the problem $5x^3 + 40x^2 = -30x$.

Procedure. Both quantitative and qualitative methods were utilized to gather data. Quantitative data were collected using a pretest Calculus Achievement Test. For the experimental group, all students received the writing activities. The treatment took approximately five weeks. Prior to instruction, the researcher conducted a series of training workshops with teachers on how to use the writing activities as a tool in teaching and learning of calculus. The experimental group learned calculus by using the writing activities for five weeks, while the control group learned mathematics by using traditional whole-class instruction. Students in the experimental classes completed activities and exercises designed by the researcher. Students worked either individually or in groups of two, or three, depending on the types of activities. To encourage students to reflect upon their activities the teacher instructed students to keep a record in their notebooks of what they did each day. In every teaching session, the researcher used writing activities for the experimental group. The students needed to do the following activities:

1. Describe each step used in solving the problem clearly.
2. Explain why those steps were used.
3. Show how they used the previous knowledge in helping them to solve the problem.
4. Describe in detail what is (are) the thing(s) the students think about while solving the problem.

For the control groups no treatment was given. Students used their own textbook for calculus and no material was provided by the researcher. The main teaching tool was a textbook. When the instructional activities were completed, all students in both experimental and control groups took the posttest or Calculus Achievement Test. For the experimental group, they took attitude inventory items and students perception questions.

Qualitative data on the other hand were collected by means of interview with twelve students. The researcher employed purposeful sampling on the assumption that the investigator wants to discover, understand, and gain insight about the belief and attitudes on writing activities as a learning tool. The interview sessions were audiotaped.

Data Analysis. Data from students' achievement in the pretest and posttest, data on students' attitude and questionnaire responses were analyzed using quantitative analysis. The SPSS program was used to analyze the data. The quantitative analyses were complemented with analyses of qualitative data gathered as a result of interviews.

RESULTS OF THE STUDY

Student Achievement in Calculus

To answer the question whether students in the experimental group using writing activities achieve significantly greater improvement on mathematic scores compared to students in the control group who do not use the instructional activities, an analysis of covariance (ANCOVA) was used.

The ANCOVA shows a main effect for usage of writing activities, F (1,167) = 56.38, p<.05, indicating significant difference on the improvement scores in mathematics between the experimental and the control groups.

To further examine the data for differences between the two groups, the adjusted mean scores of achievement posttest of the two groups were determined. Table 1 provides a summary of the adjusted means of the experimental and control groups.

Table 1. Means and Standard Deviations for Experimental and Control Groups on Pre- and Posttest Mathematics

Test		Experimental	Control
Covariate (Pretest)	*N*	85	84
	Mean	12.01	12.29
	Standard Deviation	3. 91	3.88
Dependent (Posttest)	*N*	85	84
	Mean	24.17	20.21
	Standard Deviation	3. 12	3.24
	Adjusted Means	24.51	20.52

The pretest mean for the experimental group was 12.01 (*SD*=3.91) compared to the control group mean of 12.29 (*SD*= 3.88). The posttest means for both groups increased from the pretest, with experimental group showing the greater increase. Table 1 shows that the adjusted mean of the experimental group was significantly higher than the adjusted mean of the control group.

The results of the analysis of the usage of writing activities indicated that students in the experimental group showed significantly greater improvement on calculus achievement than students in the control group.

Attitude of Students Toward Learning in Calculus

Table 2 summarizes results from 85 Form Four students. All survey questions were phrased so that an "Agree" or "Yes" answer is a favorable response and "Disagree" or "No" answer is an unfavorable response.

Table 2. Attitude of Students toward Learning in Calculus

Item	Favorable (%)	Unfavorable (%)
1. I like Calculus better now more Calculus now	79	21
2. I learned	84	16
3. I spent more time on Calculus now than before	85	15
4. I enjoy Calculus better now than before	87	13
5. It was easy to learn Calculus by writing activities	83	17
6. I learn Calculus better by reflecting instead of only with book and memorizing	83	17

The students showed positive reaction towards the use of writing activities. This is reflected in the responses they gave in the survey form. Table 2 shows that 79% of the students like Calculus better, 84% learned more Calculus, 85% spent more time on Calculus and 87% enjoy Calculus better now than before. Students also highly agreed that learning Calculus was easy and they also agreed (83%) that they learn better by reflecting through writing instead of only using a book and memorizing.

Perceptions of Students towards the Use of Writing Activities

Eighty five Form Four students completed a form with the questions as shown in Table 3. The scaled score is calculated based on 5 – strongly agree, 4 – agree, 3 – not sure, 2 – disagree, and 1 – strongly disagree.

Table 3. Students' Perception towards the Use of Writing Activities

Item	5	4	3	2	1	Mean
1. Writing activities help me in understanding the topics better.	7 (8.24)	68 (80.00)	5 (5.89)	5 (5.89)	0 (0.0)	3.91
2. I am able to interact with my teacher and friends.	12 (140.12)	59 (69.41)	10 (11.76)	3 (3.53)	1 (1.18)	3.92
3. I feel confident about trying a new problem.	11 (12.94)	66 (77.64)	6 (7.06)	2 (2.35)	0 (0.00)	4.01
4. Writing activities make me feel comfortable learning Calculus.	9 (10.59)	67 (78.82)	5 (5.89)	3 (1.18)	1 (1.18)	3.94

As shown in Table 3, most of the students showed positive reactions towards the use of writing activities. Students felt confident about trying a new problem with a mean of 4.01.

Students felt that writing activities made them comfortable learning Calculus, with a mean response of 3.94.

Beliefs and Attitudes about Writing Activities as Learning Tool

As the interview data were analyzed, it became clear that the students' beliefs and attitudes about the use of writing activities in mathematics learning actually fell into several categories:

1. Most of the respondents feel that writing activities helps students to focus on their own thinking and use their own language. The students stated that

> S01: … I need to visualize, reflect, and use my own language to answer the task given by the teacher. It's hard but it's help me to understand the concepts and more focus .
>
> S04: I am able to explore algebraic functions and describe how to integrate the function using my own words. By doing so, I had logically deduced the meaning of the gradient myself.
>
> S09: I'm sometimes not confident but through group work, it's help me to know the basic meaning and then I'll tried to explain to others in my own words.
>
> S11: Writing activity help me not just simply remembering information or follow a set of instructions, but allow me to justify using my own way.

2. When students were given tasks to solve, conceptions and misconceptions were revealed as students describe their explorations of a problem. During the interview with the student, researcher gave the problem for student to answer. It was found that seven out of twelve students were still confused about the meaning of gradient.

3. All the respondents agreed that writing activities assist them to explore multiple methods and multiple solutions. The student stated that:

> S02 & S07: … The writing activities make us think and tried to find as many methods to solve the problems.
>
> S12: It's hard to find the alternative strategy but it's really open my mind that we can solve calculus using several ways.

4. Most of the students appreciate the opportunity to become authors of their own ideas. The students stated:

S08: The writing task made me write to explain how do I solve the problem. Even though I never do this before but it did help me to share my thinking.

S10: At first I think I'm wasting time to describe and explain how to I calculate the value of intercept and the gradient but later I'm able to understand better.

5. Writing encourages students to explore content rather than merely concentrate on the mechanics of symbol manipulation. The students stated

S03: I need to know the basic meaning in order to apply suitable strategy to solve the problem. When I see $y = mx + c$, I need to know and to understand the concept and meaning of each symbol not just used the symbol mechanically.

S09: When I engaged in solving a problem, I first need to make sense of the content of an equation not just simply recall and concentrate on the symbol to solve the problem

DISCUSSION AND CONCLUSION

Students who have succeeded on a calculus task are usually eager to do more of the same kind of task. They are motivated to achieve learning goals that they consider relevant to their needs. A learning goal is an instructional purpose, aim or objective that is set before students as a means of encouraging learning (McIntosh & Draper, 2001). Students usually aim to achieve goals that they perceive as interesting, realistic and attainable. Mathematics can be a creative activity involving intuition and invention (Miller, 1992). Mathematicians often explore mathematical ideas with a specific goal and discover new and interesting relationships through writing.

In this research, students were given an opportunity to use writing activities to explore calculus materials, concepts and ideas freely to assist them to develop their own intuitive ideas about mathematics. Mathematics students often need time to think about the problem before gaining an insight into possible solutions (Pugalee, 1997). Therefore in this research, students in the experimental group were given similar opportunities. Students' insights may open up further possibilities for creative endeavor. The writing which involved reflecting on the problem-solving process is an exciting and creative process for students and teachers. When solving problems, students were involved in creative processes such as:

1) Reflecting for a moment to consider what they really learn.

2) Searching for alternative methods of solving a problem.

3) Finding evidence and making decisions.

4) Exploring and constructing explanations.

When students perceive learning to be interesting, fun, personally meaningful, and relevant and the context supports and encourages personal control, motivation to learn and self-regulation of the learning process occur naturally (Brophy, 1987; Lepper, 1988; Noraini, 1999). Learning through writing activities and experiences that interest and stimulate students is usually inherently motivating. When students' interests in prescribed learning have been aroused, there is usually little need for other incentives or reinforcers. To make learning interesting and challenging, there must be sufficient variety in the nature and type of planned activities.

The writing activities were able to motivate the learner, identifying what is to be learned, and providing active involvement. With the use of writing activities, students were also able to compare, classify, analyze errors, or construct support that they encounter in the course of problem solving. The writing activities included conceptual understanding, procedural knowledge and logical thinking is a means for transforming concepts and skills. Writing engaged all students actively express and explain meaning at their own abilities (Borasi & Rose, 1989; Noraini, 2006)

To take advantage of the vast potential of writing activities in calculus, teachers will need to make significant changes in their pedagogy. They will need to form working relationships with students that allow for emergence of creative, higher order thinking. They will need to break traditional barriers that restrict the potential benefits of writing activities use in classrooms, and they must be willing to make ongoing efforts to succeed in the implementation process.

Changes in pedagogy entail teacher proficiency in using writing as an instructional tool. Teachers may no longer follow the well-traveled road of traditional education, but must be visionary in the potential opportunities for learning and their shift from teacher to guide and facilitator. Teachers need to structure mathematics lessons differently. They have to change in terms of the entire process, focus, and outcomes of educational expectations. Teachers are not only transferring knowledge to students but they are also engaging in different roles as students experience different processes of learning.

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