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ASSESSING ALGEBRAIC SOLVING ABILITY OF FORM FOUR STUDENTS

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ABSTRACT. Mathematics researchers generally agree that algebra is a tool for problem solving, a method of expressing relationship, analyzing and representing patterns, and exploring mathematical properties in a variety of problem situations. Thus, several mathematics researchers and educators have focused on investigating the introduction and the development of algebraic solving abilities. However research works on assessing students' algebraic solving ability is sparse in literature. The purpose of this study was to use the SOLO model as a theoretical framework for assessing Form Four students' algebraic solving abilities in using linear equation. The content domains incorporated in this framework were linear pattern (pictorial), direct variations, concepts of function and arithmetic sequence. This study was divided into two phases. In the first phase, students were given a pencil-and-paper test. The test comprised of eight superitems of four items each. Results were analyzed using a Partial Credit model. In the second phase, clinical interviews were conducted to seek the clarification of the students' algebraic solving processes. Results of the study indicated that 62% of the students have less than 50% probability of success at relational level. The majority of the students in this study could be classified into unistructural and multistructural. Generally, most of the students encountered difficulties in generalizing their arithmetic thinking through the use of algebraic symbols. The qualitative data analysis found that the high ability students seemed to be more able to seek the recurring linear pattern and identify the linear relationship between variables. They were able to co-ordinate all the information given in the question to form the algebraic expression and linear equations. Whereas, the low ability students showed an ability more on drawing and counting method. They lacked understanding of algebraic concepts to express the relationship between the variables. The results of this study provided evidence on the significance of SOLO model in assessing algebraic solving ability in the upper secondary school level.

KEYWORDS. Algebraic Solving Ability, Assessment, Linear Equation, Partial Credit Scoring, SOLO, Upper Secondary School Students.

INTRODUCTION

Background

Several mathematics researchers and educators studied the investigation into the introduction and development of algebraic solving abilities which can be viewed from different approaches such as generalization, modeling and functional.

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The important role played by generalization approach in the introduction of algebraic solving abilities cannot be denied. In this approach, algebraic solving ability can be laid when students engage in the investigative processes: i) finding a pattern ii) generalizing a formula by using algebraic symbols, and iii) applying the formula to solve the problem (Fernandez & Anhalt, 2001; Friedlander & Hershkowitz,1997; Herbert & Brown, 1997; Mason, 1996). Ferrucci, Yeap and Carter (2003) have established modeling approach as a fundamental of algebraic solving abilities. Essentially, this approach uses the pictorial representation to analyze the relationship among the quantities in a problem. According to them, modeling approach consists of two phases, the first phase involves the investigation some key relationships between variables in the situation. The second phase comprises a series of mathematical transformations or operations that lead to a model expressed such as symbolic expressions, graphs or tables (Ferrucci, Yeap & Carter, 2003). In a functional approach, the emergence of algebraic solving abilities involves the representation of variables as quantities with changing values and an exploration the graphical and numerical representation that highlight the changes for different function rules (Thornton, 2001).

Obviously, the nature of algebraic solving ability inherits in each approach is sufficient to generate powerful algebraic solving ability in the classroom. Many recommendations have been made to transform algebra from a skill-drilling sequence of practice into a meaningful topic that can be approached through these different approaches. According to Bishop, Otto and Lubinski (2001), Carey (1992), Herbert and Brown (1997), the application of these approaches to introduce algebra provide concrete model and concrete experience that enable students to experience algebra in the real world. In this manner, students will be able to construct a better understanding of the algebra concept and connect the concrete experience with the abstract symbolic algebra.

However, the question of how to assess algebraic solving ability through these approaches may still be problematic for many teachers. Thus, in this study, SOLO model, known as the Structure of the Observed Learning Outcome, developed by Biggs and Collis (1982) was used to assess students' algebraic solving ability. It is a cognitive psychology model which emphasizes more on the internal process and more interested in investigating how a problem is handled by students rather than whether their answers are correct. It had provided the basic theoretical underpinning for developing the technique for assessing students' cognitive attainment. In the area of algebra, SOLO model have been used to describe students' elementary equation solving (Biggs & Collis, 1982) and made the comparison with various learning theories in describing development of algebraic ideas (Pegg, 2001) but there is no coherent description of students' algebraic solving ability sufficient to inform instruction decision. Thus, in this study, we claimed that the proposed framework enable upper secondary school students' algebraic solving ability to be described across four levels of SOLO model.

SOLO model was used to construct items which reflect the four levels of SOLO model: unistructural, multistructural, relational and extended abstract. The levels are in hierarchical manner which are increasingly complex. Students' algebraic solving ability were assessed through their performance in using linear equation to solve the problem situations across the four content domains, these include linear pattern (pictorial), direct variation, concept of function and arithmetic sequence. Based on Malaysian Integrated Curriculum for Secondary Schools syllabus, linear equation is a prerequisite to the learning of more complex topics at the upper secondary school such as straight line; gradient and area under graph; index and logarithm; matrix; variation; graph of function and quadratic equation (Teng, 2002). Therefore, linear equation became the focus of this study.

Statement Of The Problem

Clements (1999), Stacey and Macgregor (1999a), and Murphy (1999) stated that at the present time, the assessment in algebra is still focusing on getting a correct answer, symbol manipulations, rote skill and little or no application of algebraic concepts in the problem situation. Teng (2002), Tall and Razali (1993) have noted that symbol manipulation and procedural skill practice in algebra class among the secondary school students might serve to prolong the interpretation that algebra is a 'menagerie' of disconnected rules to deal with different contexts. It, therefore exhibits the poor understanding of the basic concept and cognitive obstacles among the students as this practice to algebra relies almost exclusively on written symbolic forms as the tool to make representation, generalization and interpretation to the applied problem. In this regard, it is impossible to inculcate students with algebraic solving ability if the assessment procedure is not changed. As Stacey and MacGregor (1999a) viewed that although the students have apparently learned algebra, in reality they find algebra difficult and do not know how to apply it. They still can't see the way to use what they learnt about algebra and it is still seen separately. Therefore, more complex algebraic problem items that demonstrating the efficiency and power of algebraic solving ability should be constructed and frequently used in examination and classroom practice.

Purpose Of The Study

This study aimed to assess the Form Four students' levels of algebraic solving ability in using linear equation. In order to capture the manifold nature of algebraic solving ability in using linear equation to solve the problem situations, the framework of this study incorporated four content domains of linear equation. Further, this study sought to the breath and depth of students' algebraic solving processes in using linear equation through interview method.

Research Questions

In this study, two research questions addressed are the following:

a. What are Form Four students' levels of algebraic solving ability (according to SOLO model) with regard to the use of linear equation to solve a series of tasks across the four content domains (linear pattern, direct variation, concept of function and arithmetic sequence)?

b. How do Form Four students solve the four levels items (according to SOLO model) which are constructed in assessing students' algebraic solving ability in the process of:

i. investigating the pattern?

ii. representing and generalizing the pattern?

iii. applying the rule to the related situation?

iv. generating an alternative solution for the new situation?

Significance of The Study

The results of this study might provide evidence on the significance of SOLO model in assessing algebraic solving ability in the upper secondary level. It provided a guideline for teachers who want to identify the level and the process of algebraic solving ability among their students in using linear equation across the four content domains.

Subsequently, the instrument of the study might also be used as a diagnostic assessment tool to evaluate the strengths and weaknesses of the students' conceptual understanding about linear equation.

The results of this study also provided an evidence whether SOLO model can be used as an alternative method of assessment in the upper secondary level. Thus, the findings might provide useful information to the assessment developer particularly in the subject of mathematics.

Perhaps and even more significant is that, there were very few researches conducted concerning the assessment of algebra. So far, there had been four studies concerning the learning of algebra in Malaysia (Cheah & Malone, 1996; Heng and Norbisham, 2002; Ong, 2000; Teng, 2002). They investigated and identified the conceptual understanding problems in learning and teaching of algebra. These studies however did not attempt to develop a model to assess students' algebraic solving ability in the problem situations. This lack of research suggested that an assessment of algebraic solving ability in mathematics classroom would be an added value to the research on algebra.

Theoretical Framework

The SOLO taxonomy is designed mainly as a mean to assess students' cognitive ability in school learning context (Biggs & Collis, 1982; Collis & Romberg, 1991; Chick, 1988; Collis, Romberg & Jurdak, 1986; Reading, 1998; Vallecillos & Moreno, 2002; Watson, Chick & Collis, 1988). It had been used to analyze the structure response of student's problem solving ability, mathematical thinking ability and understanding of mathematical concepts over a wide educational span from primary to tertiary levels (e.g Chick, 1988; Collis, Romberg & Jurdak, 1986; Lam & Foong, 1998; Reading, 1999; Vallecillos & Moreno, 2002; Watson, Chick & Collis, 1988; Wilson & Iventosch, 1988). SOLO provides a framework to classify the quality of response which can be inferred from the structure of the answer to a stimulus. According to SOLO model, coding a student's response depends on two features. The first is a series of five modes of cognitive development and the second is a series of levels of response.

In SOLO model, mode is closely related to the existing notion of Piaget's stage of cognitive development which proposes a number of developmental stages demonstrating increasing abstraction form sensori-motor (infancy), ikonic (early childhood of preschool), concrete-symbolic (childhood to adolescene), formal (early adulthood) through to postformal (adulthood) (Biggs & Collis, 1982).

Although the sequence of five modes followed from simple to complex, it is common knowledge that students do not always operate at the same level as their developmental age suggests they should, nor do they perform consistently (Biggs & Collis, 1982; Biggs & Teller, 1987; Collis & Romberg, 1991; Romberg, Zarinnia & Collis,1990). For example, a formal mode response in mathematics given by a student might be followed by a series of concrete-symbolic mode response in mathematics. Further, concrete-symbolic mode response in mathematics given by a student on this week might be followed by formal mode response on the next week. Was that particular student at the formal mode or concrete-symbolic mode? According to SOLO model , this kind of difficulty can be solved by shifting the label from the student to his response to a particular task (Biggs & Collis, 1982). In other words, SOLO model improves this phenomenon by describing the complexity of the structure response to a particular task within a mode.

According to SOLO model, the structure responses in solving algebra problems can be classified into four levels which include unistructural, multistructural, relational and extended abstract. Researcher hypothesized that Form Four students could exhibit four levels of algebraic solving ability. The theoretical framework had been developed along with the expected students' algebraic solving ability across the four levels for each of the four content domains. Table 1 showed the framework on the characteristic of students' algebraic solving ability incorporate four content domains of linear equation and Figure 1 represented the theoretical framework of the study.

Figure 1 represented the theoretical framework of this study. Based on Friedlander and hershkowitz's (1997) and Swafford and Langrall's (2000) views, the ability of using equation to solve and represent the problem situation involves a number of algebraic processes which consist of three phases: i) investigating the pattern by collecting the numerical data; ii) representing and generalizing the pattern into table and equation; iii) interpreting and applying the equation to the related or new situation. This study suggested that the algebraic solving ability can be assessed based on these three phases. Eight linear equation problem situations in the form of superitem were to be assigned to the students allowing them to display such abilities. The problem situations represented the following four content domains of linear equation: linear pattern (pictorial), direct variation, concept of function and arithmetic sequence. There were four items in each superitem which represent four level of SOLO model: uinstructural, multistructural, relational and extended abstract. The correct response of one of the item would indicate an ability to respond to the problem at least at the level reflected in the SOLO structure of that item.

Figure 1. *Theoretical framework of the study*

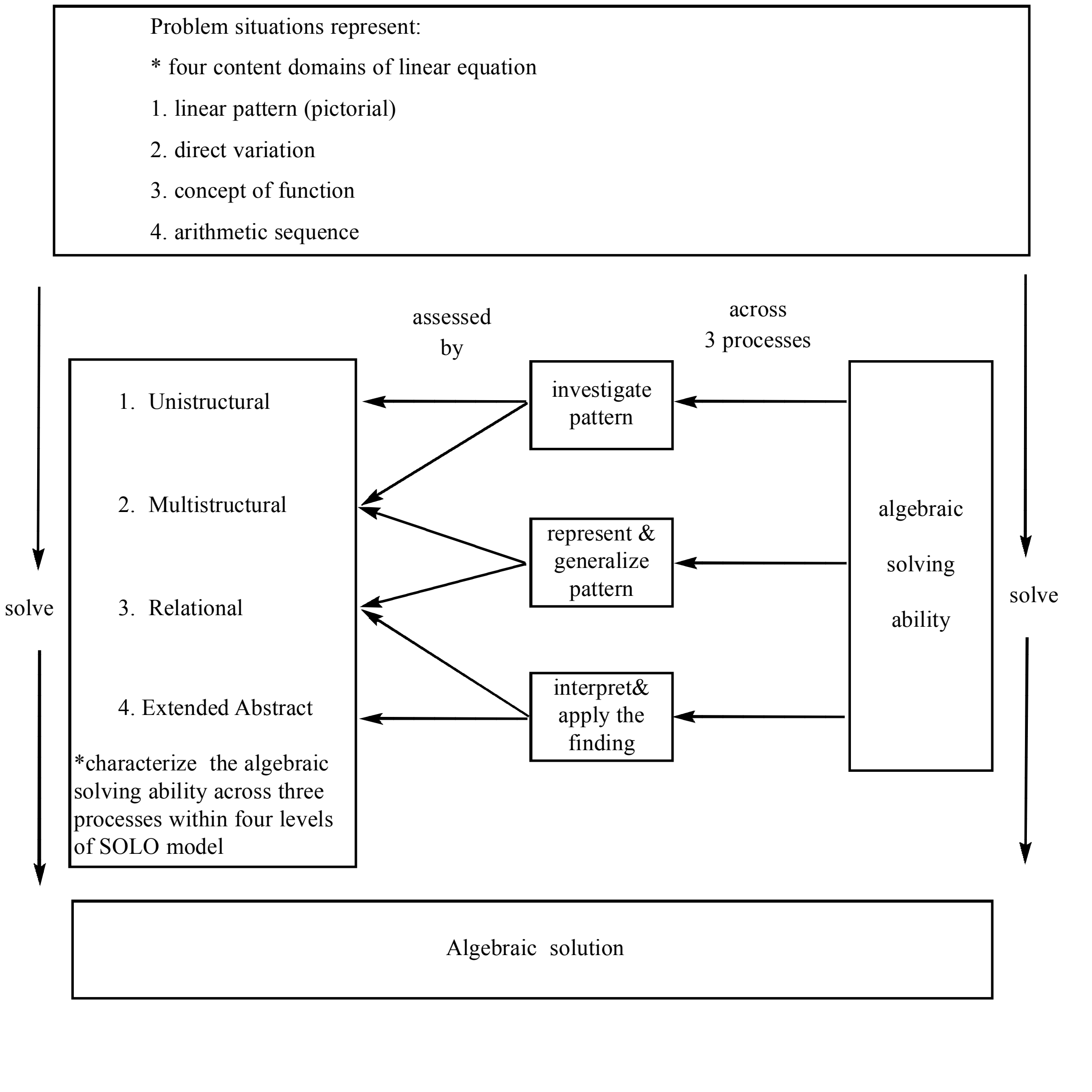


Table 1. Algebraic Solving Ability Framework

	Unistructural	Multistructural	Relational	Extended abstract
Linear pattern (pictorial)	• investigate the pictorial pattern and extend the next term of it by referring directly the information and diagram given in the stem.	• ability to see the given pattern as successive process. That is, identify the recursive relationship between the terms in the sequence. • ability to understand and use the given information serially to compute some specific cases and represent the data in table.	• generalize the linear relationship of the pattern symbolically based on all information given .• all the information given need to integrate to generate an algebraic expression and a rule for the pictorial pattern. • apply the rule to solve the related situation.	• ability to analyze the linear pattern across a wider range of cases. That is, use of linear relationship, shape and perimeter concept to form a rule for the new linear pictorial pattern. • in forming the new rule for the new pattern, attempt to make conjecture and verify the conjecture deductively.
Direct variation	• investigate the form of direct variation by identifying the constant. • use only one or relevant aspect of the information to find the answer.	• ability to use more than one numerical operation (such as addition operation and multiplication operation) to give the response.	• make generalization by using algebraic expression and linear equation. • all the necessary information given is integrated to make the such generalization. • application of the formula to solve the related problem.	• ability to extract the abstract concept from the information given and apply the related concept (percentage) into a more abstract situation.
Concept of function	• ability to use one aspect of the available information (first stage of the process input-output which involved only one operation) to find the output.	• notice the numerical relationship of variables, forming the arithmetic expression to compute the values of dependent variable and represent it in table. • all the information given is used as sequence to find the values of output that involved more than one operations.	• ability to generalize the input-output process in terms of a linear relationship between the variables. • ability to inter-relate all the available information to form an algebraic expression and linear equation to represent the situation. • working backward which requires the application of the formula.	• inter-relate all the available information and test it against appropriate abstract general principle. (functional relationship between the variables) in order to generate alternative solution. •.ability to examine the structure and consider the possibility of more than one answer to the problem situation.
arithmetic sequence	• investigate the pattern that comes next to it. It requires the understanding of the sequence of number pattern by referring directly the information given in the stem.	• identify the numerical relationship of the dependent and independent variables by computing some specific cases and represent the data in table.• the information given in the stem is still used as sequence.	• make a generalization by integrating all the information given in order to formulate an algebraic expression and formula.• involve working backward which requires the application of the rule.	• ability to use an hypothesis or an abstract general principle (concept of arithmetic sequence) to form a possible alternative solution. • ability to use of logical skill to account for the possibility. For instance, work with variables to conceive the possibility in a range of number.

METHODOLOGY

This study used quantitative and qualitative approach to assess the students' algebraic solving ability based on SOLO model. The rational of researcher chose the quantitative method was to assess the students' levels of algebraic solving ability. The dataset was submitted to partial credit analysis. Subsequently, the qualitative method was used to seek the clarification of the students' algebraic solving processes. Thus, this study was divided into two phases. In the first phase, a test was given to the students. There were eight questions which were designed according to the SOLO superitem format. Each superitem consisted of a situation or story (the stem) and four items related to it. The items represented four levels of reasoning defined by SOLO (unistructural, multistructural, relational, extended abstract). In the second phase, clinical interview was conducted for the qualitative method. The clinical interview session was carried out after the pencil-and-paper test.

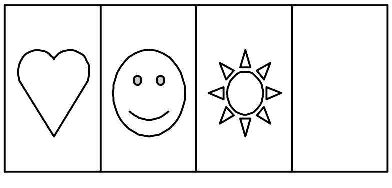
Participants

In Malaysia, basic topic of algebra is taught during lower secondary school and the topic of linear equation is taught in Form Two and Form Three (grade 8 and grade 9). Thus, the construction of an assessment to assess algebraic solving ability amongst Form Four students was important as the teachers could gain a greater awareness of students' algebraic solving abilities about this topic before they learnt the more complex topics which were required the basic algebra knowledge. The participants of this study consisted of 40 Form Four students from a secondary school. Six out of the 40 samples were selected for the clinical interview (two subjects from unistructural level, relational level and extended abstract level respectively).

Instrumentation

In this study, the instrument of data collection consisted of eight superitems. All of the superitems were open-ended questions. Two superitems were constructed for each content domain to be assessed. The following is an example of a superitem (superitem 1: linear pattern pictorial) designed for this study.

Pictures are hung in a line. The pictures that are hung next to each other share two tacks as shown below.



Level 1: Unistructural

How many tacks are needed to hang four pictures by this way?

Answer: 10 tacks

Note: The item requires the response based on referring the concrete information (given terms in the diagram) to find the next term for the given sequence. This item can be answered most simply by drawing and counting the number of tacks in the diagram to come up with the answer of 10.

Level 2: Multistructural

How many tacks are needed to hang 10 pictures, 16 pictures, and 20 pictures? Represent your answers in a table.

Answers:

the number of pictures	**the number of tacks**
10	22
16	34
20	42

Note: This item requires the given information is handled serially. That is, identify the recursive relationship between the terms in the sequence in order to compute the specific cases and represent them in table.

Level 3: Relational

i) If you have y pictures, how many tacks are needed?

ii) Write a linear equation for finding a number of tacks for any number of pictures. Let ***t*** represent the number of tacks and ***p*** represent the number of pictures.

iii) How many pictures can be hung if the number of tacks is 92? Try to apply linear equation to solve it.

Answers: i) $2 + 2y$

ii) $t = 2 + 2p$

iii) $92 = 2 + 2p$

$90 = 2p$

$45 = p$

(There are 45 pictures)

Note: This item requires the response that integrates all the information to make generalization for the pattern. In order to response, the student must identify not only the two tacks per picture but also the need for two more to hang the last picture in the series (e.g. $t = 2 + 2p$). If the student provided this response, it would demonstrate his/her algebraic solving ability in identifying the linear relationship between variables and applying algebraic symbols to make the representation. Besides, the student may involve working backward which requires the application of rule.

Level 4: Extended Abstract

'I don't have enough tacks to hang the such a lot of pictures by this way!', Said Lisa. Try to create a new linear equation which represent the number of tacks (t) for any number of pictures(p) to help Lisa.

Answer: $t = p + 1$ or

$t = p$ or

$t = 2p$

Note: This level represents the highest level of algebraic solving ability. The response shows an ability to extend the application of the given information in the new situation (new pattern) and recognize an alternative approach which is distinguishable by the abstract features (linear relationship).

The clinical interview session was carried out after the pencil-and-paper test had been conducted. Clinical interview which is also known as 'flexible interviewing' is available for the purpose of assessing any kind of mathematical solving ability. It is flexible, responsive and open-ended in nature. The samples who were involved in the clinical interview session were selected based on their performance in written assessment. Before the interview started, the samples were shown their test papers to enable them to recall the methods and procedures they used to solve the items. The interview questions were structured based on the items in the test.

Data Analysis

The data analysis had been done based on the findings from pencil-and-paper test and interview. The data analysis procedures were categorized into two levels.

Level 1: The test paper results were analyzed by using Partial Credit Model. Partial Credit Model (Wright & Masters, 1982) is a statistical model specifically incorporates the possibility of having different number of steps or levels for the items in a test (Bond & Fox, 2001). For example, the ordered values 0, 1, and 2 might be applied to an item which has three ordered performance category levels as follow: 0 = totally wrong, 1 = partially correct and 2 = completely correct. In this study, the ordered values 0, 1, 2, 3, 4, 5 and 6 might be applied to the superitem as follow: 0 = totally wrong or no response, 1 = unistructural level, 2 = multistructural level, 3 = lower relational level, 4 = relational level, 5 = higher relational level and 6 = extended abstract level. Codes 0, 1, 2, 3, 4, 5, 6 covered all the response possibilities in the test. Code 0 as the code for the lowest possible response level and 6 as the code for the highest possible response level in each superitem.

WINSTEP software program was used to run the analysis. It computed probability of each response pattern in which took into account the ability of the learner and the difficulty of the questions. The purpose for this computer analyze was also estimating the value of validity, the reliability index, difficulty of the items and the levels achieved by the students based on the different content domain items.

Level 2: The information from the clinical interview session was transcribed into writing form. Six students were selected to be interviewed. Each interview was audiotaped and lasted between 30 minutes to an hour.

RESULTS

Quantitative Results

In the Partial Credit model, reliability is estimated both for person and for items. The item reliability index indicates the replicability of item placements along the pathway if these same items are given to another sample with comparable ability levels. Person reliability index indicates the replicability of person ordering that could expect if this sample are given another set of items measuring the same construct. In this analysis, the item reliability index and person reliability index were 0.91 and 0.73. The values fall within the acceptable range.

Valid is referred to the reliability index and the success of this evaluation to fit. If the fit statistics (infit and outfit) of an item is acceptable, the expected value of the mean square (variation in the observed data) is shown between 0.7 and 1.3. In the analysis, the infit and outfit mean square for each superitem fall within the acceptable range. Besides, the means for all the infit mean square and outfit mean square were considered reasonably well: 1.06 (mean for infit mean square) and 0.98 (mean for outfit mean square). (see Table 2)

Table 2. Partial Credit Analysis Of SOLO Item Dataset

Superitem	Infit MNSQ	Outfit MNSQ
1	0.99	0.94
2	1.31	1.30
3	0.75	0.83
4	0.84	0.88
5	1.16	1.15
6	1.14	1.10
7	0.88	0.82
8	0.99	0.87
Mean	1.06	0.98

Analysis of Levels and Difficulty of Superitem

Figure 2 showed that the number of students who have 50% and above probability of success at six levels. From the findings, the difference of the number of students who have 50% and above probability of success between the higher relational level and the highest level (extended abstract) were only 4, compared with the difference of the number of students who have 50% and above probability of success between the unistructural level and multistructural level, there were 10. Besides, the difference of the number of students who have 50% and above probability of success between the multstructural and lower relational level were 8. These findings indicated that 62% of the students have less than 50% probability of success at relational level. Majority of the students in this study could be classified into lower level of algebraic solving ability especially unistructural level and multistructural level. Generally, most of them were only able to numerically solve a variety of problems involving specific cases. They encountered difficulties in generalizing the arithmetic through the use of algebraic symbols.

Figure 3 depicted that the number of students who have the 50% and above probability of success at each superitem. Findings from this study showed that superitem 3 and 4 (direct variation) were the easiest to respond. There were 34 and 29 students who have 50% and above chance to respond successfully to superitem 3 and 4 respectively. However, there was some range in the difficulty of the same content domain of superitems, especially superitem 1 and 2 (linear pattern pictorial).

Qualitative Results

Six subjects were interviewed. They were selected from three different levels of algebraic solving ability: 1) Subjects from Unistructural Level (**Sul1** and **Sul2**), 2) Subjects from Relational Level (**Srl1** and **Srl2**) 3) Subjects from Extended Abstract (**Seal1** and **Seal2**).

1.Sul1 and Sul2: Samples who were only able to answer most of the unistructural level of items successfully. The item responses include representation of one specific value that comes next for the pattern.

2. Srl1 and Srl2: Samples who were able to answer most of the relational level items successfully. The item responses include making connection between the information given to form an algebraic expression and linear equation.

3. Seal1 and Seal2: Samples who were able to answer most of the extended abstract level items successfully. The item responses include extracting the abstract general principle from the information given to form an alternative rule for the new situation.

The four processes of algebraic solving ability in solving superitem 1 (linear pattern pictorial) were investigated as below :

1a) Investigating the numerical pattern that comes next to it

Six subjects were able to solve and respond correctly by seeking out the sequence of pattern which comes next to it or extend the sequence by referring directly to the diagram and information given. The following dialogues depicted how the three subjects responded to item 1a: (In the following dialogues, R denotes researcher)

R **:** Please tell me how did you get the answer for item1a?

Sul1 **:** (counting the number of tacks that he drew in the diagram given). I draw to get the answer for the number of tacks.

R **:** How did you get 10 tacks for item 1a?

Srl1 **:** 1, 2, 3…10. (pointing to the diagram given)

R **:** How did you solve the item 1a?

Seal2 **:** (pointing to the diagram). I count from here. 1, 2, 3…10.

Figure 2. Difficulty of SOLO levels and ability levels

Level		Person
	0.65	x
	0.60	
	0.55	
	0.50	
	0.45	
	0.40	
	0.35	
	0.30	
	0.25	x
	0.20	
EA	0.15	
	0.10	xxx
HR	0.05	x
	0	x
	-0.05	x
	-0.10	x
R	-0.15	xx
	-0.20	xxx
LR	-0.25	x
	-0.30	xxx
	-0.35	
	-0.40	xxx
M	-0.45	xx
	-0.50	xxxxxx
	-0.55	xxxx
U	-0.60	
	-0.65	x
	-0.70	
	-0.75	
	-0.80	
	-0.85	xx
	-0.90	x
	-0.95	x
	-1.00	x
	-1.05	
	-1.10	
	-1.15	
	-1.20	x
easy	logits scale	least able

U-unistrucrtural HR - higher relational
M-multisturctural EA - extended abstract
LR- lower relational
R-relational

Figure 3. Difficulty of superitems and ability levels

Level		Person
	0.65	x
	0.60	
	0.55	
	0.50	
	0.45	
	0.40	
	0.35	
8	0.30	
	0.25	x
	0.20	
	0.15	
7	0.10	xxx
	0.05	x
	0	x
	-0.05	x
2	-0.10	x
	-0.15	xx
	-0.20	xxx
	-0.25	x
6	-0.30	xxx
	-0.35	
5 & 1	-0.40	xxx
	-0.45	xx
4	-0.50	xxxxxx
	-0.55	xxxx
	-0.60	
	-0.65	x
3	-0.70	
	-0.75	
	-0.80	
	-0.85	xx
	-0.90	x
	-0.95	x
	-1.00	x
	-1.05	
	-1.10	
	-1.15	
	-1.20	x
easy	logits scale	least able

1b) Investigate the pattern by computing specific cases

When subjects were confronted with various number tasks as a way to assess and refine their understanding of the pattern, subjects began to notice the pattern and understand the linear relationship involving an arithmetic operation. Subject Srl1, Srl2, Seal1 and Seal2 did not use the manipulative to get the solution instead of substituted the specific values into the arithmetic expression. These were shown in the extracts below:

R **:** How did you solve the item 1b?

Srl1 **:** (Recalling) I multiply the number of pictures by two then I add two to it.

R **:** Can you try to explain the method you used?

Srl1 **:** There are at least two tacks needed for each pictures. So, I multiply the number of pictures by 2. There are two tacks in the end. So, I add two to it. (see Figure 4)

R **:** How did you find the number of tacks for the different number of pictures (item 1b)?

Seal2 **:** (Pointing to the solution to find the number of tacks for 10 pictures). 2(10) + 2 = 20 + 2 = 22. '2' means two tacks, each pictures at least has two tacks, '10' means the number of pictures and ' + 2' means two tacks in the end. The same method I use to get the answer for 16 and 20 pictures.

R **:** Can you explain how you solve the item 16?

Seal1 **:** (16-1)2 + 4 = 34. (16-1) means I subtract the first pictures. The first pictures has 4 tacks so (+4). The others pictures have 2 tacks each, so (15 x 2).

Sul1 and **Sul2** were unable to notice the linear pattern in the diagram. They solved the items by counting method. They drew the pictures to count the number of tacks. They noticed the pattern by working with manipulative to get the answers. They relied on the drawing and counting. Their explanation were shown in the following dialogue:

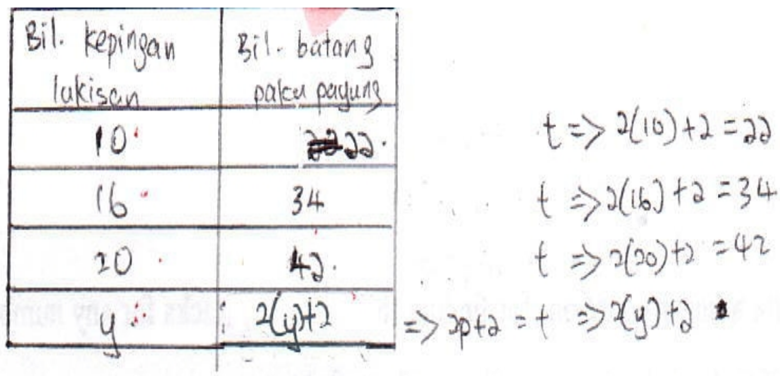
R **:** How about item 1b? How did you solve it?

Sul1 **:** I draw the number of pictures that the question asked and count the number of tacks are needed.

R **:** Can you show me how you get the answer for 10 pictures?

Sul2 **:** 1,2,….22 (counting from the pictures).

Figure 4. SRL1's arithmetic expression and algebraic expression



Bil. kepingan lukisan	Bil. batang paku payung
10	22
16	34
20	42
y	2(y)+2

t => 2(10)+2 = 22
t => 2(16)+2 = 34
t => 2(20)+2 = 42
=> 2p+2 = t => 2(y)+2

2a) Representing the data into table

Subjects represented their data in a table and classified it into categories which reflected the different ways of solving ability about the pattern. Subject Srl1, Srl2, Seal1 and Seal2 who were able to comprehend the pattern. They used the arithmetic rule to compute the table. Whereas subject Sul1 and Sul2 who failed to comprehend the pattern and the linear relationship between the variables, made the computational errors along the way he represented the findings in the table.

2b) Representing the unknown value with the use of letter

Subject Srl1, Srl2, Seal1 and Seal2 were able to gather the information and findings at the concrete level into more abstract level. In other words, they were able to transfer the meaning from arithmetic expression to an abstract conjecture. They linked their interpretation and mathematical findings to connect the counting action with an accurate symbolic representation in the form of algebraic expression. The following dialogues shown Srl1, Seal1 and Seal2 explained how the algebraic expression was formulated:

R **:** How many tacks are needed for y pictures (item 1b)? Try to explain your answer.
Srl1 **:** 2(y) + 2 (referring to the table). I substitute the figure with y.
R **:** What is y?
Srl1 **:** y is an unknown… can be a certain number.

R **:** For y pictures (item 1b), how many tacks are needed?
Seal2 **:** 2y add to 2. (2y + 2).
R **:** How did you form this expression? Which part of information did you refer?
Seal2 **:** Based on the data in the table (pointing to the table). I substitute the figure with y, an unknown. So, 2(y) + 2.
R **:** What is unknown?
Seal2 **:** (Thinking) …Eh… represent a quantity, a number but don't know how many.

R **:** For y pictures, how many tacks are needed?
Seal1 **:** (y-1)2 + 4. I refer this (pointing to arithmetic expression)

The problem about lack understanding of algebraic concepts such as unknown and misconception that 'letter has a unique value' as opposed to an ability to make transition from arithmetic method to the application of algebraic symbols . For example, Sul1 and Sul2 showed that they were unable to use algebraic concept such as algebra expression to express or describe the numerical relationship that existed in the pattern into the abstract situation. The problem about lack understanding of unknown and misconception can be seen in the extract below:

R **:** For item 1b, if y pictures, how many tacks are needed? How did you get 52?
Sul1 **:** I simply write the answers.
R **:** What does it mean ' y pictures'?
Sul1 **:** I don't understand actually.

R **:** If y pictures, how many tacks are needed?
Sul2 **:** If y is 8, the number of tacks is 18 (counting the number of tacks).

2c) Writing linear equation to make generalization of pattern

Srl1, Srl2, Seal1 and Seal2 were able to generate the linear equation for the problem situation based on the information given and the data from their table. Srl1 and Seal2's explanation were shown in the following extracts:

R **:** Try to explain your equation for item 1c(i)? How did you get it?
Srl1 **:** t = p x 2 + 2, t = 2p + 2. I got it from the findings in question b (pointing to the table).

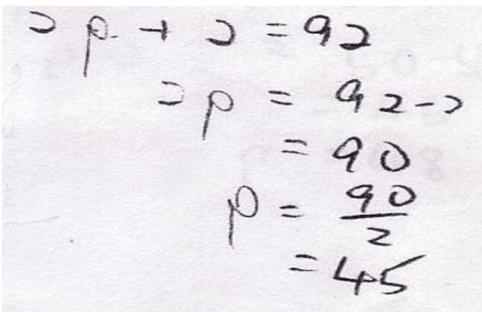
R **:** Ok, for item 1c(i), try to write a linear equation to represent the situation?
Seal2 **:** p(2) + 2 = t, p multiply by 2 and then add to 2 equal to the number of tacks (t).
R **:** How did you form this equation?
Seal2 **:** I got it based on the answer of item 1a and 1b.

3) Application of the rule to solve the related problem

The equation was applied to represent the relationship between dependent variable and independent variable of the problem. Srl1, Srl2, Seal1 and Seal2 were able to analyze the problem into the rule that they formed. In the following extract, Seal2 tried to explain the application of linear equation:

R **:** If you have 92 tacks, how many pictures can be hung (item 1cii: linear pattern)? How did you solve the problem?
Seal2 **:** I use equation to solve it.
R **:** How did you solve it?
Seal2 **:** 92 = 2p + 2. 2p =90, so p = 45. I find p to get the number of pictures.
R **:** Any method that you can use?
Seal2 **:** I think this is the fastest. (see Figure 5)

Figure 5. SEAL2's use of linear equation to solve problem



However, Srl1 was unable to apply the rule due to the misunderstanding of the use of reverse operation between multiplication and division. This weakness had obstructed to the progress of mapping the steps appropriately to the solution.

4) Making generalization for the new pattern or new situation by forming the alternative solution

Seal2 was able to consistently generalize the new pattern or new situation by forming the alternative solution for each problem situation, with the exception of the concept of function problem situation. Seal1 was able to solve most of the items in this level with the exception of the direct variation problem situation. In solving the item 1d, Seal1 and Seal2 were able to extract the abstract concept (linear pattern) from the information given to form the linear equation to generalize and represent the new situation that they created. For example, Seal2 apply the concept of linear relationship to assess the causal and effect relationship between the number of pictures and the number of tacks that he created. Then, he proved the decision by designing a new rule to the new situation. The extract below demonstrated how Seal2 provided an alternative solution to another problem situation:

R : How did you solve the item 1d ?

Seal2 : From item 1c, the equation is 2p + 2 = t. In order to reduce the number of tacks in each picture, I divide 2p + 2 by 2, try to reduce half of the quantity of tacks.

R : How did you prove your equation?

Seal2 : For example, if there are two pictures $t = \frac{2p+2}{2}$

$$t = p + 1$$
$$t = 2 + 1$$
$$t = 3$$

There are only 3 tacks needed compared with previous equation in which 6 tacks are needed.

Six subjects gave the detailed responses in explaining their solving abilities within the four processes. They added more to their initial responses when probed and prompted in an interview situation but in many instances, their responses still at the same level rather than increasing the level of the responses.

DISCUSSION AND CONCLUSION

Six levels of sophistication in algebraic solving ability can be found in learner's response to the tasks: unistructural, multistructural, lower relational, relational, higher relational and extended abstract. The quantitative data analysis demonstrated that majority of students (62%) achieved algebraic solving ability at unistructural level and multistructural level. Most were able to numerically solve a variety of problem involving specific cases. They encountered difficulties in making generalization through the use of linear equation. This finding are consistent with previous research findings (Swafford & Langrall, 2000; Orton & Orton, 1994) that majority of the middle grade students were able to solve the problems involving specific cases, explain the sequence of pattern only in terms of difference between successive terms and very few of them able to generalize the problem into algebraic form. In the qualitative data analysis, researcher found that the high ability students seemed to be more able to seek out the recurring linear pattern and identify the linear relationship between the variables. They were able to co-ordinate all the information given to generalize the pattern algebraically, (forming an algebraic expression and linear equation), ability to use the linear pattern concept in more abstract situation such as forming a rule for the new linear pattern that they created. Also, they used their methods more consistently to find the solutions. Whereas the low ability students showed an ability more on counting method in where the task given is performed and understood serially. They failed to inter-relate the features of linear pattern given in the question due to the lack understanding of algebraic concepts especially unknown and linear equation.

However, this study did not reveal the cognitive obstacles encountered in using linear equation to solve the problem situation. Further research might address these issues so that the framework will be more effective for supporting instructional programs that build on students' prior knowledge, foster their solving ability and monitor their understanding. Further studies are also needed to investigate whether the framework is appropriate for students in other grade levels to determine the extent to which it can actually be used to inform instructional and assessment programs in secondary school algebra.

As noted above, the framework not only suggest an item writing with the format of superitem, it also can be used to score the item, can allow for crediting partial knowledge. Thus, it provides teachers an indication of some of the levels of algebraic solving ability they can expect to encounter in their classroom. Notwithstanding, this framework has the potential to contribute to both instruction and assessment. In the instructional perspective, it would seem prudent for teachers to use solving ability level descriptors as broad guidelines for organizing instruction and building problem task. From an assessment perspective, it appears to be valuable in providing teacher with useful background on students' initial solving ability, and in enabling them to monitor general growth in algebraic solving ability.

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