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Efficiency of understanding some mathematical problems by means of Pascal's triangle

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ABSTRACT

Various features have been found hidden in the Pascal triangle. In this paper, some very well-known properties of the Pascal triangle will be presented, as well as the properties related to different extensions of the triangle, namely the Pascal pyramid. Given that in the textbooks of the tenth grade, respectively in the school, where we realised the research but also in general in other schools, the importance of the Pascal triangle is not at the right level, then in this paper it has been shown very well that many different exercises in mathematics. The purpose of this paper is to look at the difference and importance of explaining mathematical units against units that students do not have knowledge of, namely the explanation of Pascal's triangle in the efficiency of solutions to various mathematical exercises. This research is mainly based on descriptive and quantitative method, while research instruments are two tests. From the study of Pascal's triangle, many solutions of problems in mathematics emerged through this triangle, starting from the binomial formula, the extension of the binomial formula, and the combinatorics as well as the probability. So the students realized that Pascal's triangle enables the solution of all these exercises in an easier and more understandable way and this also came from the results of two tests. Also, the students were open and motivated for the idea of using Pascal's triangle for other exercises but what now remains for them to find other possible solutions to the exercises through Pascal's triangle.

Keywords: Pascal's triangle, Pascal's pyramid, mathematical problem, SPSS program, binomial formula, combinatorics

INTRODUCTION

Pascal's triangle and pyramid are very important in solving math problems. The classic configuration of Pascal's triangle can be described as a triangle table of numbers, where each number is equal to the sum of the two numbers above it (Gardner, 1974). The highest entry in the triangle is 1. Pascal's triangle is considered to be one of the most elegant mathematical schemes. It also provides the link between combinatorics theory and probability theory and can be used on the demonstration of a Gaussian distribution (Gardner, 1974 ; Kuz'min, 2000a, 2000b).

In our mathematics classrooms as well as in our curriculum, we are usually used to encounter a special role of solving mathematical problems, which are usually presented to students by teachers, various textbooks and many other sources of information for students. We are also taught, especially in the subject of mathematics, that for each new chapter the students should first be informed about that lesson and then solve different exercises. In this paper, the importance of providing important information of the unit, respectively of the chapter that will be explained, is shown. So, the students, still without information about Pascal's triangle, were subjected to the first test and after a few weeks to the second test and the results were analyzed.

Pascal's triangle finds application in solving many exercises in mathematics, starting from the Binomial formula, finding coefficients, then in combinatorics and also in solving exercises in probability.

LITERATURE REVIEW

Pascal's triangle is a three-dimensional set of numbers (Hosch, 2013). This gives you a way to solve problems in combinatorics, number theory and even algebra using this method. Flusser and Francia (2000) published a theoretical paper on the history of the Binomial theorem from the time of Euclid to Newtonian mathematics. Their goal was to use visuals to derive both the binomial and multinomial theorems for use in high school and the first two years of undergraduate mathematics. They started with Euclid's elements, book II's specific expansion of $(a+b)^2$, and ended with a general formula for the expansion of $(a+b)^n$. Finally, the authors presented the derivation and proof of the polynomial theorem for the positive integers m and n, $(x_1 + x_2 + \cdots + x_m)^n$. The study added to the knowledge of the origin and derivation of the binomial theorem, the binomial series expansion was not obtained, but was treated as a subset of the binomial theorem. This was achieved by replacing n with one in the formula for the expansion of $(1 + b)^n$, with no explanation given for the transition from finite to infinite expansion.

Finding the coefficients of $(x + y)^n$ is the most typical application of the triangle, where n is the row number of Pascal's triangle containing the coefficients in the same order (from left to right) (Hosch, 2013). It is also worth it is mentioned that these lines start counting from zero instead of one.

Starting with one, we use our traditional numbering system. Despite the fact that the aforementioned triangle was given a name after Pascal, a French mathematician who is also credited as one of the first to invent mechanical calculators, there is evidence that it was used in civilizations long before Pascal's time in 1654 (Jerphagnon & Orcibal, 2020). This triangle has proven to be a useful tool for determining the outcome of situations such as coin tosses, the number of coin tosses can be represented by a row in Pascal's triangle, which contains the possible outcomes in its coefficients. Pascal's triangle is a gold mine of combinatorics, number theory and mathematics in general, with everything from prime numbers to the Fibonacci sequence. Besides Pascal, there is another famous mathematician from antiquity who made such an important contribution to this field that his theorems are still used today and have been rigorously studied by mathematicians for millennia. This paper is mainly based on the book, *Pascal's triangle* (Uspenskiĭ, 1976), which was translated and adapted from Russian language to English language by David J. Sookne and Timothy McLarnan.

Definition of the Research

Pascal's triangle is a special case of solving mathematical exercises and refers to the solution of binomial exercises and combinatorics exercises by means of Pascal's triangle. Specifically, the importance of this triangle is based on the fact that it enables finding the coefficients of the binomial formula, the various combinations in the combinatorics part, probability, etc.

Purpose of the Problem

The main purpose of this research is to examine and analyze the solution of the exercises by the students themselves for Pascal's triangle, where the exercises are also closely related to the Binomial formula, and at the same time this research shows that the importance of Pascal's triangle is extremely great, especially in the part of combinatorics.

METHODOLOGY

This research is quantitative research, which aims to explore students' understanding of the efficiency of solving mathematical problems using Pascal's triangle.

Samples

The research takes place in the high school of medicine, "Resonanca", City Pristina, Kosovo, namely in the tenth grade, therefore, the selection of participants was deliberate. One class was randomly selected from eight tenths of medical classes in Pristina. So, the sample of this research consists of 25 students of the "Resonanca" high school of medicine in Pristina who have no knowledge of Pascal's triangle and also neither the binomial formula nor the part of combinations nor probability.

Data Collection Methods

The data collection methods of this research will be two tests in written form, where the students will undergo the first test and also the second test.

First, students in a lesson will be shown that students from a randomly selected class will be given a test and then a second test after the binomial formula, combinatorics and trigonometry will be explained to them. of Pascal as well as probability. In this selected class, they will be subjected to the test, which consists of exercises that can be solved with the help of Pascal's triangle, but since the students have no knowledge of this triangle, the exercises are formulated in such a way that in addition to triangle may have other ideas to solve, so it will not be an exercise that the students have no idea to solve. To complete this test, the students had 40-50 minutes to complete it. The students worked well and during the completion of the test they were observed by the teacher, and they were also given instructions for anything unclear.

In the following hours after the test, the students begin to be informed first about the combinatorics chapter, then the binomial formula and finally about Pascal's triangle and probability.

After the explanation of these units, which took approximately four weeks to explain, and also for the students to have these concepts completely clear, the same student will then undergo the second test in which there will be exercises similar to the test exercises first but not the same. The second test will last 40-50 min.

Method of Formulating the Test

The idea of this research is that by means of these two tests grades are not necessarily assigned to the students, but to see the impact of the teacher's explanation on the solution of the test exercises as opposed to the solution of the exercises without having knowledge of these units and also of Pascal's triangle.

The first test and also the second test are formulated in such a way that the test contains a total of five questions, where the first two questions are related to the expansion and also the assignment of the coefficients of the given exercise, where the expansion and coefficients are easily assigned by means of the triangle Pascal. While the last two questions are about combinations, i.e., combinations that can also be solved with Pascal's triangle, and the last exercise will be with probability.

The exercises presented in the test are normally exercises that students with general knowledge can solve, but of course after the explanation of Pascal's triangle, the solution of these exercises becomes even easier and clearer.

The way of evaluating the exercises will be done in this form: students who do not have the correct solution will receive one point, if the solution is partially correct they will receive one point and if the solution is correct they will receive they get two points.

These data will first be placed in Excel but will be analyzed with SPSS, where in the first column the results for the first test will be entered, in the second column the results for the second test and by means of the t-test we will see that does the explanation of Pascal's triangle have an impact on the solution of the exercises.

Research Questions

1. Examining and analyzing the solutions of the exercises related to Pascal's triangle, as the students have no knowledge of this triangle.
2. Reviewing the solutions and analyzing the solutions of the exercises related to Pascal's triangle after gaining knowledge of Pascal's triangle.

Pascal's Triangle

We consider the sequence of numbers $d_0, d_1, d_2, \dots, d_n$ for some $n = 1, 2, 3, \dots, n$ (for $n = 0$ this sequence consists of the only number d_0. From here let's now generate a new sequence for numbers $s_0, s_1, s_2, \dots s_{n+1}$, from the following rules:

$$s_0 = d_0. \tag{1}$$

$$s_k = d_{k-1} + d_k \ (1 \leq k \leq n). \tag{2}$$

$$s_{n+1} = d_n. \tag{3}$$

We see that this new set of sequences is derived from α the original Pascal's relation. For example, from the sequence $2, 0, -2$, a new sequence can be obtained (using Pascal's relations) $2, 2, -2, -2$, from this sequence, then the next sequence can be obtained, which is $2, 4, 0, -4, -2$.

French mathematician and philosopher Pascal (1623-1662) began to investigate the properties of a triangular table of numbers, such that each row follows from Eq. (1)-Eq. (3). This table, which we will examine further, is known as "Pascal's triangle".

Remark 1: If the sequence β is derived from α the sequence by Pascal's relations, then the sum of the elements of the sequence β is equal to twice the sum of the elements of the sequence α. Using relations Eq. (1)-Eq. (3), we get:

$$\begin{aligned} s_0 + s_1 + s_2 + s_3 + \cdots + s_n + s_{n+1} &= d_0 + (d_0 + d_1) + (d_1 + d_2) + \cdots + (d_{n-1} + d_n) + d_n \\ &= (d_0 + d_0) + (d_1 + d_1) + (d_2 + d_2) + \cdots + (d_n + d_n) = 2(d_0 + d_1 + \cdots + d_n). \end{aligned} \tag{4}$$

Remark 2: We say that the series of numbers $d_0 + d_1 + \cdots + d_n$ is symmetric if for every integer k from 0 to n holds:

$$d_k = d_{n-k}. \tag{5}$$

For example, the sequence of four numbers $1, 0, 0, 1$ is symmetric. The series $s_0 + s_1 + \cdots + s_{n+1}$ of numbers obtained from the symmetric series $d_0 + d_1 + \cdots + d_n$ from the relations of Pascal's triangle is symmetric to itself. To show this, we must first prove the following relation:

$$s_k = s_{(n+1)-k}. \tag{6}$$

For $k = 0, 1, 2, \dots, n + 1$. But for $k = 0$ and $k = n + 1$ Eq. (6) follows from Eq. (1) and Eq. (3) and the condition $d_{0=}d_n$, (which follows from Eq. (5) for $k = 0$). For $(1 \leq k \leq n)$, we have:

$$s_k = d_{k-1} + d_k = d_{n-(k-1)} + d_{n-k} = d_{(n+1)-k} + d_{[(n+1)-k]-1} = d_{[(n+1)-k]-1} + d_{(n+1)-k} = s_{(n+1)-k}. \tag{7}$$

In the case of our example, applying Pascal's relations to the array 1 0 0 1, gives the array of five elements 1 1 0 1 1, that is symmetric to itself. Let's consider the case when the sequence contains the only number one. We can call this "Pascal zero sequence." From here, with the help of Pascal's relations, the sequence can be generated, which we can call Pascal's first sequence. By applying Pascal's sequence, one can generate the second Pascal's sequence from the first Pascal's sequence, and so on.

As long as on each pass to the new array, the number of elements of the array increases by one, then it can be $n + 1$ numbers in the -th array. Without performing any calculations, we see that from **remark 1** and **remark 2** we conclude:

1. The sum of the numbers n-th in Pascal's sequence is 2^n (while in the procedure from the first sequence to the next sequence, the sum of the numbers is doubled, and the sum of the zero sequence is $2^0 = 1$).

														1														
													1		1													
												1		2		1												
											1		3		3		1											
										1		4		6		4		1										
									1		5		10		10		5		1									
								1		6		15		20		15		6		1								
							1		7		21		35		35		21		7		1							
						1		8		28		56		70		56		28		8		1						
					1		9		36		84		126		126		84		36		9		1					
				1		10		45		120		210		252		210		120		45		10		1				
			1		11		55		165		330		462		462		330		165		55		11		1			
		1		12		66		220		495		792		924		792		495		220		66		12		1		
	1		13		78		286		715		1287		1716		1716		1287		715		286		78		13		1	
1		14		91		364		1001		2002		3003		3432		3003		2002		1001		364		91		14		1

Figure 1. Pascal triangle (Source: Authors' own elaboration)

Example 1: Calculate $(x\text{-}7)^7$

Figure 2. Example 1 of the first test (A picture taken from student work in the classroom, reprinted with permission)

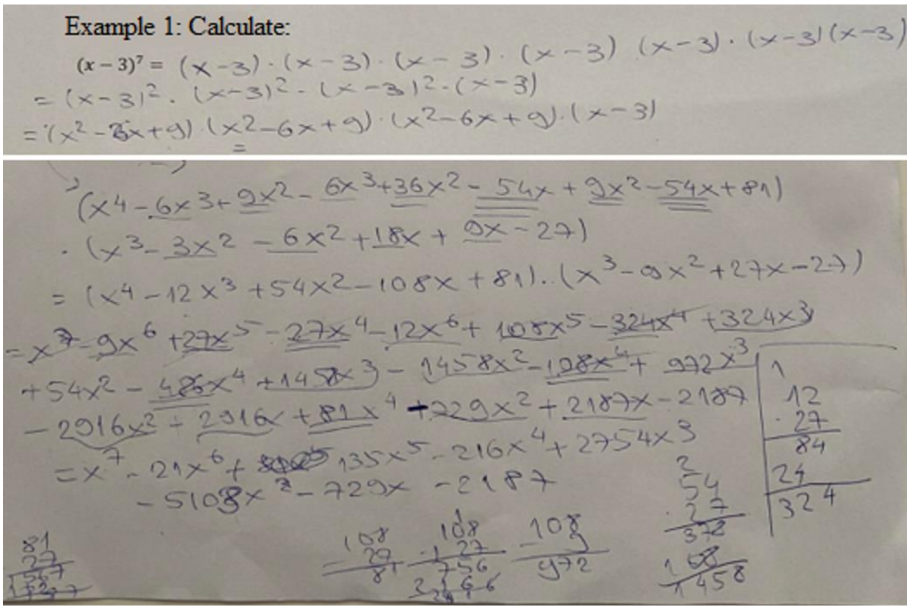

Figure 3. Correct student solution of example 1 (A picture taken from student work in the classroom, reprinted with permission)

2. All sequences of Pascal are symmetric (as long as the property of symmetry is preserved from the past from one sequence to another and the zero sequence is symmetric).

Let us write Pascal's series, one after the other, so that each number of each series is between and below those numbers of the previous line from which it is calculated. We obtain an infinite table called Pascal's triangle, which fills the interior of an angle of each of its segments, consisting of the first row to the n-th row that forms the triangle. The segment of Pascal's triangle containing the first 15 lines (from zero to the fifteenth) is shown in **Figure 1**.

RESULTS

Analysis and Interpretation of the Results

Through the results that are presented in the following tables as well as their detailed analysis, we will answer the research questions. Students were evaluated with two points if the solution was completely correct, with one point if the solution was not completely correct and with zero points if the solution was wrong or was not solved at all. Below we will present some exercises of some students who solved the exercise correctly, those whose solution was incomplete and those whose solution was wrong.

Presentation of some exercises solved from the first test (**Figure 2**).

Next, we present correct student solution of example 1, solved in the test (**Figure 3**).

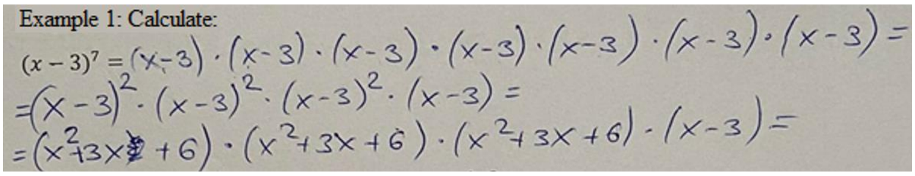

Figure 4. Partially correct student solution of example 1 (A picture taken from student work in the classroom, reprinted with permission)

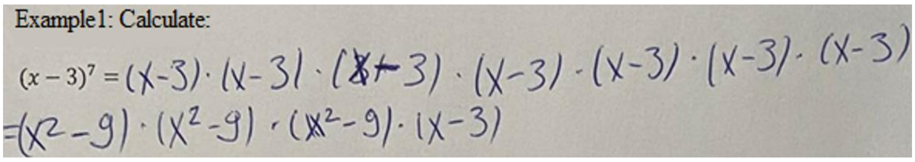

Figure 5. Incorrect student solution of example 1 (A picture taken from student work in the classroom, reprinted with permission)

Example 2: Find the adjacent coefficient x^3y^2 for $(2x+y)^5$

Figure 6. Example 2 of the first test (A picture taken from student work in the classroom, reprinted with permission)

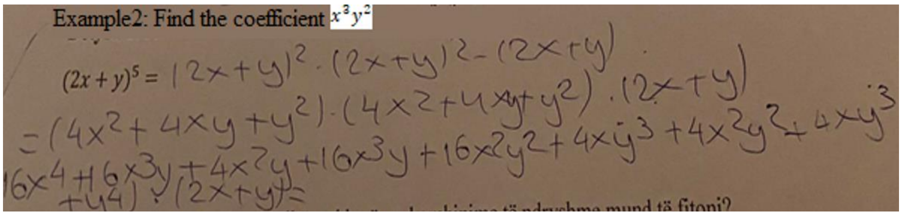

Figure 7. Partially correct student solution of example 2 (A picture taken from student work in the classroom, reprinted with permission)

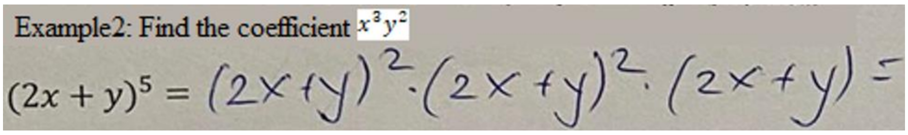

Figure 8. Uncompleted student solution of example 2 (A picture taken from student work in the classroom, reprinted with permission)

The exercise in **Figure 3** is about the student who correctly solved the exercise with the knowledge he had before Pascal's triangle was explained and received two points. Mostly the solution of the exercise does not have any errors and is evaluated as complete.

Other solutions from the students, but in this case incorrect and partially correct solutions are presented in **Figure 4**.

The decomposition by the student started correctly, but then the decomposition of the square of the binomial was wrong and is considered an incorrect answer (**Figure 5**).

Another also incorrect solution of the first exercise is presented below. It is noticed that there are students who, if they multiply the brackets, multiply only the first and second expressions. So, even in the above case, the student made this mistake, and then normally the solution to this exercise will be incorrect.

While **Figure 6** presented the solution of the second exercise by the student so that the solution was incomplete and the correct solution of this exercise from my side is shown.

Figure 7 shows how the exercise was solved by the student.

This exercise is partially correct. The method of starting the solution of the exercise is correct up to the decomposition of the square of the binomial. Also in the part of multiplying the brackets, but then the student did not continue. Apparently, he did not know how to continue the remainder of the multiplication.

Another solution to this exercise but in this case incorrect solution (**Figure 8**).

Solving the exercise started correctly, but then the student did not continue solving this exercise.

While the correct solution to the exercise of finding the coefficient near x^3y^2 is given in **Figure 9**.

The adjacent coefficient of x^3y^2 is 80.

Also, we presented some solutions of the third exercise, which were mostly incorrect solutions (**Figure 10**).

The student's incorrect solution is shown in **Figure 11**.

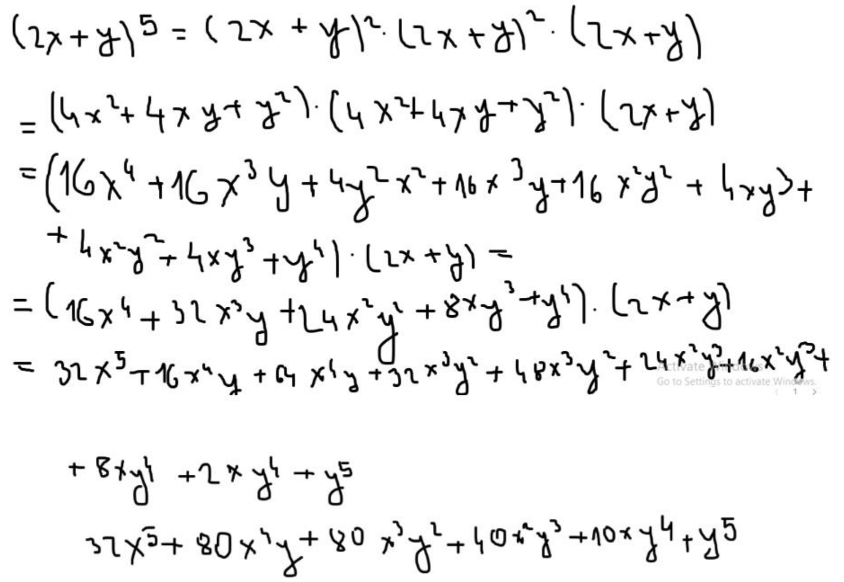

Figure 9. Correct solution of example 2 (A picture taken from student work in the classroom, reprinted with permission)

Example 3: If you toss the coin three times, how many possible combinations can you win?

Figure 10. Example 3 of the first test (A picture taken from student work in the classroom, reprinted with permission)

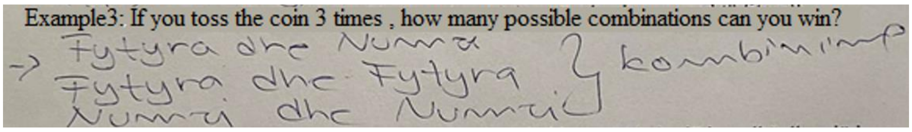

Figure 11. Incorrect student solution of example 3 (A picture taken from student work in the classroom, reprinted with permission)

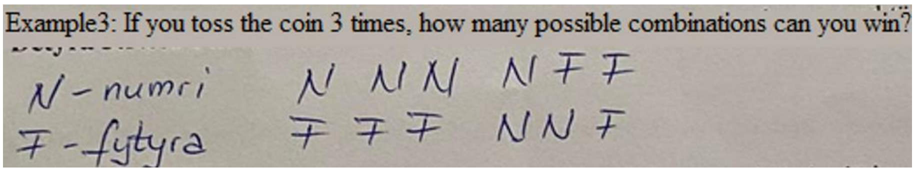

Figure 12. Uncompleted student solution of example 3 (A picture taken from student work in the classroom, reprinted with permission)

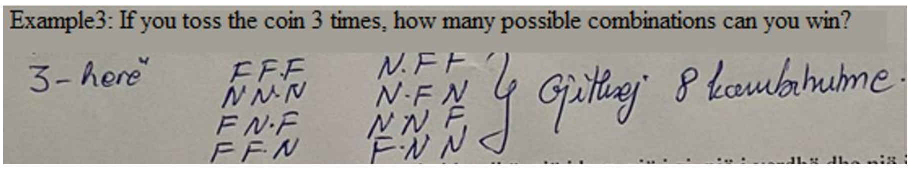

Figure 13. Correct student solution of example 3 (A picture taken from student work in the classroom, reprinted with permission)

The way of wording to answer, that is, how he divided the numbers and the figures of the presentation of the coin are in order, but the student gave only three possible cases, but the combinations were not correct because if the coin is tossed three times then there are not only two possible combinations but three. It may happen that the student has not read the exercise carefully or it may happen that the knowledge of solving exercises of this type or the sense of solving these types of exercises is insufficient.

Another incorrect solution to this exercise is shown in **Figure 12**.

The student has submitted only four combinations.

While a correct solution by a student is shown in **Figure 13**.

The Internet resources as well as other illustrative and demonstration methods to make the as attractive as possible (Kamberi et al., 2022). While the correct solution to find these combinations using coins is shown in **Figure 14**.

So, a total of eight different combinations.

Figure 14. Correct student solution of example 3 with illustration (Source: https://d3k6u6bv48g1ck.cloudfront.net/coin-image-1_Euro-a-Germany-bGF_AAEBBAcAAAEl4E_AFZux.jpg, Accessed: 19 September 2022)

Example 1: Calculate $(x+1)^6$

Figure 15. Example 1 of the second test (A picture taken from student work in the classroom, reprinted with permission)

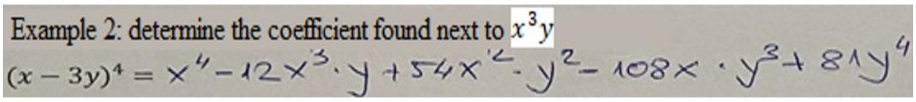

Figure 16. Partially correct student solution of example 2 (A picture taken from student work in the classroom, reprinted with permission)

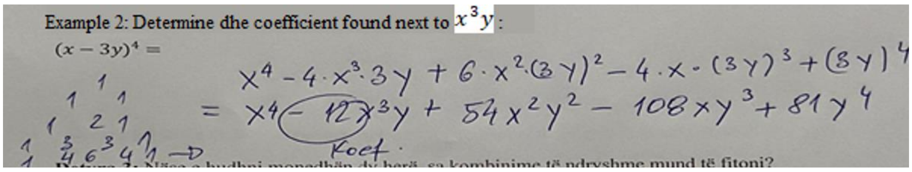

Figure 17. Correct student solution using Pascal's triangle of example 2 (A picture taken from student work in the classroom, reprinted with permission)

These were some of the exercises solved by the students in the first test. While after the explanation of the units, they are presenting some photos of the students who have solved the Easter triangle exercises. The exercises were almost the same, but they differed in the part of the numbers or in the wording of the exercises.

Presentation of Some Exercises Solved from the Second Test

Figure 15 shows solved exercise from the second test after explaining Pascal's triangle.

The student has a complete solution of the exercise using Pascal's triangle as well and has determined the adjacent coefficient of x^3y . So, based on the coefficients of the triangle, the exercise given in the second test has been expanded. The student in question also correctly solved the first exercise in the first test.

Another correct solution of the student in the second test, but the adjacent coefficient of x^3y was not determined, therefore, the solution was taken as partially correct (**Figure 16**).

Another completely correct solution the second exercise is shown in **Figure 17**.

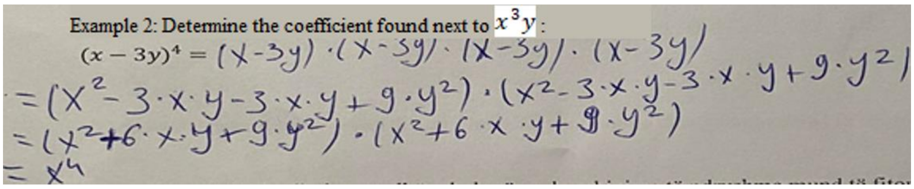

Figure 18. Incorrect student solution of example 2 (A picture taken from student work in the classroom, reprinted with permission)

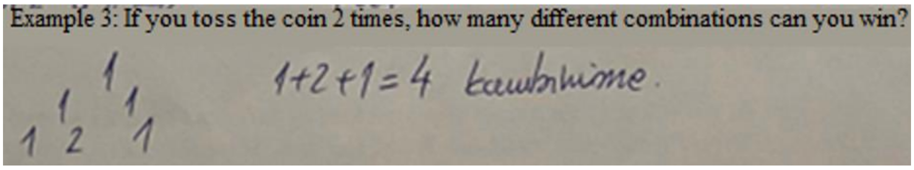

Figure 19. Correct student solution using Pascal's triangle of example 3 (A picture taken from student work in the classroom, reprinted with permission)

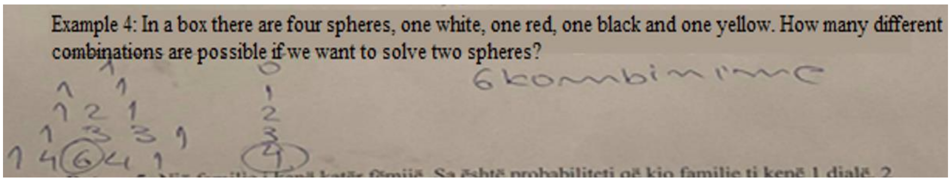

Figure 20. Correct student solution using Pascal's triangle of example 4 (A picture taken from student work in the classroom, reprinted with permission)

Table 1. Student results in the first & the second test

n	FT	ST	n	FT	ST	n	FT	ST	n	FT	ST	n	FT	ST
Nx. 1	4	7	Nx .6	6	6	Nx. 11	10	10	Nx. 16	3	5	Nx. 21	8	10
Nx. 2	6	8	Nx. 7	5	9	Nx. 12	5	4	Nx. 17	5	8	Nx. 22	7	8
Nx. 3	2	5	Nx. 8	4	7	Nx. 13	6	10	Nx. 18	3	8	Nx. 23	4	8
Nx. 4	8	10	Nx. 9	1	3	Nx. 14	7	7	Nx. 19	4	8	Nx. 24	5	9
Nx. 5	4	8	Nx. 10	4	4	Nx. 15	7	9	Nx. 20	5	9	Nx. 25	6	10

Note. n: Number of students; FT: First test; & ST: Second test

Table 2. Presentation of exercise solutions in the first test

Correct answers	PCA	Wrong answer	No answer	Correct answers	PCA	Wrong answer	No answer
1	2	1	1	3	1	1	0
2	2	1	0	2	3	0	0
0	2	1	2	1	1	2	1
3	2	0	0	2	1	2	0
1	2	2	0	1	1	3	0
2	2	0	1	2	0	3	0
1	3	1	0	1	3	1	0
1	2	2	0	3	2	0	0
0	1	2	2	3	1	1	0
2	0	3	0	2	0	2	1
5	0	0	0	1	3	1	0
2	1	1	1	3	0	1	1
2	2	1	0				

Note. PCA: Partially correct answers

While at the end we present a solution of the student who solved the second exercise not with Pascal's triangle but like the solutions in the first test. But the solution of the exercise was not correct (**Figure 18**).

The third and fourth exercise solved correctly by the students (**Figure 19** and **Figure 20**).

Table 1 shows a database with student data for scores on the test and scores on the second test.

For the first test on solutions of exercises and their analysis, the students have managed to answer a total of 125 questions, 46 answers were correct, 37 were partially correct answers, 32 were wrong answers, and 10 exercises were not solved at all (**Table 2**).

While data in **Table 3** were obtained from the second test: from a total of 125 questions, 77 answers were correct, 35 answers were partially correct, nine wrong answers, and four were not answered at all.

Table 3. Presentation of exercise solutions in the second test

Correct answers	PCA	Wrong answer	No answer	Correct answers	PCA	Wrong answer	No answer
3	1	1	0	3	1	0	1
3	2	0	0	4	1	0	0
2	1	1	1	1	3	1	0
5	0	0	0	3	2	0	0
3	2	0	0	3	2	0	0
2	1	1	1	3	2	0	0
4	1	0	0	4	1	0	0
2	3	0	0	5	0	0	0
0	3	2	0	3	2	0	0
1	2	1	1	3	2	0	0
5	0	0	0	4	1	0	0
1	2	2	0	5	0	0	0
5	0	0	0				

Note. PCA: Partially correct answers

Table 4. t-test: Group statistics

Group	n	Mean	Standard deviation	Standard error mean
1	25	5.1600	2.01412	.40282
2	25	7.6000	2.06155	.41231

Table 5. t-test: Independent samples test

	Levene's test for equality of variances		t-test for equality means						
	F	Sig.	t	df	Sig. (2-tailed)	MD	SED	95% CI of difference	
								Lower	Upper
Equal variances assumed	.053	.819	-4.233	48.000	.000	-2.44000	.57643	-3.59898	-1.28102
Equal variances not assumed				47.974				-3.59900	-1.28100

Note. MD: Mean difference; SED: Standard error difference; & CI: Confidence interval

Looking at **Table 2** and **Table 3** without analyzing the data yet, we see that the number of correct answers in the second test has doubled, while there were more partially correct answers in the first test, which shows that from the explanation of these units the students have managed to improve in solving exercises with Pascal's triangle, making most of the exercises that were partly correct answers in the first test, switch to correct answers in the second part.

Interpretation of Statistical Data for the First & Second Test

Table 2 and **Table 3** with data for the first and second test is given above, so, the students' results are presented in **Table 4** and **Table 5**. In order to analyze the statistical data with the data of the first and second test, the SPSS program was used to the linear regression model (Aliu et al., 2021). With this program, by comparing means, specifically independent samples t-test, the results in **Table 4** and **Table 5** were obtained. The groups are defined by the results of the first and second test out of 25 results for each group. The average in the first group is 5.1600 while the average in the second group is 7.6000, the standard deviation and standard errors of the averages are also presented. While in the second table it is important to see the significant e that simultaneously represents the value of p. In our case, the significance is 0.000103, which is smaller than $p<0.05$, which means that the difference between the results of the first and second test is quite large, and thus we conclude that the influence on the part of the teacher's explanation, specifically the triangle of Pascal has resulted in improvement and increase in results after explaining Pascal's triangle.

CONCLUSIONS

The purpose of this research was to analyze the importance of explaining Pascal's triangle in solving different exercises in mathematics, such as exercises with the binomial formula, with combinations and probability.

The results of this research were analyzed based on the students' tests and the results achieved in the tests. The students' solutions were also analyzed that some of the solutions were correct, some incomplete, but also incorrect or empty solutions.

What is important in this paper, based on the test results, of course, the explanation of Pascal's triangle has influenced the easier and faster solution of the exercises, resulting in better results in the second test. There were some of them who, even though Pascal's triangle was explained to them, solved the exercises using the method they solved the exercises in the first test.

There have also been those who, as in the first test, as well as in the second, have shown maximum results. But mainly a large percentage show that the results in the second test were very good compared to the first one, which also results in proving the purpose of the research on the efficiency of solving exercises by means of Pascal's triangle.

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Ethical statement: The author stated that, according to applicable institutional regulations, the study does not require approval from an ethics committee since it is educational research based on standardized tests. Informed consent was obtained from the participants before the research was conducted; and they were informed about the research, anonymity and their rights about withdrawing from the study without adverse repercussions. All students participate in the study with desire, will and full awareness.

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Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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