



International Electronic Journal of Mathematics Education

Volume 3, Number 2, July 2008

www.iejme.com

DEVELOPMENT OF A COMPUTERIZED NUMBER SENSE SCALE FOR 3rd GRADERS: RELIABILITY AND VALIDITY ANALYSIS

De-Ching Yang

Mao-neng Fred Li

Wei-Jin Li

ABSTRACT. This study was to develop a computerized number sense scale (CNST) to assess the performance of students who had already completed the 3rd-grade mathematics curriculum. In total, 808 students from representative elementary schools, including cities, country and rural areas of Taiwan, participated in this study.

The results of statistical analyses and content analysis indicated that this computerized number sense scale demonstrates good reliability and validity. Cronbach's α coefficient of the scale was .8526 and its construct reliability was .805. In addition, the 5-factor number sense model was empirically and theoretically supported via confirmatory factor analysis and literature review..

KEYWORDS. Computerized Testing, Confirmatory Factor Analysis, Number Sense, Reliability, Validity.

RATIONALE AND PURPOSE

Developing children's number sense is considered, internationally, to be a key ingredient in mathematical skills (Emanuelsson & Johansson, 1996; McIntosh, Reys, Reys, Bana, & Farrell, 1997; National Council of Teachers of Mathematics [NCTM], 2000; Sowder, 1992; Yang, 2003). All strongly suggest that this topic should be integrated into school mathematics curricula, in order to help children develop number sense. For example, the Number and Operations Standard, in the NCTM's Principles and Standards for School Mathematics, emphasizes the importance of number sense, by stating "...central to this Standard is the development of number sense."(p. 32). Indeed, number sense has become a major topic, in mathematics education, in the 21st century.

Due to its importance, number sense has recently attracted a growing amount of attention and research in Taiwan. More specifically, a major mathematics curricula reform is currently under discussion in Taiwan. The guidelines for a nine-year integrated mathematics curricula plan (Ministry of Education in Taiwan, 2003) also underline the importance of the teaching and learning of number sense. During the past decade, most of number sense studies have focused on either the performance of students or the instruction of teachers. However, few studies have discussed methods of assessing number sense. The traditional paper-pencil instrument used to assess number sense plays a role in measurement and evaluating children's number sense development. Yet, there is one possible advantage over a computerized number sense test: without paper or pencil available, children are less likely to use paper and pencil algorithms. Therefore, the purpose of this study was to develop a computerized number sense scale to assess the performance of students who had just completed the mathematics curriculum of grade 3.

BACKGROUND

Number sense implies that an individual has a very good understanding on numbers, operations, and their relationships. It also includes the ability to develop and use the characteristics of number sense efficiently (such as mental computation, estimation, judging the reasonableness of computational results, and so on) to handle numerical problems or daily-life situations which include numbers (McIntosh et al., 1997; Reys & Yang, 1998; Sowder, 1992; Yang, 2003).

Due to its' complex, therefore, there is a lack of consistent theoretical consensus defining number sense (Berch, 2005; Case, 1998; Sowder, 1992; Verschaffel, Greer, & De Corte, 2007). Although number sense is a relatively new term as far as the Taiwanese mathematics curriculum is concerned, the emphasis on meaningful learning and understanding has been widely discussed, and is already accepted in mathematics education (Burns, 1994; Hiebert & Grouws, 2007). Because number sense is a complex process, it may involve different components in which mathematics educators, educational psychologists, researchers and curricula developers are interested during the past two decades. As a result, different psychological perspectives and characteristics of number sense have been described (Case, 1989; Howden, 1989; Resnick, 1989); theoretical frameworks of number sense proposed (Greeno, 1991; McIntosh, Reys, & Reys, 1992); and finally essential components of number sense enumerated (Markovits & Sowder, 1994; NCTM, 2000; Yang, 2003; Yang, Hsu, & Huang, 2004).

Based on related research reports and documents (Markovits & Sowder, 1994; McIntosh et al., 1992; NCTM, 2000; Reys & Yang, 1998; Sowder, 1992; Yang, 2003;), this study defined number sense components as below:

(1) Understanding the meanings of numbers and operations

This implies an understanding of the base ten number system (whole numbers, fractions, and decimals), including place value, number patterns and the use of multiple ways to represent numbers (McIntosh et al., 1992).

(2) Recognizing relative number size

This implies the recognition of the relative size of numbers. For example, when students compare fractions, they do not need to depend on standard written methods (such as finding the least common denominator suggested in the mathematics curriculum). They are able to use meaningful ways, such as the same numerator, same denominator, transitive, and residual (Cramer, Post, & delMas, 2002) to compare the fractions.

(3) Being able to compose and decompose of numbers

This means that an individual is able to decompose and compose numbers flexibly for the convenience of computational fluency. For example, when students are asking to solve 32×25 , they know how to decompose 32 to 8×4 and $8 \times 4 \times 25$ equal to 8×100 , then the answer is 800. This can help children to solve problems easily.

(4) Recognizing the relative effect of operations on numbers

This means that an individual is able to recognize how the four basic operations affect the results. For example, when asking children to find the best estimate for 591×0.95 or $196 \div 0.95$, they do not need to depend on written methods to find the answers. However, they should be able to decide that 591×0.95 resulted in a smaller number and $196 \div 0.95$ resulted in a larger number.

(5) Judging the reasonableness of computational results

This implies that individuals can mentally apply estimation strategies to problems without using written computation (McIntosh, et al., 1992; Sowder, 1992). At the same time, they should also be able to judge the reasonableness of the result. For example, when children were told that the digits 156116 correctly represented the product of 629.5 and 0.248 but that the decimal point was missing, they should not need to rely on paper-and-pencil or memorize rules to find the answer. However, they should know that 600 multiplied by 0.254 (about $1/4$) is about 150 and that an answer of 16.2179 is unreasonable.

The design of test items in the number sense scale was based on the above number sense components, as well as related mathematical textbooks used for 1st to 3rd graders in Taiwan.

Technology and Mathematics Assessment

Technology not only plays a key role in mathematics teaching and learning, but also aids in mathematics assessment (NCTM, 2000). Mathematics assessments should not be the end of instruction and learning, but should support future learning and instruction (NCTM, 2000). Over the past decade, most assessments of children's number sense have used paper-and-pencil tests (Markovits & Sowder, 1994; McIntosh et al., 1997; Reys & Yang, 1998; Yang, 2003). Paper-and-pencil tests, however, can waste time and paper, require strenuous effort to correct papers, and be restricted to classrooms or testing centers. Assessment through the Internet can eliminate these limitations imposed on paper-and-pencil tests and be more efficient and effective than paper-and-pencil tests. Although number sense is considered to be an internationally important topic in mathematics education, no math assessment presently exists that uses on-line number sense tests via the Internet. There is one possible advantage over a computerized number sense test: without paper or pencil available children are less likely to use paper and pencil algorithms. This motivated and encouraged us to proceed with this study.

METHOD

Sample

Participants consisted of 808 students who just completed the 3rd-grade mathematics curriculum, from representative elementary schools in the cities, country or rural areas of Taiwan. These students were selected from north, south, east, and west areas of Taiwan based on the amount of population in each area. Due to the schools must have computer rooms and can connect to website, therefore, students were purposively selected from a wide range of family backgrounds, covering different parent occupations, incomes, and a variety of educational levels. They all volunteered to participate into the on-line tests when they were selected.

Instrument development

Based on the number sense framework, described in theoretical and related math textbooks presently in use in Taiwan, 90 test questions were initially drafted. To ensure

representative and balanced content, they were then reviewed by several math educators and five math teachers in elementary schools. As a result of their careful review, sixty items were selected for the 1st-run pilot study. After this had been done, to derive frequently occurring errors for distractors and possible correct reasons or misconceptions, 150 3rd-graders were informed of the reasons for the study and asked to participate, with 20 students being interviewed for further investigation. After the questions had been posed and after picking the correct answer, they were asked to give the reasons they chose this option. With careful screening, fifty test questions, along with relevant reasons, were selected to make up the formal on-line test. Due to a limitation of forty minutes per class, and the persistence of these young children, the test questions were divided into two parts, making up a two-stage online test.

Once again, in order to understand whether these participating students could fully understand each question and be able to complete the on-line test, a 2nd-run pilot study was given to 68 students. It was found that the two-stage on-line test worked very well and the time limit of ninety seconds for each question was deemed appropriate. Because some of the item descriptions were found to be unclear, three questions were further revised. The pre-specified test item numbers, for the five number sense components, are presented in Table 1.

Table 1: An initial proposed 5- factor number sense model and related test question numbers

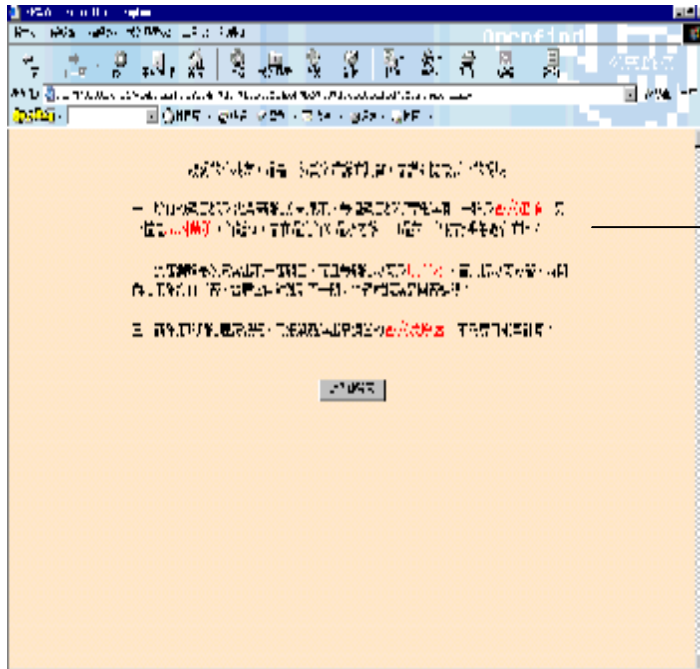
| Preliminary Factors | Question number | # of items | # of items after deletion |
|--|--|------------|---------------------------|
| 1.) Understanding the meanings of numbers and operations | Q1*.Q7.Q13.Q19.Q25*.Q31.Q37.Q43.Q48 | 9 | 7 |
| 2.) Recognizing relative number size | Q2*.Q8*.Q14.Q20.Q26*.Q32.Q38.Q44*.Q49* | 9 | 4 |
| 3.) Being able to compose and decompose of numbers | Q3.Q9.Q15.Q21.Q27.Q33.Q39.Q50 | 8 | 8 |
| 4.) Recognizing the relative effect of operations on numbers | Q4*.Q10.Q16.Q22.Q28.Q34.Q41.Q46.Q45* | 9 | 7 |
| 5.) Judging the reasonableness of computational results | Q5*□Q11*□Q17□Q23□Q29*□Q35*□Q40□Q42 | 8 | 4 |
| | Total | 43 | 30 |

Note. * indicates that a poor item deleted after the 1st-run item analysis and expert review

Procedures

All of the tests were conducted in the computer rooms. Students were asked to connect the internet and enter the website of the number sense testing system. They first entered the welcome website and asked students to select the section 1 or section 2.

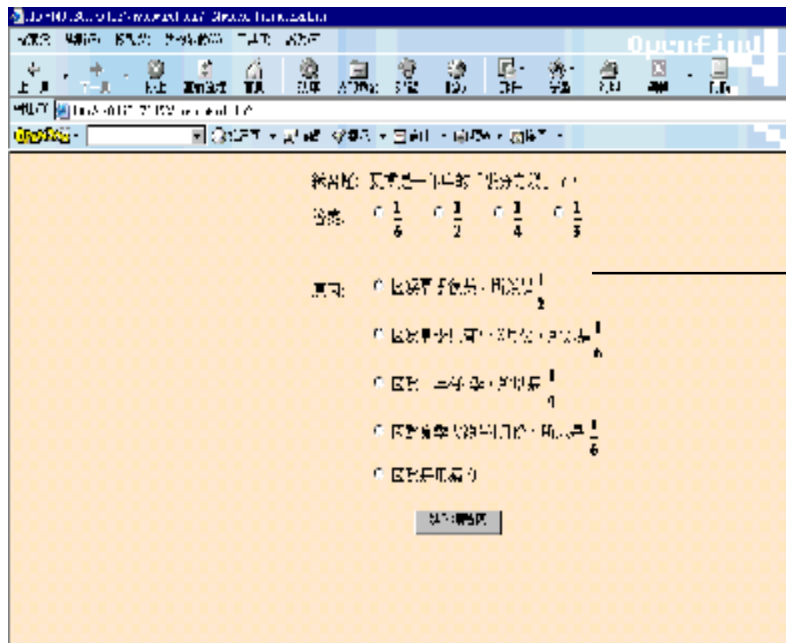
After the selection of personal information, the screen will enter the area of testing explanation as follows:



The testing explanations include :

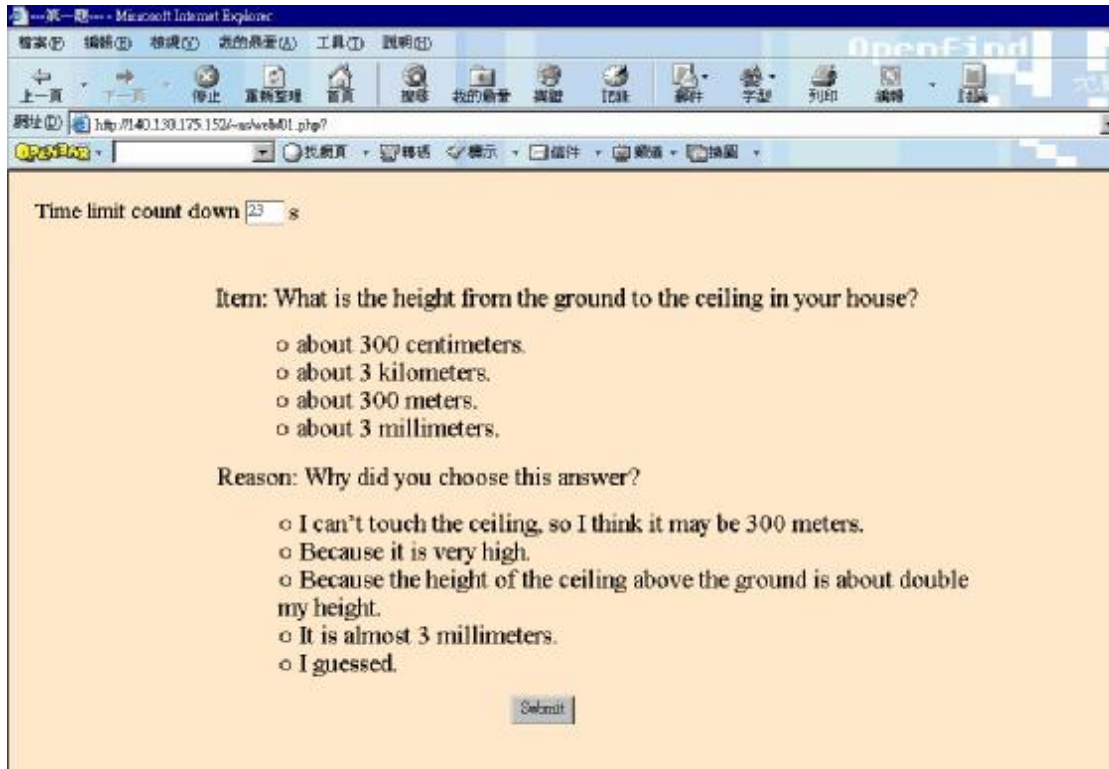
1. All test questions are multiple-choice items. You are required to choose an answer and a reason to pick it for each item. ...
2. The time limit for picking an answer and the choice of a reason is 90 seconds for each item.

After the explanation for the test, one item for practice will show on the screen. The children can learn how to use the system. The screen is as follows:



One item for kids to practice

After the practice, the formal test begins and the format is as follows:



Scoring rules

The number sense on-line test was composed of multiple-choice questions. For each test question the students had to provide a correct answer and the reason for their choice. Because of these unique features, a special scoring rule was determined (Yang & Li, 2003) as described below:

- 1) If both the answer and the corresponding reason were correct, 4 points were given;
- 2) If the answer was correct, but the corresponding reason was incorrect, 1 point was given;
- 3) If the answer was incorrect but the corresponding reason was correct, 2 points were given;
- 4) If both the answer and the corresponding reason were incorrect, a score of zero was given.

RESULTS

Below, we show the item analysis used for development of the scale, the analysis of the underlying structure and statistical analysis for reliability and validity of the number sense test.

Item analysis

Based on the item difficulty indices (.0495~.7723), item discrimination power (.2723~.5941) and a given item deleted α , we were able to decide which items should be deleted, in order to achieve a desirable Cronbach's α . For example, since the corrected correlation between Q5 and the total score (with Q5 deleted) indicates a low discrimination power (.1018), Cronbach's α will increase to 0.8866 (compared to .8845 for the total test) after Q5 has been deleted. Accordingly, four questions: Q5, Q35, Q44 and Q49 were deleted, due to their discriminatory indices being less than 0.2. In addition, seven questions: Q1, Q2, Q8, Q11, Q25, Q26 and Q45 were deleted due to they were either too difficult, unclear or had less discriminatory power. This study retained 30 test questions for the underlying structure analysis. These 30 items reflected the original 5-factor number sense model. (See Table 1 for details).

Underlying structural analysis

To empirically search for the underlying structure of the 30-item number sense test, a Principal Component Analysis (PCA) was conducted using SAS. The results of the oblique rotation of the factor loading matrix through covarimin show that most of the factor loading patterns appear consistent with Thurstone's (1947) principle of simple structure. For example, the factor loading of Q22 in Factor 1 is .70485, while the factor loadings are quite small (<.14762) in the other Factors. Therefore, Q22 can be unambiguously assigned to Factor 1. However, substantial factor loading (<.30) was found across more than one factor, for a few questions, such as, Q41 and Q42. Judging from their item content, we decided to assign Q41 to Factor 2 and Q42 to Factor 4.

Since Kaiser's decision rule of an eigenvalue >1 often retains too many factors (Enzmann, 1997), we applied Lautenschlager's parallel analysis method (1989) to decide which factor should be kept, using a computer program called RanEigen written by Enzmann(1997). Results show that there are three factors with eigenvalues larger than those generated by the RanEigen program. Thus, these should definitely be retained. Due to the trivial differences found

on factors 4 and 5, we decided to keep them as well, after a careful review of their item content. Consequently, 5 factors were retained after theoretical and empirical consideration. In total, 25 items remained in the final version of the on-line number-sense test.

Second order factor analysis

If the correlations between the factors were substantial, then second-order factor analysis was performed. The belief is that some math problems can only be solved through a combination of multiple skills or abilities. Therefore, these five subscales were treated as indicators (by summing up the item scores in each subscale). The inter-scale correlation found between the five subscales range from .383 to .509. Thus, we can factorize them using PCA. Results indicate only one dominant factor with an eigenvalue of 2.798. The eigenvalues for the other factors are .622, .614, .503, .463, respectively. Moreover, data also shows that these five subscales all have substantial factor loadings ($> .70$) on the first dominant factor. This implies that these inter-correlated factors can be represented by a higher level factor; this factor was named “number sense”.

Confirmatory Factor Analysis (CFA)

To double check our 1-factor number sense model, using the five-subscale as indicators, a CFA was performed. In order to examine the goodness of fit the 1-factor CNST model, the 5-subscale CNST model was tested using AMOS. Data suggest that the proposed model fits the data quite well ($\chi^2= 6.745$, $df= 5$, $P= .240$, $RMSEA=.021$, and $NFI= .994$). This confirms that the five subscales tap into the same underlying trait (number sense). Moreover, the standardized coefficients (.62, .75, .70, .68 and .61), for the five subscales also reflect that the convergent validity of the newly-constructed scale is satisfactory.

Reliability analysis

To obtain preliminary information concerning the reliability of the newly developed scale, the Cronbach's α and construct reliability results were investigated. The Cronbach's α coefficients for the five components were .7097, .6984, .6051, .5633 and .5488, respectively. The reliability of the last two subscales was below satisfaction, perhaps due to the length of the test or difficulties stemming from the restricted range of items. Except for factors 4 and 5, the reliability

of the other factors was acceptable (above .6). Besides, Cronbach's α coefficient for the entire test was 0.8526. This indicated that, in general, the test items included in the final version of CNST test were generally homogeneous in content. Furthermore, the construct reliability index was computed using Korchia's Excel macro program (2001) and the construct reliability was .805, reflecting that the CNST's construct reliability was quite satisfactory.

Validity Information

Information about the scale validity included content validity, specialist validity and construct validity. To analyze whether the constructed questions were representative and not beyond the curriculum usually taught within 3rd graders, three elementary school teachers and two mathematics educators were invited to carefully review the items; all agreed that the content of the 25 test items was representative and appropriate. They also agreed that the 25 items exactly reflected the corresponding five components, in terms of item content.

The twenty-five items were then subjected to PCA and CFA for construct validity analysis (see Table 2). Once more, the 5 identified factors were reconfirmed as having eigenvalues larger

Table 2: Factor loadings for the 25-item factor analysis with oblique rotation

| | Factor 1 | Factor 2 | Factor 3 | Factor 4 | Factor 5 |
|-----|----------|----------|----------|----------|----------|
| Q13 | 0.65007 | | 0.13205 | | |
| Q15 | 0.44988 | 0.17267 | 0.23246 | | 0.27392 |
| Q16 | 0.69911 | | | | |
| Q17 | 0.46225 | | 0.11833 | 0.20458 | 0.25008 |
| Q19 | 0.62160 | 0.26524 | | | |
| Q22 | 0.72324 | 0.13574 | | | |
| Q27 | | 0.59622 | | | |
| Q31 | | 0.63861 | | | |
| Q33 | 0.19009 | 0.44917 | 0.19449 | | |
| Q34 | 0.19608 | 0.47520 | | 0.16916 | 0.23108 |
| Q41 | | 0.47090 | | | 0.24380 |
| Q46 | | 0.49479 | | 0.21712 | 0.13693 |
| Q9 | | | 0.62533 | | 0.18742 |
| Q24 | 0.35489 | | 0.57173 | 0.11934 | |
| Q28 | | | 0.55337 | 0.18628 | 0.21346 |
| Q36 | | 0.18910 | 0.43254 | 0.10694 | 0.12039 |
| Q39 | | 0.11680 | 0.44999 | 0.31187 | |
| Q50 | | 0.12138 | 0.46329 | 0.24964 | |
| Q37 | 0.13276 | 0.16618 | 0.15875 | 0.26355 | 0.28043 |
| Q40 | | | | 0.74516 | |
| Q42 | 0.11543 | 0.27478 | | 0.36942 | 0.17574 |
| Q43 | | | 0.12097 | 0.56204 | 0.15064 |
| Q12 | 0.19400 | | | 0.23615 | 0.58509 |
| Q32 | | 0.10896 | 0.17933 | | 0.58619 |
| Q38 | | 0.39075 | 0.21293 | | 0.46327 |

The table just lists factor loadings above 1.

than 1. A variance of 45.8% was extracted. This indicated that there were five significant components underlying the number sense skills. With previously demonstrated reliability and validity, we concluded that the newly-constructed number sense scale, with 5 components, was satisfactory; the test items within each component were conceptually and empirically related to the corresponding construct they were intended to measure.

SUMMARY

The results of reliability and validity analyses indicate that the five-factor number sense model seems to be reasonable. In comparing the difference between the initial model and the present empirical findings, the results shows that the five-factor number sense model has a close correlation with earlier research (Hsu, Yang, & Li, 2001). However, two number sense components: “Recognizing the relative effect of operations on numbers” and “Developing and using benchmarks appropriately” disappeared, but a different factor “Using multiple representations of numbers and operations” appeared after the factor analysis. In reviewing these questions, we believe that the two factors probably disappeared because the children used different strategies to solve the problems. For example, initially Q36 was designed to measure “Developing and using benchmarks appropriately”; we originally thought that students might solve this question using \$500 or \$1000 as a benchmark. After factor analysis, this question was statistically assigned to “Being able to decompose and compose numbers”. In reviewing this item, and the reason children selected it, this was reasonable, as students may solve this problem by “decomposing and composing numbers”.

DISCUSSION

The major contribution of this study is to demonstrate empirical support of the 5-factor number sense model which can be used to assess 3rd graders’ number sense performance and misconceptions. According to this model, main number sense skills are five-dimensional, comprising understanding the meanings of numbers and operations, using multiple representations of numbers and operations, recognizing relative number size, judging the reasonableness of computational results, and being able to compose and decompose numbers. This was consistent with the earlier study of Hsu, Yang, and Li (2001). Four factors emerged in both studies: understanding the meaning of numbers and operations; using multiple

representations of numbers and operations; judging the reasonableness of computational results; and recognizing relative number size. The dominant factor in Hsu, Yang, and Li's study (2001) was "Recognizing relative number size". However, the dominant factor in our study turned out to be "Understanding the meanings of numbers and operations". It is believed that key mathematical knowledge is qualitatively different between these two different grade levels. For example, the dominant factor, "Understanding the meanings of numbers and operations", is more important for students in grades 1 to 3, so that they can establish good basic knowledge; this is presumed to have already taken place for those in grades 5 and 6. Yet, one aspect, that of being able to decompose and compose numbers vs. understanding the relative effect of operations on numbers was different from Hsu, Yang, and Li's study (2001). This may be due to the different mathematical content of the different grades. For example, the math textbooks for grades 1 to 3 devote more space to the decomposing and composing of numbers. However, the textbooks for fifth and sixth graders focus more on the relative numerical effects of operations.

In addition, two benefits of using computers as number sense assessment tools are pinpointed below:

1. A key feature of this test was not only asking students to decide the correct answer for each question, but also asking them to determine the reasons they chose that answer. This is different from traditional multiple-choice tests. Teachers and students can be informed immediately of the test results. They can also examine why they chose each answer. This can give teachers better insight into children's number sense development.
2. Feedback and reinforcement is immediate: teachers and their students, who participated in this test were able to know the results immediately. Students could better understand their number sense weaknesses and strengths.

Bringing technology into full play of number sense, through such integrated testing methods, supports the statement: "assessment should support the learning of important mathematics and furnish useful information to both teachers and students" (NCTM, 2000, p. 22).

However, several possible limitations of the computerized number sense testing are listed below:

1. The unique problems (such as, personal unique way of thinking, different solution methods, and so on) may not be easily detected via computerized testing.

2. We are not sure all of the students working hard during the computerized testing. During the test, we found some students responded to these questions too quickly. They might answer these questions by guessing.
3. The schools involved in the study must have computer rooms and the capacity to hook up to the www website. If the on-line testing system does not work well, the on-line test is required to do once.

REFERENCES

- Burns, M. (1994). Arithmetic: the last holdout. *Phi Delta Kappan*, 75(6), 471-476.
- Berch, D. B. (2005). Making sense of number sense: Implications for children with mathematical disabilities. *Journal of Learning Disabilities*, 38(4), 333-339.
- Case, R. (1989). Fostering the development children's number sense: Reflection on the conference. In J. T. Sowder & B. P. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference*. San Diego, San Diego University, Center for Research in Mathematics and Science Education.
- Case, R. (1998). *A psychological model of number sense*. Paper presented at the annual meeting of the American Educational Research Association, San Diego, CA.
- Cramer, K. A., Post, T. R., & delMas R. C. (2002). Initial fraction learning by forth- and fifth-grade students: A comparison for the effects of using commercial curricula with the effects of using the rational number project curriculum. *Journal for Research in Mathematics Education*, 33(2), 111-144.
- Enzmann, D. (1997). RanEigen: A program to determine the parallel analysis criterion for the number of principal components. *Applied Psychological Measurement*, 21(3), 232.
- Emanuelsson, G., & Johansson, B. (1996). *Kommentarer till kursplan I matematik Lpo 94*.
- Greeno, J. G. (1991). Number sense as situated knowing in a conceptual domain. *Journal for Research in Mathematics*, 22(3), 170-218.
- Hiebert, J., & Grouws, D. A. (2007). The effects of classroom mathematics teaching on students' learning. In F. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 371-404). Greenwich, CT: Information Age Publishing.
- Howden, H. (1989). Teaching number sense. *Arithmetic Teacher*, 36, 6-11.
- Hsu, C. Y., Yang, D. C., & Li, M. N. (2001). The design of the fifth and sixth grade number sense rating scale. *Journal of Science Education (In Chinese)*, 9(4), 351-374.
- Korchia, M. (2001). *SEM Stats*. Retrieved July 2, 2004, from the World Wide Web: http://mkorchia.free.fr/sem/sem_sfats.xls
- Lautenschlager, G. J. (1989). Parallel analysis criteria: revised equations for estimating the latent roots of random data correlation matrices. *Educational and Psychological Measurement*, 49(2), 339-345.
- Markovits, Z. & Sowder, J. T. (1994). Developing number sense: An intervention study in grade 7. *Journal for Research in Mathematics Education*, 25(1), 4-29.
- McIntosh, A., Reys, B. J., & Reys, R. E. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, 12, 2-8.

- McIntosh, A., Reys, B. J., Reys, R. E., Bana, J., & Farrel, B. (1997). *Number Sense in School Mathematics: Student Performance in Four Countries*. MASTEC: Mathematics, Science Technology Education Centre.
- Ministry of Education in Taiwan. (2003). *Nine-year-integrated mathematics curriculum standards for national schools from grade 1 to 9 in Taiwan*. Taiwan, ME.
- National Council of Teachers of Mathematics. (2000). *The Principles and Standards for School Mathematics*. Reston, VA: NCTM.
- Resnick, L.B. (1989). Defining, assessing, and teaching number sense. In J. T. Sowder & B. P. Schappelle(Eds.), *Establishing foundations for research on number sense and related topics: report of a conference*(pp.35-39). San Diego: San Diego University, Center for Research in Mathematics and Science Education.
- Reys, R. E., & Yang, D. C. (1998). Relationship between Computational Performance and number sense among sixth- and eighth-grade students in Taiwan. *Journal for Research in Mathematics Education*, 29, 225-237.
- Sowder, J. T. (1992). Estimation and number sense. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp.371-389). New York: Macmillan.
- Thurstone, L. L.(1947). *Multiple factor analysis*. Chicago, University of Chicago.
- Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 557-628). Charlotte, NC: Information Age Publishing.
- Yang, D.C. (2003). Teaching and learning number sense—an intervention study of fifth grade students in Taiwan. *International Journal of Science and Mathematics Education*, 1(1), 115-134.
- Yang, D. C., Hsu, C. J., & Huang, M. C. (2004). A study of teaching and learning number sense for sixth grade students in Taiwan. *International Journal of Science and Mathematics Education*, 2(3), 407-430 .
- Yang, D. C. & Li, M. N. (2003). The Development of a Computerized Number Sense Scale for 3rd-graders, A number sense project supported by National Science Council, Taiwan.

Author : **Der-Ching Yang**

E-mail : dcyang@mail.ncyu.edu.tw

Address : Graduate Institute of Mathematics Education, National Chiayi University
85, Wen Lung, Ming-Hsiung, Chiayi, 621, Taiwan, R. O. C.

Phone : 011-886-5-2263411 ext 1924

Fax : 011-885-5-2266531