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Analyzing experimental and theoretical probabilities in Japanese 7th and 8th grade textbooks

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ABSTRACT

Probability is a difficult concept, which is not always taught accurately. This study aims to clarify how experimental and theoretical probabilities are taught in Japanese 7^{th} and 8^{th} grades through a textbook analysis. We analyzed seven, government approved Japanese 7^{th} and 8^{th} grade textbooks each. Focusing on the definition and explanation of experimental and theoretical probabilities and the law of large numbers, we identified eight discrete perspectives. Findings revealed that, first, some textbooks aim to teach students to distinguish between experimental and theoretical probabilities, whereas others aim to teach students to be aware of the connection between the two. Second, after explaining theoretical probability, some textbooks asked questions to clarify the scope of theoretical probability, aiming to teach students to distinguish between the two probabilities. Third, in explaining the law of large numbers, textbooks do not adopt the perspective that experimental probability converges on true probability, but that it converges on theoretical probability.

Keywords: experimental and theoretical probability, true probability, law of large numbers, teaching probability in Japan, 7^{th} and 8^{th} grade, textbook analysis

INTRODUCTION

As illustrated by the spread of COVID-19, modern society is uncertain. The importance of studying probability is increasing so that people can quantify uncertainty and make decisions in all situations, including in everyday society and professional situations where chance is present (Batanero & Chernoff, 2018; Batanero et al., 2016). However, Konold et al. (2011) and Stohl (2005) reported that the experimental and theoretical probability pedagogy is not always accurate in school mathematics.

In Japan, experimental probability is taught in the 7^{th} grade (1^{st} grade in lower secondary school in Japan, from 12 to 13 years old) and theoretical probability in the 8^{th} grade (2^{nd} grade in lower secondary school in Japan, from 13 to 14 years old) (Ministry of Education Culture, Sports, Science and Technology [MEXT], 2018a). In Japan, a new course of study has been in effect since April 2021. It has been determined as the broad standard for all schools, from kindergarten through upper secondary, for organizing their programs ensuring a fixed standard of education throughout the country. However, whether the course is being followed accurately. Considering that understanding experimental and theoretical probabilities is difficult (e.g., Kazak et al., 2011; Prodromou, 2012) and that effective intervention by teachers is essential for students to understand the relationship between them (Ireland & Watson, 2009), it is important to clarify how experimental and theoretical probabilities are taught in Japan.

Thus, this study aims to clarify how experimental and theoretical probabilities are taught in Japanese 7^{th} and 8^{th} grades using a textbook analysis approach as textbooks play an intermediate role between the intended and the implemented curricula (Valverde et al., 2002). If the Japanese course of study, which is the intended curriculum, were analyzed, it would not fully capture the real-world practices of teaching probability because the level of abstraction is too high. Alternately, if we analyze a class, which represents the implemented curriculum, it will not fully capture the actual teaching practices of probability because they are highly dependent on the competence of individual teachers and students. Therefore, textbooks, which are a part of both intended and implemented curricula, were used for analysis.

LITERATURE REVIEW

Borovcnik and Kapadia (2014) state that classical a priori theory, frequentist theory, and subjectivist theory are central to the nature of probability, and thus, are relevant for school mathematics. In the following literature review, first, we provide an overview of probabilities of frequentist theory (experimental probability[1]) and classical a priori theory (theoretical probability[2]), which are taught in Japan. Next, we provide an overview of true probability which is estimated through experimental and theoretical probabilities, followed by an overview of the relationship between the three probabilities.

Experimental Probability

The frequentist theory defines experimental probability. According to Gillies (2000), in frequentist theory, probability is associated with events or a set of elements, is objective, and is independent of the individual who evaluates the probability, just as the mass of a thing is independent of the measurer in mechanics.

In the Japanese course of study, the aims and goals of probability in the first grade of lower secondary school (7th grade) are set, as follows:

> To teach students to acquire the following matters regarding the possibility of uncertain events through mathematical activities.
>
> a. To acquire the following knowledge and skills:
>
> (i). To understand the necessity and meaning of experimental probability.
>
> b. To acquire the ability to think, judge, and express the following:
>
> (ii). To read and express the tendency of the possibility of uncertain events based on the results of many observations and trials (MEXT, 2018a, p. 68).

In the commentary on the Japanese course of study (MEXT, 2018b) prepared by lower secondary school teachers, university teachers, and senior specialists for the curriculum, experimental probability is explained, as follows:

> For example, consider the case where you throw a plastic bottle cap and find out how many times it lands face up. Let us increase the number of times n throws the cap, find the number of times r lands face up, and calculate the value of the relative frequency $\frac{r}{n}$ that it lands face up. As n is gradually increased, r also increases, but the value of $\frac{r}{n}$ gradually converges on a certain value. This constant value that $\frac{r}{n}$ converges on is called the probability of throwing the plastic bottle cap and it landing face up. (MEXT, 2018b, p. 93)

Theoretical Probability

Theoretical probability has been defined in the classical a priori theory. According to Gillies (2000), in classical a priori theory, probability cannot be inherent in objective properties but is rather correlated to human ignorance. With respect to three or more events, we regard each as equally possible when there are no conditions to convince us that one of them occurs in preference to the others. If there are n possible cases and m of them are favorable for outcome A to occur, then the probability of A occurring is defined as $\frac{m}{n}$.

In the Japanese course of study, the aims and goals of probability in the second grade of lower secondary school (8th Grade) are set as follows:

> To teach students to acquire the following matters regarding the possibility of uncertain events through mathematical activities.
>
> a. To acquire the following knowledge and skills:
>
> (i). To understand the necessity and meaning of theoretical probability in relationship to experimental probability.
>
> (ii). To calculate probability for simple cases.
>
> b. To acquire the ability to think, judge, express, and so on, as follows:
>
> (i). To focus on equally likely events, and to consider and express how to calculate the theoretical probability.
>
> (ii). To grasp uncertain events using probability, and to consider and express them (MEXT, 2018a, p. 71-72).

In the commentary on the Japanese course of study, theoretical probability is explained, as follows:

> For example, if we assume that there are six possible ways to throw a die and that it is equally likely that the die will land on any side, we find that the theoretical probability that it will land on any side is $\frac{1}{6}$ (MEXT, 2018b, p. 123).

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[1] Experimental probability is sometimes referred to as empirical probability or objective probability (Chernoff, 2008).
[2] Theoretical probability is sometimes referred to as model probability (Pfannkuch & Ziedins, 2014).

True Probability

Experimental probability is defined as the ratio of the number of favorable trials to the total number of trials (Konold et al., 2011; MEXT, 2018b). Defined in this way, experimental probability is bound to change with each sample (Konold et al., 2011). Theoretical probability is the probability assigned under a given model. For example, assuming the complexities of physics and the apparent (and probably imperfect) symmetry of a die, the probability of rolling a "four" is $\frac{1}{6}$ (Stohl, 2005). Therefore, experimental and theoretical probabilities are estimates of something—of the actual but ultimately unknowable, probability of the outcome of interest (Konold et al., 2011; Stohl, 2005). Konold et al. (2011) refer to this probability as the "true probability."[3]

The Connection Between Experimental and Theoretical Probability

The perspective that experimental probability converges on theoretical probability

Experimental and theoretical probabilities have been taught in many countries, including Japan (Batanero & Borovcnik, 2016), and the relationship between them is often debated. Experimental and theoretical probabilities are connected by the law of large numbers (Batanero et al., 2016). For example, in the commentary on the Japanese course of study, this has been described, as follows:

> For example, if we assume that there are six possible ways to throw a die and that it is equally likely that the die will land on any side, we find that the theoretical probability is $\frac{1}{6}$. If we throw the die many times, the ratio of the number of times each side appears tends to stabilize at $\frac{1}{6}$ for each side. Thus, when all possible cases are equally likely, it is known that the experimental probability converges on the theoretical probability as the number of trials increases (the law of large numbers) (MEXT, 2018b, p. 123).

We should pay attention to the meanings of "convergence" and "close" (Stohl, 2005). For example, given a sequence $x_n = \frac{1}{n}$, there is only one value it can take. The convergence point is determined to be $x_\infty = 0$. However, when we consider the example of repeated coin tossing, this is not easy. If we place the random variable corresponding to the i-th coin toss as X_i, the random variable X_i can take two values, 0 or 1. In this case, the average percentage of *n* coin tosses that result in heads puts $Y_n = \frac{X_1 + \cdots X_n}{n}$. When *n* is sufficiently large, the ratio of the number of tosses that results in heads Y_n seems to converge on $\frac{1}{2}$, but there is still the possibility that all coin tosses result in heads and $Y_n = 1$. The implication of the law of large numbers is that experimental probability rarely differs significantly from theoretical probability if several trials are conducted (Stohl, 2005). Stohl (2005) argues that we should pay attention to the teachers' and the textbooks' explanations. He picks examples of explanations from the textbooks to make his case: "With a large number of trials, the experimental probabilities will converge on the theoretical probability" and "the experimental probability approaches, or gets closer and closer to, the theoretical probability" (Stohl, 2005, p. 348). Ireland and Watson (2009) report that many students continue to believe that fairness means obtaining the expected distribution of outcomes in the short term.

The perspective that experimental probability converges on true probability

Konold et al. (2011) and Pfannkuch and Ziedins (2014) argue against the perspective that experimental probability converges on theoretical probability. Konold et al. (2011, p. 83) argue that: "From this perspective, it is misleading and incorrect to claim that data from throwing a real die will converge on the theoretical probability, which is how many curricula currently portray it." This perspective maintains that experimental and theoretical probabilities are positively considered to be estimates of true probability. We explain these three probabilities using a fair coin as an example with reference to Pfannkuch and Ziedins (2014). First, the theoretical probability of a coin coming up as heads is $\frac{1}{2}$. However, because there is a slight bias in the actual coin, the true probability is not exactly $\frac{1}{2}$. If we assume that each trial is an independent trial and that whenever a coin is tossed, it has the same probability of showing up heads, the true probability of tossing the coin and it coming up heads can be estimated by the experimental probability, which is the proportion of the coin coming up heads over many trials. Thus, if we can relate the theoretical probability to the experimental probability, we can correctly estimate the true probability (which converge on $\frac{1}{2}$ but not exactly $\frac{1}{2}$). However, if one fails to properly recognize the relationship between the three probabilities and confuses, for example, the theoretical probability with the true probability, one might estimate the true probability incorrectly, such as considering the true probability to be exactly $\frac{1}{2}$.

Accordingly, Konold et al. (2011) state that empirical probability converges only on true probability and converges on theoretical probability only if it happens to be a perfect model of the coin. Furthermore, they argue that activities that require students to explore both experimental and theoretical probabilities of throwing a die or tossing a coin should be avoided because they obscure what they really want to know, which is true probability.

Research Questions

Experimental and theoretical probabilities are different. They are connected by the law of large numbers. In this case, we need to pay attention to the meanings of "convergence" and "close." In addition, if we consider that experimental and theoretical probabilities are estimates of true probability, then experimental probability converges on true probability, and the discourse that

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[3] Konold et al. (2011) and Pfannkuch and Ziedins (2014) call it true probability, while Stohl (2005), for example, calls it the actual probability.

Table 1. Textbooks analyzed in this paper

Textbook	Grade	Label in this paper
Fujii et al. (2021a)	7	Textbook 7a
Fujii et al. (2021b)	8	Textbook 8a
Ikeda et al. (2021a)	7	Textbook 7b
Ikeda et al. (2021b)	8	Textbook 8b
Okabe et al. (2021a)	7	Textbook 7c
Okabe et al. (2021b)	8	Textbook 8c
Okamoto et al. (2021a)	7	Textbook 7d
Okamoto et al. (2021b)	8	Textbook 8d
Sakai et al. (2021a)	7	Textbook 7e
Sakai et al. (2021b)	8	Textbook 8e
Shigematsu et al. (2021a)	7	Textbook 7f
Shigematsu et al. (2021b)	8	Textbook 8f
Yoshida et al. (2021a)	7	Textbook 7g
Yoshida et al. (2021b)	8	Textbook 8g

experimental probability converges on theoretical probability is incorrect. Based on the above, our research questions (RQs) are as follows:

(RQ1) In Japan, are experimental and theoretical probabilities taught with the awareness that they have different probabilities?

(RQ2) Which perspective is employed in teaching probability in Japan, “experimental probability converges on theoretical probability” or “experimental probability converges on true probability”?

METHODOLOGY

To analyze the explanations in textbooks, this study used content analysis—one of the textbook analysis approaches—because it is the primary research method employed in mathematics education research for the analysis of textbook explanations (Rezat & Sträßer, 2015). Krippendorff (2004) defined this method as “a research technique for making replicable and valid inferences from texts (or other meaningful matter) to the contexts of their use” (p. 18).

In Japan, the course of study is revised about once every 10 years. The current Japanese course of study for lower secondary schools was implemented in April 2021. It stipulates considerations for the overall curriculum and the number of class hours, while also broadly specifying the goals, content, and handling of content for each subject. Textbooks are created based on them. For 7th and 8th grade textbooks, publishers collaborate with university teachers and lower secondary school teachers to produce textbooks. The textbooks are then submitted to the MEXT for approval, and only those that are accepted can be published as MEXT-approved textbooks[4]. In 2021, seven MEXT-approved textbooks were published for 7th and 8th grade mathematics each[5]. In this study, all 14 textbooks (Fujii et al., 2021a, 2021b; Ikeda et al., 2021a, 2021b; Okabe et al., 2021a, 2021b; Okamoto et al., 2021a, 2021b; Sakai et al., 2021a, 2021b; Shigematsu et al., 2021a, 2021b; Yoshida et al., 2021a, 2021b) that have passed the MEXT assessment were analyzed (**Table 1**).

We are interested in the explanation of experimental and theoretical probabilities and the law of large numbers. Therefore, we first examined the relevant explanations in all textbooks for each of these concepts. To examine experimental probability, the applicable sections in which experimental probability is first explained in the 7th grade textbooks were examined. In addition, because Japanese textbooks require students to reflect on previous, related, learning content before starting new content, we also examined applicable sections in which students reflect on experimental probability learned in the 7th grade when studying it again in the 8th grade textbooks. Next, we examined applicable sections in which theoretical probability was first explained in the 8th grade textbooks. These findings correspond to RQ1. Then, we examined applicable sections in which the law of large numbers was explained after the explanation of theoretical probability in the 8th grade textbooks; this corresponds to RQ2.

As a result, eight analytical perspectives were created (**Table 2**). Krippendorff (2004, p. 180) mentions the “application of standards” in applying the analytical structure. In this case, our standards for creating the textbook analysis perspectives and judging their relevance were based on our professional mathematical knowledge and their consistency with the RQs, as seen in Stacey and Vincent (2009).

[4] For information on courses of study and textbook certification in Japan, see MEXT (n. da, n. db).
[5] The author of this paper was not involved in the preparation of any textbook.

Table 2. Eight perspectives on textbook analysis

No	Perspective
1	Subject of explanation of experimental probability
2	What to call experimental probability
3	Teaching materials to explain theoretical probability (i.e., die, coin, playing card)
4	What to call theoretical probability
5	Question about the probability of a fair die
6	Explanation of the law of large numbers
7	Question on the law of large numbers
8	Teaching true probability

Table 3. Results of throwing a plastic bottle cap (Textbook 7f, p. 249)

Number of times thrown (times)	20	40	60	80	100	200	400	600	800	1,000
Number of times facing forward (times)	7	16	21	26	30	57	98	151	205	260
Number of times turned sideways (times)	5	7	7	8	11	19	36	50	65	77
Number of times facing backward (times)	8	17	32	46	59	124	266	399	530	663
Relative frequency of face-up	0.350	0.400	0.350	0.325	0.300	0.285	0.245	0.252	0.256	0.260

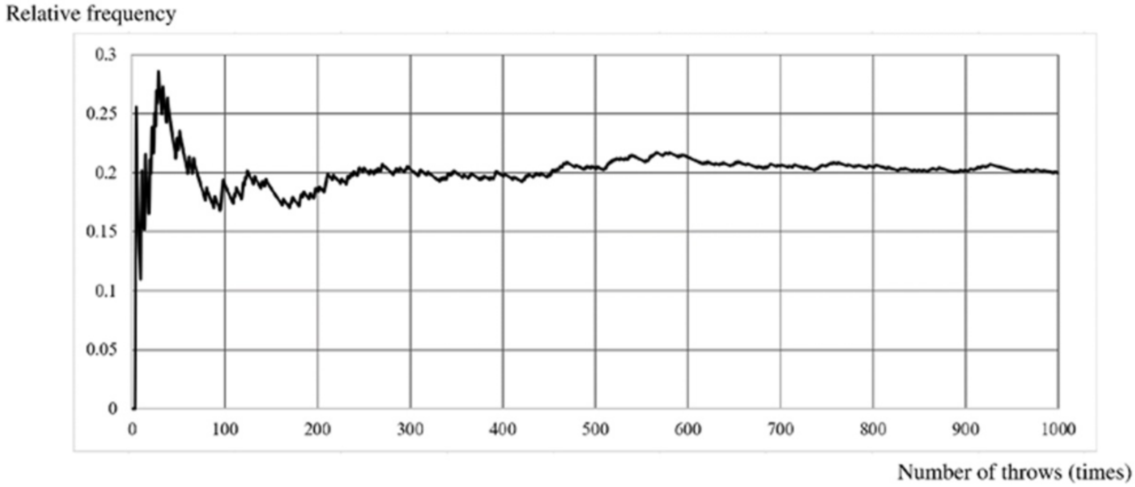

Figure 1. Example of a graph of relative frequency (The vertical axis represents its value and the horizontal axis represents the number of times the plastic bottle cap was thrown. The experimental probability of landing face up is approximately 0.2)

RESULTS

Explanation of Experimental and Theoretical Probabilities in Japanese Textbooks (RQ 1)

In this subsection, we analyze the explanation of experimental and theoretical probabilities in Japanese textbooks to examine RQ 1.

Experimental probability

In five 7th grade textbooks, the experiment of throwing a plastic bottle cap is used as teaching material. For the other two, an experiment of throwing a Shogi piece is used in Textbook 7d and an experiment of throwing a die is used in Textbook 7g. In all textbooks, the process of explaining experimental probability begins with a table summarizing the results of the experiment (see **Table 3**). Next, the results are shown in a graph, where the horizontal axis represents the number of throws and the vertical axis represents the relative frequency (see **Figure 1**). Based on the table and graph, it is explained that when the number of throws is small, the variation in the relative frequency is large; however, when the number of throws is large, the variation becomes small, and the relative frequency converges on a certain value. Although it differs from textbook to textbook, the number of throws fell between 1,000 and 3,000. Subsequently, experimental probability is defined. For example, the explanation written in Textbook 7c is as follows:

> As the number of times the plastic bottle cap is thrown increases, we find that the relative frequency of it landing face-up converges on a constant value of 0.190. This value is thought to indicate the degree to which the event of landing face-up is likely to occur. When making an experiment or observation, the number that represents the degree of possibility of a certain thing occurring is called the *probability* of that thing occurring (Textbook 7c, p. 246) (emphasis in original).

Textbook 7g, which illustrates experimental probability through the number of times a die is thrown and a 3 is obtained, shows a table similar to **Table 3**, summarizing the results of the experiment, and a figure similar to **Figure 1**, summarizing the change in the relative frequencies. The textbook states,

Table 4. Results of throwing a plastic bottle cap (Textbook 8a, p. 249)

Number of times thrown	Number of times obtaining a 1	Relative frequency of obtaining a 1
50	7	0.140
100	13	0.130
200	32	0.160
400	70	0.175
600	89	0.148
800	125	0.156
1,000	165	0.165
1,200	202	0.168
1,400	239	0.171
1,600	269	0.168
1,800	299	0.166
2,000	334	0.167

> As the number of times a die is thrown increases, the relative frequency at which a 3 is obtained gradually converges on a certain value. The following table shows the results. The constant value can be thought of as a number that expresses the degree of possibility of obtaining a 3. In this case, the value is approximately 0.167. Thus, when conducting an experiment or making an observation, the number that represents the degree of possibility of a certain thing occurring is called the *probability* of that thing occurring (Textbook 7g, p. 255) (emphasis in original).

As observed from the explanations in Textbook 7c and 7g, experimental probability is defined as probability, not experimental probability. The same is true for the other textbooks.

After experimental probability is defined, questions evaluating students' understanding of it are presented. In Textbook 7c, 7d, 7e, 7f, and 7g, for example, a table showing the number of births by sex per year in Japan is presented, and questions are asked to find the probability that a "girl is born" in Japan based on this table. In Textbook 7c, an experiment wherein the crown cap of a bottle is thrown 1,000 times is presented, with a table presenting the number of times the head of the crown appears for each 100 times; the question is to find the probability that a head appears. In Textbook 7a, a problem wherein students find the probability that a certain direction of a plastic bottle cap will appear is presented.

In all 8th grade textbooks, experimental probability is reviewed with the experiment of throwing a die. In Textbooks 8a, 8f, and 8g, a graph and a table of the relative frequencies resulting from many throws of a die are shown. Students are asked to determine the experimental probability of a certain outcome. For example, based on **Table 4**, Textbook 8a asks about the probability of obtaining a 1 when throwing a die. In other textbooks, a graph and a table of relative frequency, similar to **Figure 1**, **Table 3**, and **Table 4**, and sample results of calculating the experimental probability are shown, such as: "Therefore, the probability of obtaining a 3 is considered to be 0.17" (Textbook 8b, p. 180); "Therefore, the probability of obtaining a 1 is about 0.167" (Textbook 8c, p. 188); "From the experiment on the previous page, we can find that the probability of obtaining a 1 is about 0.167, which converge on $\frac{1}{6}$" (Textbook 8d, p. 160); and "For example, we can assume that the probability of obtaining a 1 is about 0.167" (Textbook 8e, p. 184).

Theoretical probability

As mentioned already, in all 8th grade textbooks, a die-throwing experiment is used as teaching material to explain experimental probability. After determining the experimental probability, the textbooks illustrate students saying: "Isn't it possible to find the probability from the fact that there are six faces without making an experiment?" (Textbook 8c, p. 188) or "It's hard to find the probability by making an experiment" (Textbook 8d, p. 159). Next, the motive for finding the probability without an experiment is presented, and theoretical probability is explained. For example, in Textbook 8c, the following explanation is given:

> When a correctly made die is thrown, the results are 1, 2, 3, 4, 5, and 6. What occurs in each case is equally certain. In such cases, each case is said to be *equally likely* to occur. When each case is equally likely to occur, the probability can be found, as follows: Suppose there are *n* possible cases in an experiment or observation, wherein each case is equally likely to occur. If there are *a* cases in which event A occurs, then the probability *p* that A occurs is $p = \frac{a}{n}$ (Textbook 8c, p. 189) (emphasis in original).

Textbook 8e explains as follows:

> When a correctly made die is thrown, a certain number is not considered particularly likely to appear. Therefore, equal certainty can be expected for any of the numbers to appear. In such cases, that a number from 1 to 6 will appear on the die is said to be equally likely to occur. Based on this idea, we can find the probability that the number 1 will appear, as follows.
>
> [1] There are six possible outcomes and it is equally likely that any of them will occur.
>
> [2] There is only one case in which the number 1 appears.

> [3] From [1] and [2], the probability that the number 1 appears is $\frac{1}{6}$ (Textbook 8e, p. 184).

As observed from the explanations in Textbooks 8c and 8e, theoretical probability is defined as probability, not theoretical probability. The same is true for the other textbooks.

After defining theoretical probability, in some textbooks (Textbook 8c, 8d, and 8f), cards or die are used to find probability, but not in others. In the former, for example, “Find the following probabilities when throwing a single die. (i) the probability of getting a 4, (ii) the probability of getting an even number, and (iii) the probability of getting a number less than or equal to 2” (Textbook 7d, p. 190) is asked. In the latter, a picture of an unfair die is shown and the following question is asked: “For a die shaped like the one on the right, can you find that the probability of obtaining a 1 is $\frac{1}{6}$? Also, explain why you think so” (Textbook 8a, p. 163).

Explanation of the Law of Large Numbers and True Probability in Japanese Textbooks (RQ 2)

In this subsection, we analyze the explanation of the law of large numbers and true probability in Japanese textbooks for examining RQ 2.

The explanations for the law of large numbers can be divided into three patterns. First, there are explanations about the convergence of experimental probability on theoretical probability. Second, there are questions regarding the law of large numbers. Third, there are neither explanations nor questions.

In Textbooks 8b and 8e, the convergence of experimental probability on theoretical probability is explained. In Textbook 8b, it is explained that the probability of obtaining any number from 1 to 6 on a die is $\frac{1}{6}$ for each of them. Furthermore, in Textbook 8b, it is explained that “0.17, which is the probability of obtaining a 3 in the experiment on the previous page is almost equal to $\frac{1}{6}$” (Textbook 8b, p. 181). In Textbook 8e, it is explained that the probability of getting a 1 on a die is $\frac{1}{6}$. In Textbook 8e, it is explained that: “This value of $\frac{1}{6}$ is almost identical to the experimental probability of 0.167” (Textbook 8e, p. 184).

Next, Textbooks 8a, 8b, 8d, and 8f include questions about the law of large numbers. For example, Textbook 8a has the following question:

> When throwing a die, Mr. A thought about the probability of obtaining a 1 as follows: Do you think this idea is correct?
>
> The probability of getting a 1 is $\frac{1}{6}$, so, if I throw a die six times, at least one of them will be 1 (Textbook 8a, p. 164).

Textbook 8b has the same question. In Textbook 8f, students are also asked to select the correct statement out of five sentences describing how to roll a die with a $\frac{1}{6}$ probability of obtaining a 1:

> There is a die for which the probability of obtaining a 1 is $\frac{1}{6}$.
>
> Choose the correct answer from [a] to [e] that describes how this die rolls.
>
> [a] If you throw the die six times, one of the trials will always roll a 1.
>
> [b] If 1 has not appeared even once after 5 throws, it will always appear on the 6th throw.
>
> [c] After 6 throws, numbers from 1 to 6 will always appear once.
>
> [d] If you throw 30 times, 1 will always appear 5 times.
>
> [e] If you throw the dice 6,000 times, 1 will appear about 1,000 times (correct answer; Textbook 8f, p. 177).

In Textbook 8d, it is asked whether the following statement is correct or not: “When taking out a ball from a box containing four red balls, two yellow balls, and three blue balls, and putting it back 900 times, you will always get a red ball 400 times.” Textbooks 8c and 8g did not explain the law of large numbers.

None of the textbooks included an explanation that experimental probability may not converge on the value of theoretical probability even when the number of trials is large, as noted by Stohl (2005). According to Stohl’s (2005) assertion about the law of large numbers, if a coin is tossed 10,000 times, it would be extremely rare for it to land face up each time, but not entirely impossible. Moreover, none of the textbooks included explanations about true probability, such as experimental and theoretical probabilities are estimates of true probability.

DISCUSSION

In this section, based on the results of the textbook analysis in the previous section, we will answer the RQs.

The Relationship Between Experimental and Theoretical Probabilities

RQ1 is "In Japan, are experimental and theoretical probabilities taught with the awareness that they have different probabilities?"

First, in the explanation of experimental probability in all textbooks for the 7th and 8th grade, except Textbook 7g, teaching materials for which theoretical probability could not be found, such as a plastic bottle cap, are used. Thus, it is being taught that the probability of an event can only be determined through experimental probability, which implies that experimental and theoretical probabilities are taught as different probabilities (Konold et al., 2011).

Next, in the explanations of experimental probability in textbooks for 8th grade, a die is used as teaching material, with an awareness of the connection with theoretical probability that follows in all textbooks. The experimental results are shown graphically, and then the theoretical probability is explained. If we can have students perform this activity, they can conduct a learning activity that is consistent with the results of Aspinwall and Tarr (2001) and Stohl and Tarr (2002). Accordingly, students can deepen their understanding of the role of sample space and number of trials in the relationship between experimental and theoretical probabilities by conducting their own trials and expressing the results graphically. However, the explanation of experimental probability was different in one textbook, which is, as follows: "From the experiment on the previous page, we can find that the probability of obtaining a 1 is about 0.167, which converge on $\frac{1}{6}$" (Textbook 8d, p. 160). We believe this is not an appropriate explanation because students who have only learned experimental probability do not necessarily think that the probability of obtaining a 1 converge on $\frac{1}{6}$ after they find the probability to be approximately 0.167. This can be considered an explanation that will lead to the learning of theoretical probability later on. However, before learning theoretical probability, it is preemptive for students to think that the probability of obtaining a 1 on a die converge on $\frac{1}{6}$ after they found the probability to be approximately 0.167. Therefore, we believe that the explanation in Textbook 8d may cause students to confuse experimental probability with theoretical probability.

In the explanation of theoretical probability for the 8th grade, after defining theoretical probability, a picture of a die that was not fair is presented, and the question of whether theoretical probability could be used to determine probability is asked in some textbooks. This question is intended to clarify the scope of theoretical probability; therefore, it can be said that textbooks have been written with the awareness that experimental and theoretical probabilities are different.

The Perspective About Converges of Experimental Probability

RQ2 is "Which perspective is employed in teaching probability in Japan, "experimental probability converges on theoretical probability" or "experimental probability converges on true probability"?

The results of the analysis showed that true probability was not taught in teaching probability in Japan. We believe there are two reasons for the same. First, it is not explicitly stated that experimental and theoretical probabilities are estimates of true probability in textbooks. If textbook authors are actively considering this, they are expected to clarify this in writing. Second, all textbooks have developed teaching that converges on the probability of a certain value (e.g., 1) on a die from both experimental and theoretical probabilities. Konold et al. (2011) state that such teaching is not suitable for teaching true probability. It causes students to equate theoretical probability with true probability, obscuring the idea that what they really wish and strive to know is the true probability (Konold et al. 2011).

The explanation of the law of large numbers differed among textbooks. First, although the explanations in Textbooks 8b and 8e are not incorrect, they are somewhat oversimplified as pointed out by Stohl (2005). Based on Stohl's (2005) point, it is suggested that the explanation of "almost" "almost equal" (Textbook 8b, p. 181) and "almost identical" (Textbook 8e, p. 184) needs to be enhanced.

Next, Textbooks 8a, 8d, and 8f have questions about the law of large numbers, although they do not explain "converging," "closing," or "corresponding" as Textbooks 8b and 8e did. In these questions, it can be considered that the goal is for students to understand that relative frequency does not converge on theoretical probability in a small number of trials and that even in a large number of trials, relative frequency does not exactly match theoretical probability. In light of this, we believe that problems related to the law of large numbers provided in the textbooks aim to help students understand the meanings of "closing" and "close" through problem solving. Thus, we can consider that Textbook 8b is attempting to help students understand the meaning of "almost" through problem solving in relation to the law of large numbers.

However, no explanation was given for the fact that experimental probability may not converge on the value of theoretical probability even when the number of trials is large. This implies that the textbook does not correctly explain the law of large numbers. Alternatively, it is possible that rigorous explanations of something that rarely happens could interfere with students' understanding of the law of large numbers, and thus, the textbooks do not strictly explain it.

The discussion is summarized as shown in **Table 5**.

CONCLUSION

The purpose of this study was to clarify how experimental and theoretical probabilities are taught in Japanese 7th and 8th grades by employing textbook analysis. We analyzed all Japanese 7th and 8th grade mathematics textbooks that were written based on the Japanese course of study implemented in April 2021 and approved by the MEXT. Regarding the explanation of experimental probability, some textbooks are written to teach students to distinguish between experimental and theoretical probabilities, while

Table 5. Summary of textbook analysis and discussion

Textbook	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
7a, 8a	Plastic bottle cap	Probability	Die	Probability	Included	Not included	Included	Not included
7b, 8b	Plastic bottle cap	Probability	Die	Probability	Included	Included	Included	Not included
7c, 8c	Plastic bottle cap	Probability	Die	Probability	Not included	Not included	Not included	Not included
7d, 8d	Shogi piece	Probability	Die	Probability	Not included	Not included	Included	Not included
7e, 8e	Plastic bottle cap	Probability	Die	Probability	Included	Included	Not included	Not included
7f, 8f	Plastic bottle cap	Probability	Die	Probability	Not included	Not included	Included	Not included
7g, 8g	Die	Probability	Die	Probability	Included	Not included	Not included	Not included

Note. From (1) to (8) are perspectives on textbook analysis (**Table 2**)

others taught them to be aware of experimental probability converges on theoretical probability. Regarding the explanation of theoretical probability, some textbooks have questions intended to clarify the scope of theoretical probability and are written with the goal of teaching students to distinguish between experimental and theoretical probabilities. Regarding the explanation of the law of large numbers, the perspectives of Konold et al. (2011) and Pfannkuch and Ziedins (2014) that experimental probability converges on true probability were not taught. Those textbooks that do have explanations and questions about the law of large numbers based on the perspective that experimental probability converges on theoretical probability, however, have inadequate explanations of the law of large numbers.

The significance of this study is twofold. First, we discussed the Japanese course of study and the textbooks that were implemented in April 2021. Since the course of study is revised every 10 years, this study can serve as a basis for teaching probability in Japan for the next nine years. It is also expected to make an impact on the next revision of the Japanese course of study and textbooks. Second, we discussed the teaching of probability in Japan, in English. Batanero et al. (2016) reviewed the teaching of probability worldwide; however, Japan was not included in their review. We believe one of the reasons is that research published in English regarding the teaching of probability in Japanese schools is scarce. Therefore, given that this study will be published in English, there is a chance that the teaching of probability in Japan will now be included in the global probability teaching research.

A future task is to clarify how the differences in textbooks revealed in this study affect teachers and students. We plan to conduct classroom observations at lower secondary schools using different textbooks and compare the results. Finally, it should be noted that this study was not written with the intent of recommending any particular textbook to learners or teachers; instead, they were analyzed and discussed from a neutral standpoint

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