International Electronic Journal of Mathematics Education
2024, 19(2), em0776
e-ISSN: 1306-3030
https://www.iejme.com

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Research Article

OPEN ACCESS

Advancing students' achievements in multivariable calculus education through CSCL

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Citation: Milenković, A., & Vučićević, N. (2024). Advancing students' achievements in multivariable calculus education through CSCL. *International Electronic Journal of Mathematics Education, 19*(2), em0776. https://doi.org/10.29333/iejme/14472

ARTICLE INFO

Received: 02 Feb. 2024

Accepted: 07 Apr. 2024

ABSTRACT

The contents of calculus, known for their complexity, present significant challenges for students, particularly in mastering multiple integrals and effectively visualizing related concepts. The transition to distance learning prompted by the COVID-19 pandemic has further complicated the learning process in multiple integrals. In line with this and considering the potential of computer-supported collaborative learning (CSCL), in this research we explore the impact of teaching in a CSCL environment on student achievements, focusing on students from the computer science study program. Through data analysis using ANOVA and Bonferroni post-hoc tests, it was found that students exposed to collaborative learning in GeoGebra environment demonstrated higher levels of theoretical and practical knowledge compared to peers who acquired knowledge without using GeoGebra. Additionally, this group of students achieved results comparable to those of students who attended traditional in-person teaching, showing noticeable improvements in solving complex tasks. Our findings show the effectiveness of CSCL approach in context of distance learning and highlight potential of collaborative environments enhanced with technology in facilitating student understanding and achievements in calculus education.

Keywords: multivariable calculus, CSCL, distance education, integrals

INTRODUCTION

Calculus is one of the most challenging areas of mathematics, and teaching multivariable calculus in higher education to students from non-mathematical programs poses a challenge. This is why many researchers delve into this topic, attempting through various studies to determine which methodological approaches lead to better calculus learning outcomes (Dorko & Weber, 2014; Huang, 2015; Martínez-Planell & Trigueros, 2021). In many of these studies, the impact of different digital devices is specifically examined, along with the varied organization of the calculus teaching process and student learning (Božić et al., 2019, 2023; Milenković et al, 2020). Additionally, certain studies have investigated the influence of collaborative learning on students' achievements in mathematics (Bringula & Atienza, 2023; Mullins et al., 2011).

An additional challenge that prompted education in general, including the teaching of multivariable calculus in higher education, is the COVID-19 pandemic. Teaching in various higher education institutions worldwide shifted to online instruction. After the initial shock experienced by both students and their instructors, designing and implementing education in higher education became genuinely complicated, requiring significant efforts from all participants in the process.

Considering the development of dynamic software that enhances 2D and 3D visualization, providing an intuitive platform for students to engage in exploration and experimentation with dynamic materials (which students can create without the assistance of instructors), as well as the development of platforms that enable computer-supported collaborative learning (CSCL), we decided to examine the impact of implementing a dynamic environment within collaborative calculus learning in higher mathematics education. For this purpose, we planned and implemented the teaching of the mathematics 3 course, a second-year computer science course, online. Students were required to acquire both theoretical knowledge and practical knowledge and skills regarding multiple integrals. This study was conducted in Faculty of Science at University of Kragujevac, Kragujevac, Serbia.

Teaching in the experimental group was carried out using MS Teams platform. Part of the lessons took place in meetings attended by all students, while the more intensive part of practical classes occurred in MS Teams chats attended by four-members student groups, heterogeneous in terms of their achievements in mathematics. Technological support for collaborative student learning involved the use of GeoGebra software package. Through joint efforts, students created dynamic materials that aided in visualizing geometric objects defining the integration domain in specific tasks. These materials also facilitated learning and understanding how introducing variable substitutions transforms the corresponding coordinate system through collaborative work.

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To examine the impact of implemented CSCL using GeoGebra environment on the achievements of computer science students, the results obtained by students in solving three tasks (**Appendix A**) after completing the teaching process on multiple integrals were compared across three (one experimental and two control) groups of students. The test consisted of one double integral and two triple integrals (with one task requiring variable switch). Students' results were analyzed using ANOVA and Bonferroni post-hoc tests in SPSS statistics software package.

THEORETICAL BACKGROUND

Multivariable Calculus Education & Technology

The focus of current learning materials on multivariable calculus emphasizes students' comprehension of calculus theory, visualization of multivariate calculus concepts, problem-solving skills, and providing insight into the practical applications of these concepts in science and engineering (Guichard, 2017; Stewart, 2008).

In the past decade, considering the development of technology and other educational resources, the didactic triangle related to teaching, which was set on three pillars–student, teacher, and instructional content, has been complemented by a fourth element–the artifact (Rezat & Sträßer, 2012). This fourth element is related to both digital and non-digital resources that instructors use during the instructions.

One of the outcomes of teaching multivariable calculus is the development of spatial literacy. According to Moore-Russo et al. (2013), spatial literacy requires skills and abilities from three spheres: visualization, reasoning, and communication. These three domains are interconnected, with non-empty intersections and it is important to impact all three domains when planning and implementing teaching and learning strategies.

Over the past few years, the development of dynamic software has led to an increase in the availability of interesting, well-prepared, and helpful materials aimed at visualizing various mathematical concepts and ideas. These resources can be found in recently published books and on the internet, including graphs of multivariate functions, surfaces in space, and their intersections. The impact of new technology on students' achievements in calculus has been extensively studied, particularly in the realm of 2D visualization (Božić et al., 2019; Huang, 2015). However, research on 3D visualization is notably less prevalent (Delice & Ergene, 2015; Mahir, 2009).

Multiple integrals find extensive applications in mathematics, such as determining plane areas, calculating mass, finding volumes, moments of inertia, and determining surface areas of 3D objects. Developing proficiency in handling multiple integrals is both necessary and crucial. However, mastering multivariate calculus, especially multiple integrals, often poses challenges for students (Hamidreza et al., 2010). Additionally, the transformation of integrals from one to another coordinate systems requires strong visualization skills and spatial ability to conceptualize regions in the plane and objects in space.

The findings from research conducted by Zengin and Tatar (2015) show that computer-assisted instruction approach, incorporating dynamic mathematics software, had a positive impact on the proficiency of pre-service teachers in comprehending the topic of polar coordinates. Additionally, it was observed that pre-service teachers endorsed the utilization of this method in lessons, citing its benefits in terms of visualization, enhanced retention, concretization of abstract mathematical structures, improved understanding and learning, and the creation of an engaging and interactive learning environment (Zengin & Tatar, 2015).

Mahir (2009) notes that visualizing and sketching figures in 3D, as well as understanding the relationship between graphical and algebraic representations of space, is frequently the most challenging aspect of solving problems involving multiple integrals. Researchers have consistently noted that students encounter persistent challenges in successfully completing tasks regarding translation from one representation of the object to another (Afriyani et al., 2018; Duval, 2006; Rahmawati et al., 2017).

Various challenging areas in learning multivariable calculus have been identified. In a study on multivariable calculus, Kashefi et al. (2011, 2012) discovered that for many students, finding domains and ranges, sketching graphs, understanding partial derivatives, and issues related to multiple integrals were the most difficult aspects of multivariable calculus. Students faced challenges due to their past mathematical experiences, the adverse impact of their mathematical knowledge construction, insufficient prior knowledge rooted in calculus or a lack of practice, inappropriate choices in representing concepts across three worlds of mathematics, the transition between different mathematical domains, algebraic manipulations, and constraints related to memory. In a study conducted by Kashefi et al. (2012), the authors highlight that strategies such as a blended learning environment assist students in fostering their individual mathematical reasoning abilities, aiding them in constructing fresh mathematical knowledge and essential skills, notably communication, teamwork, problem-solving, and technology skills.

As stated earlier, for solving multiple integral problems, finding the domain of the integration is quite an important part of the problem solution. Dorko and Weber (2014) in their research studied students' understanding of domain and range of one-variable functions and how they generalized it to two-variable functions. Dorko and Weber (2014) claim that domain and range are disregarded in textbooks and that they require explicit attention during the teaching process.

Henriques (2006) conducted research on the instruction and comprehension of multivariable integrals, specifically exploring their application in calculating volumes of objects in 3D. During this research, the author concentrated on problems associated with determining the volume of bodies defined with different mathematical objects to examine both graphical and analytical strategies, as well as the role of technology in addressing challenges faced by students. The findings revealed that educators and teaching materials tended to facilitate calculations based on geometric representations. However, even with the integration of technology, students encountered difficulties in generating and interpreting these representations. Students required assistance in configuring the surfaces to generate images on the preparation screen, which then need to be calculated, and establishing connections between these parameters to determine integration limits. Moreover, the author (Henriques, 2006) suggests using graphing technology to visualize parts of space defined with the intersection of different 3D surfaces and to explicitly discuss parametrizations and limits of integration.

In their meta study Martínez-Planell and Trigueros (2021) discuss that more research on the use of technology in the learning and teaching of two-variable calculus is needed, particularly studies considering technological advances.

Computer-Supported Collaborative Learning & Computer-Supported Collaborative Learning in Mathematics Education

CSCL refers to the activity of students, involving peer interaction for the purpose of their learning, with the assistance of information and communication technologies (ICT) (Suthers & Seel, 2012). CSCL is considered as an effective strategy in teaching and learning (Lehtinen et al., 1999), and it combines two ideas–computer support and collaborative learning. ICT used in CSCL regard internet resources, mobile phones, desktop and laptop computers, and other handheld devices (Suthers & Seel, 2012). In collaborative learning, students work together on a task, where each student is responsible not only for their own learning but also for the learning of other students in the group, with the aim of achieving a common goal. The learning theory that provides a theoretical framework for collaborative learning is constructivism in which one of the assumptions is that students are placed at the center of the learning process to create their own knowledge through discussions (von Glasersfeld, 1995). In such a learning environment, more capable students, i.e., those who achieve better results, can provide the necessary assistance to students with lower achievements to help them understand relevant concepts and ideas.

CSCL could be considered as an interdisciplinary research field that investigates how collaborative learning, assisted by technology, can improve peer interaction and their work, as well as how collaboration and technology facilitate the sharing and distribution of knowledge among peers (Jeong et al., 2019).

The technology tools employed in CSCL exert a positive influence on learning processes and collaboration dynamics (Hamid et al., 2015; Molinillo et al, 2018). The selection of technological resources should align with the targeted learning objectives and be in harmony with the pedagogical, cognitive, and social activities (Lyons et al., 2021; Tarun, 2019). Technology supporting collaborative learning must have the capability to organize tasks and make group analysis easier and negotiations between peers essential for task resolution (Strijbos et al., 2004).

CSCL leads to a synergy of students because student groups exhibit higher levels of thinking and can retain essential information for longer durations compared to individual students (Rao, 2019). Additionally, it has been demonstrated that students entrusted with higher levels of responsibility achieve better learning outcomes (Laal & Laal, 2012).

In general, the adaptive features of CSCL contribute to students' individual learning advantages (Sung et al., 2017). Chen et al (2018) conclude that collaboration in CSCL has significant positive effects on knowledge gain, skill acquisition, and student perceptions (Chen et al., 2018).

From conclusions in different studies, it could be noticed the positive effects of CSCL on students' mathematics (e.g., Lin et al., 2011; Mullins et al., 2011) and STEM academic achievements. These positive effects are regarding positive impact of CSCL on the process of learning (individual tasks, collaborative process), on the cognitive aspect of learning (understanding of mathematics concepts and principles, on generating a design solution, critical thinking skills, and students' grades), and on affective aspects of learning (e.g., attitudes, perceptions, motivation, interests, confidence, and satisfaction). Moreover, when it comes to collaborative learning in mathematics, this learning approach provides students with the opportunity to discuss appropriate strategies for solving mathematical tasks (Davidson, 1990).

When it comes to the type of digital devices students often use, it usually involves mobile devices such as laptops, tablets, and mobile phones. Therefore, in the literature, the term mCSCL has become established, referring to the use of mobile devices in learning that takes place with the aid of technology and is based on collaborative learning. Bringula and Atienza (2023) indicate that mCSCL in mathematics education has the potential to enhance students' cognitive capacities, social skills, and attitudes towards the course. Bringula and Atienza (2023) discuss that the most preferred subject for the use of mCSCL in mathematics education is elementary mathematics compared to other mathematics domains, such as calculus in higher education.

In various studies, the positive impact of CSCL has been confirmed compared to individual computer supported learning (Lou, 2004; Lou et al., 2001). On the other hand, instructors favor CSCL over collaborative learning without the use of technology in mathematics with the aim to influence learning through technology-driven exploration by students. They aimed to provide students with the opportunity to visualize and analyze problems, create strategies for solutions, and predict the direction of the problem-solving process before engaging in simple calculations. Researchers in education have explored ways to organize mathematics teaching to increase student engagement and achievements (Božić et al, 2019; Milenković et al, 2020). GeoGebra is a tool facilitating the mentioned student activities, as its use allows students to explore, collaborate, and construct their knowledge (Božić et al, 2019; Vasquez, 2015).

In a study, where the authors aimed to examine the impact of CSCL on high school students' achievements in the field of functions (exponential and logarithmic), a statistically significant difference in the students' achievements was identified. Those who used GeoGebra during their collaborative learning outperformed their peers who acquired relevant knowledge through direct textbook instructions (Birgin & Acar, 2022). Birgin and Acar (2021) conducted another study, where the positive impact of GeoGebra on students' achievements and the retention of acquired knowledge in the field of linear equations and slope was confirmed. In this study as well, students who used GeoGebra outperformed their peers who acquired the relevant knowledge through textbook-based direct instruction.

RESEARCH QUESTION

Many studies have shown numerous positive effects of using computers in mathematics teaching, especially for the visualization of abstract mathematical concepts. Furthermore, based on the literature, it is evident that CSCL positively influences students' readiness to delve into knowledge acquisition and problem-solving, and it also has a positive impact on students' achievements in mathematics.

In our research, the main question is: Does a software application for visualization of multivariable functions with online CSCL environment contribute to better student achievement in solving multiple integrals?

The hypotheses for our research are, as follows:

H1. GeoGebra applications for visualization while solving double and triple integrals in an online environment with small collaborative groups of students lead to better student achievements in comparison with online learning with less student engagement and without students' collaboration.

H2. Learning multivariable calculus, more precisely multiple integrals, using GeoGebra and MS Teams platform, within the Teams Chat and later through discussion with other students, leads to student achievements that do not significantly differ from the achievements students attain during in-person instruction in circumstances that they are already accustomed to.

METHODOLOGY

General Background

The transition to online teaching from in-person classes at universities was a significant challenge for higher education instructors in Serbia. Unlike the second year of the pandemic, when teaching was also conducted online and instructors had already provided teaching tools such as a graphic tablet and other teaching aids, in the first year, when the interruption of in-person classes was announced midway through the semester, instructors largely scanned or created PDF documents that they analyzed together with students.

During the multivariable calculus course for computer science students, it was up to the students to study the course materials, attempt to grasp the essence of the problems, understand basic mathematical principles and concepts, and then discuss these topics with the instructor (teacher). The instructor put in tremendous efforts to convey the content to students and ensure their understanding. Initially, students struggled with this method of teaching because they were accustomed to a traditional approach to teaching and learning mathematics. In the traditional approach, instructors would deliver the material, and students would carefully follow along, engaging in discussions with the instructor to comprehend the subject matter. Despite the students' dedication during the first year of online teaching, it became evident that they could not achieve the intended learning outcomes.

Therefore, for the second year of online learning, a methodological approach was devised, leveraging the use of MS Teams platform. However, this time, specific tasks were assigned to students in smaller, four-member, heterogeneous groups. The teaching sessions involved simultaneous meetings on MS Teams students attended, as well as separate meetings for these four-member chat groups in MS Teams. During practical teaching sessions, the instructor introduced concrete tasks to the students, ensured that they understood the expectations, and then engaged with individual student groups to monitor their progress. Students were able to manipulate images, rotate objects and view them from a different perspective when they practiced double and triple integrals, including their own attempts to determine the integration domain. Minor corrections and suggestions were provided by the teacher, with an emphasis on avoiding being overly directive and instead providing support to empower students to collaborative learning.

Participants

In our research, sample consisted of 98 second year students from Faculty of Science, University of Kragujevac, Kragujevac, Serbia. They were divided into three groups, as follows:

1. In control group 1, there were 32 computer science students in 2020.
2. In control group 2, there were 34 computer science students in 2021.
3. In the experimental group, there were 32 computer science students in 2022.

Instruments & Procedures

The purpose of the research that is the subject of this paper is to investigate the effect that GeoGebra usage for students' collaborative learning has on the students' achievements. As per our experience, students experience some issues in assessing definite integrals, or at least, in deciding a crude capability. However, when we move to multiple integrals in the second year of higher education, students experience issues when they define the integration domain and limits for variables of the double and the triple integrals. An experimental approach was used with the students in the experimental group to ease the difficulty of determining those limits. In GeoGebra, the instructor used materials he had created for specific assignments.

In control group 1, the students learned the teaching content in the classroom, in a traditional way without using a computer.

In control group 2, the teaching was organized online. Teaching content was presented using scanned materials and presentations, without the use of GeoGebra. The tasks that were used by both the groups were identical and they were carefully chosen.

In the experimental group, classes were also held online. The teaching was organized, as follows. We first presented the students with general concepts related to the double integral, the method of calculating the double integral, as well as some simpler examples of determining limits for the variables. The software package GeoGebra was presented to the students, in general, as well as specifically which functions are needed for solving double integrals.

During the teaching and learning process, students and teacher used GeoGebra. Teaching took place within MS Teams platform. Students were divided into groups on MS Teams, of four members each. For the work of the students in the group to be adequate, the students were divided into groups observing several parameters: based on grades from the subjects like mathematics 1, mathematics 2, as well as based on comments about whether they would like to work with someone in a group, i.e., whether they were against working with someone in a group.

When we planned and organized students activities we were implementing Kagan's (1994) key criteria necessary for successful implementation of collaborative work: creating positive interdependence, where individuals connect and distribute roles for successful collaboration; developing individual responsibility of each group member for collective work and goal achievement; ensuring equal participation and an even distribution of responsibilities within the group; providing simultaneous interaction, i.e., creating conditions in which all participants can act simultaneously.

Within the framework of the formed group, we could join at any time, to monitor their work. Students shared their screens, sketched graphics, and commented together within the group. In the first week, students dealt with double integrals. The first examples were related to determining limits and solving integrals (**Figure 1**, **Figure 2**, and **Figure 3**).

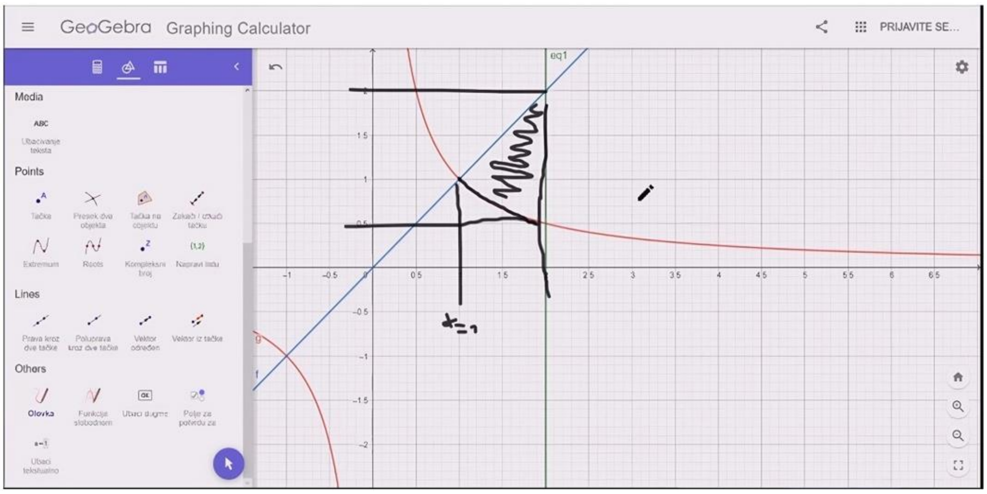

Figure 1. Students drawing & writing on dynamic worksheet shared with members of group for a concrete double integral task (Source: Field study)

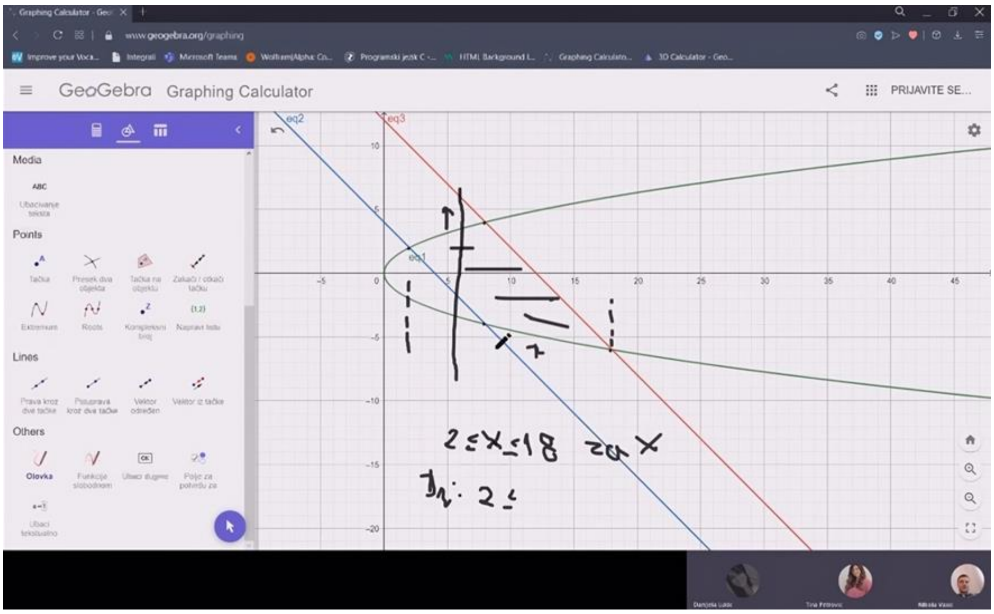

Figure 2. Students drawing & writing on dynamic worksheet shared with members of group for a concrete double integral task in which domain of integration must be divided (first way for solution) (Source: Field study)

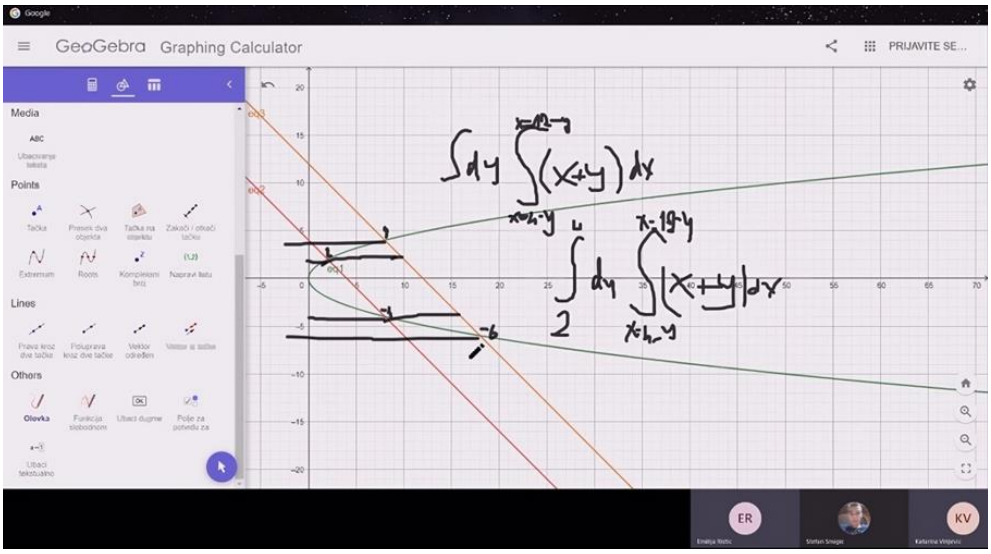

Figure 3. Students drawing & writing on dynamic worksheet shared with members of group for a concrete double integral task in which domain of integration must be divided (second way for solution) (Source: Field study)

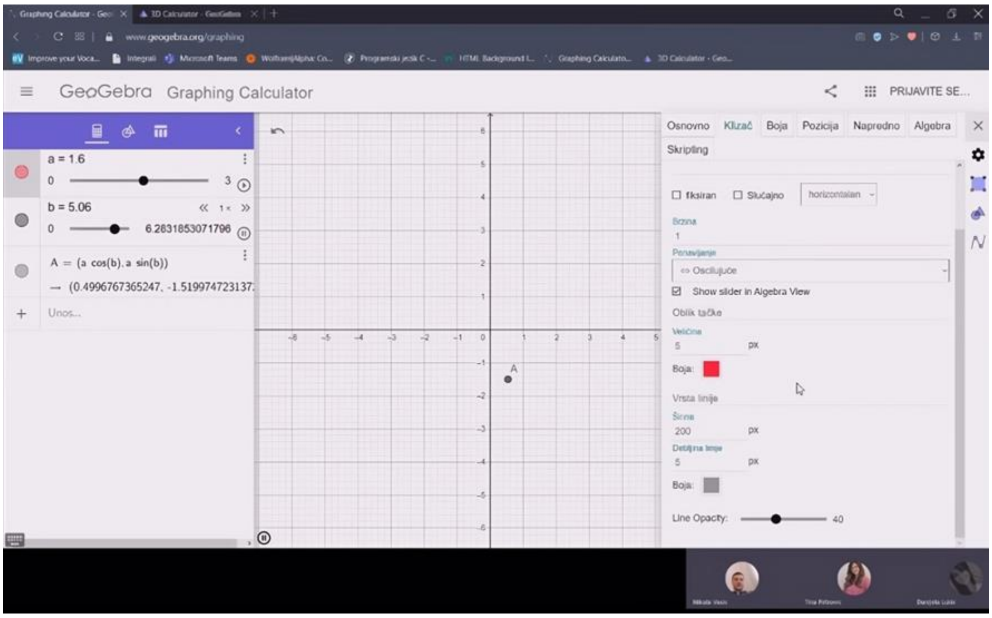

Figure 4. Students creating dynamic material shared with members of group for solving problem for double integral with usage of switch of polar coordinates & slider (Source: Field study)

Then, using the sliders in GeoGebra, with the given instructions, the students graphically displayed the meaning of the transition to polar coordinates, where they saw the meaning of the parameters in polar coordinates (**Figure 4**).

In the second week, students solved triple integrals. The first learning goal was to apply the idea of determining limits in the same way as for the double integral. Students used GeoGebra 3D online version (3D calculator). The advantage of GeoGebra compared to other tools proved to be very significant, especially because the students directly entered the equations in the form in which they were given, to get the appropriate picture. In other words, GeoGebra allows multiple representations of mathematics objects. Then, the emphasis was on moving from the calculation of the triple integral to the calculation of the double integral, also with the use of GeoGebra. This process was quite efficient, because they could rotate the body in space, which represented the integration domain and determine the 2D projection of that body on the proper plane. After that, a variable shift was made using cylindrical and spherical coordinates (**Figure 5**).

At the end of each example, we approached chats in MS Teams for different students and the students explained how they solved the task. After that all the students returned to the main team, where we again gave instructions for the next problem. The emphasis was on their collaborative work within the group, without the instructor's help. What we noticed is that when one student failed to determine the solution, the other colleagues got involved and then worked together to come up with a solution.

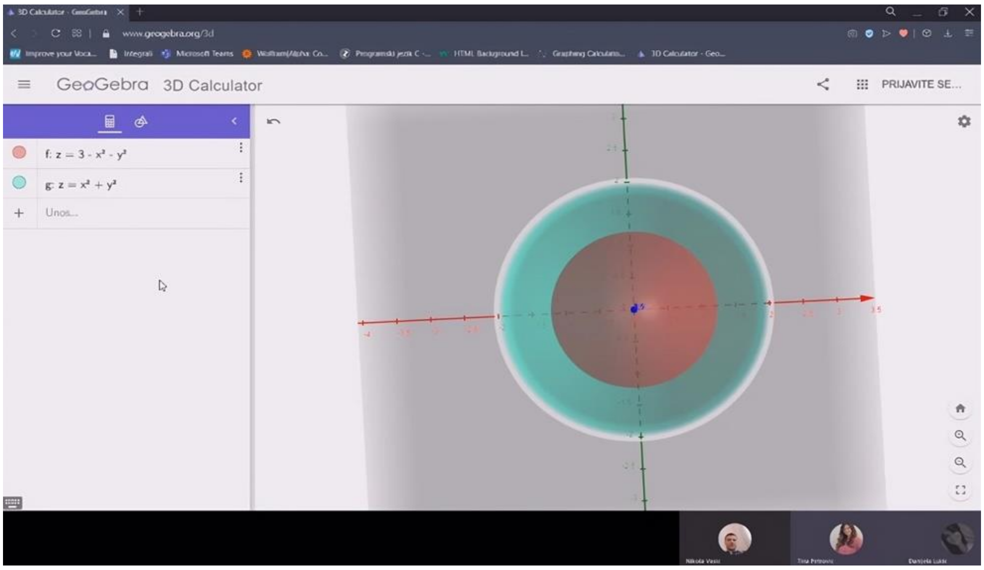

Figure 5. Students creating dynamic material in GeoGebra 3D, shared with members of group for solving triple integral task (Source: Field study)

Data Analysis

Statistical analysis was carried out by using SPSS Statistics software package. The Kolmogorov-Smirnov test was used for determining normality of the sample distribution. To analyze differences between three data sets (one experimental and two control sets) we used ANOVA test and Bonferroni post-hoc test for determining the difference between two out of three groups. The level of significance for all the calculations carried out in the SPSS software was 0.05.

RESULTS

At the beginning of each mathematics 3 course (in 2020, 2021, and 2022), students take an entrance test consisting of five tasks, three of which are related to definite integrals. To solve the first of those three tasks, students need to use their knowledge of definite integral properties (the integral of a sum or difference, the property that allows coefficients to be factored out of the integral), along with their familiarity with the table of integrals. The second task pertains to the application of the substitution method, while the third involves the use of partial integration. To analyze the students' performance, an ANOVA test was employed for the experimental group and two control groups. The results obtained from this analysis indicate that there were no statistically significant differences in students' prior knowledge of definite integrals ($F_{2,95} = 0.456, df = 2, p = 0.6200$). This was a prerequisite for conducting the research in the year when students learned about multiple integrals using CSCL approach, with the use of MS Teams platform and GeoGebra software package. The test consisted of three tasks regarding multiple integrals. The first task was the double integral, and to solve this task, students needed to determine the limits of variables and solve the integral using graphic representation for the domain of integration. The second task on the test was the triple integral, and to solve it, students needed again to determine the limits using the graphical method and to reduce it to a double integral. The third task was again triple integral, and students needed to conduct the same procedures as in the second task, but also apply a shift of variables using spherical coordinates and switch to a double integral.

Analyzing students' success in solving the first task, where students were required to solve a double integral by sketching graphs of elementary functions and lines, by observing the graphs in **Figure 6**, we can observe that the students in the experimental group achieved the best results.

However, the difference in the arithmetic means of the points obtained by students in the first task is not drastic. Based on the values in **Table 1**, we can observe that the difference in the average number of points obtained by students in the three different groups is not statistically significant.

When it comes to the achievements of students in three different groups in solving the second task related to the triple integral, based on **Figure 7**, it can be observed that students from the second control group achieved the worst results, while students from the experimental group achieved the best performance.

Based on the results of ANOVA test presented in **Table 2**, it can be concluded that there are statistically significant differences in the students' achievements. Post-hoc tests indicate that statistically significant differences exist between the achievements of the experimental group and the second control group (p=0.0270).

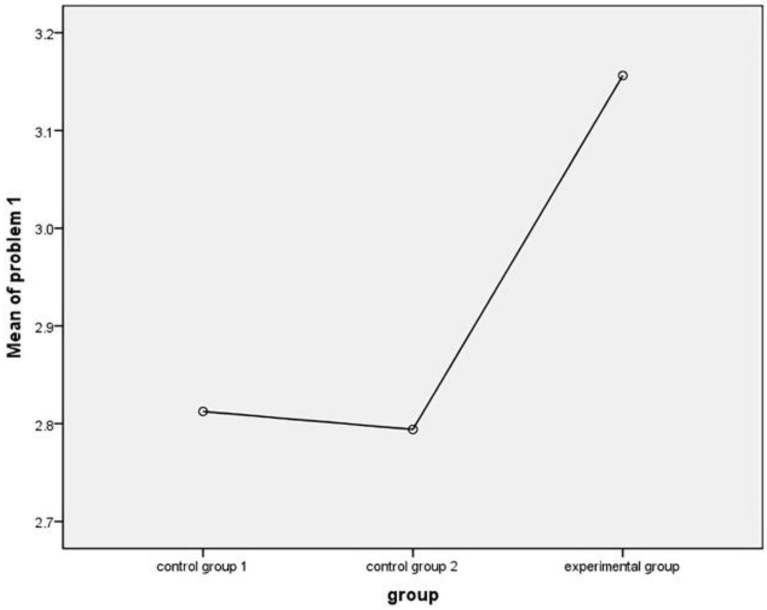

Figure 6. Students results while solving task 1 (Source: Authors' own elaboration)

Table 1. Statistical results for task 1 for three different groups of students

ANOVA	Sum of squares	df	Mean square	F	p
Between groups	2.694	2	1.347	0.394	0.675
Within groups	324.653	95	3.417		
Total	327.347	97			

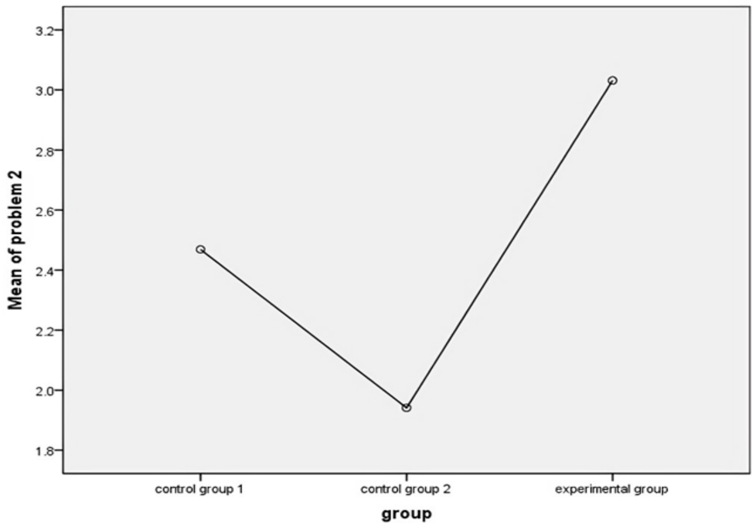

Figure 7. Students results while solving task 2 (Source: Authors' own elaboration)

Table 2. Statistical results for task 2 for three different groups of students

ANOVA	Sum of squares	df	Mean square	F	p
Between groups	19.588	2	9.794	3.540	0.033
Within groups	262.820	95	2.767		
Total	282.408	97			

Similarly, as in the second task, by analyzing **Figure 8**, we can observe that in the third task (which also involved the triple integral), where knowledge and introduction of the switch using spherical coordinates were necessary, students from the experimental group achieved the highest average score, followed by students from the first control group, while the lowest mean score was recorded in the second control group of students.

That this difference is statistically significant has been confirmed by ANOVA test (**Table 3**).

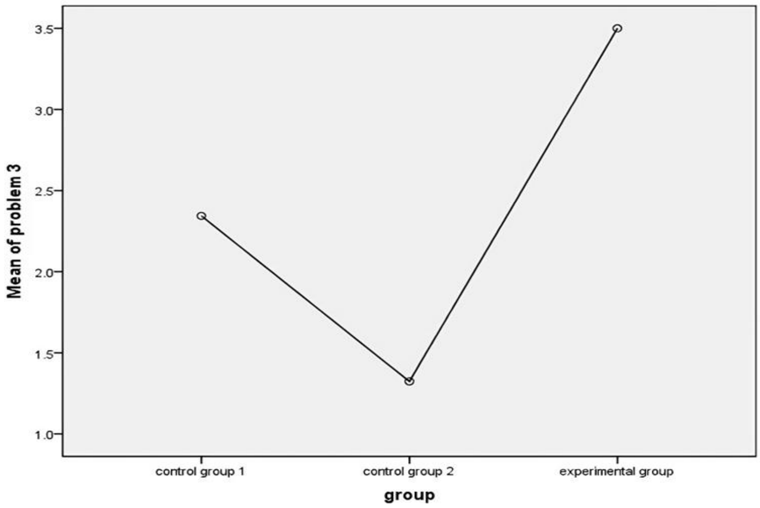

Figure 8. Students results while solving task 3 (Source: Authors' own elaboration)

Table 3. Statistical results for task 3 for three different groups of students

ANOVA	Sum of squares	df	Mean square	F	p
Between groups	78.116	2	39.058	10.828	<0.005
Within groups	342.660	95	3.607		
Total	420.776	97			

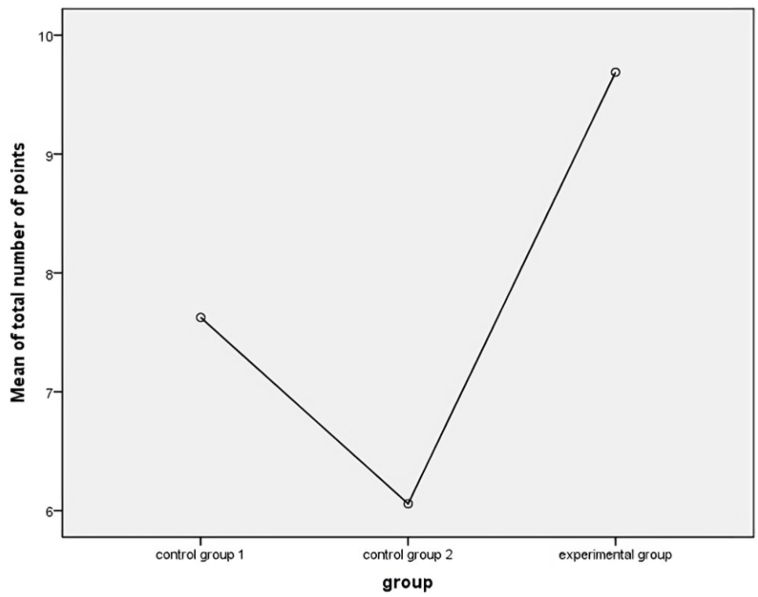

Figure 9. Students results while solving three problems (Source: Authors' own elaboration)

To determine in which groups exhibit statistically significant differences, we conducted post-hoc tests, which revealed differences in the achievements of the experimental group compared to the second control group ($p<0.0005$), as well as differences in the achievements of the experimental group and the first control group ($p=0.0400$). This practically means that students of the experimental group outperformed their peers who learned about triple integrals directly in the classroom when solving a triple integral that required the introduction of spherical coordinates.

Finally, we wanted to analyze the students' overall performance on the entire test, i.e., the total number of points students achieved. The average number of points obtained by students in all three groups is shown in **Figure 9**. The differences that were present in individual tasks also extended to the total number of points, and ANOVA test confirmed that the differences in the average number of points obtained by students in all three groups are statistically significant.

Based on the post-hoc test, it has been established that the differences achieved by students during testing are statistically significant between the experimental group and the second control group ($p=0.0020$) (**Table 4**). On the other hand, there are no statistically significant differences in the achievements of students between the experimental and the first control group, as well as between the two control groups of students on the test.

Table 4. Statistical results for task 4 for three different groups of students

ANOVA	Sum of squares	df	Mean square	F	p
Between groups	217.865	2	108.933	6.499	0.002
Within groups	1,592.257	95	16.761		
Total	1,810.122	97			

DISCUSSION & CONCLUSIONS

The methodological approach in calculus teaching with computer science students in Serbia exposed in this paper has its foundation in recognizing a significant fourth pillar that expands the didactic triangle, which is the artifact through which we strive to convey the necessary knowledge to students (Rezat & Sträßer, 2012). More precisely, instead of attempting to deliver that knowledge to them following a behavioristic learning theory, this approach places students in a role, where they discover new knowledge and connect it to existing knowledge, putting them at the center of the teaching process, which aligns with constructivist learning theory (von Glasersfeld, 1995). Additionally, students used technology through interaction with their peers, divided into groups to ensure equality in their work, a division of responsibilities, along with constant communication (Kagan, 1994).

Also, in creating this approach, we aimed not only for students to acquire appropriate educational content but also to positively impact the development of their spatial literacy (Moore-Russo et al., 2013) by emphasizing visualization in the integration domain, intensive communication in their collaborative learning, and, of course, influencing their reasoning.

Comparing the success results in solving the first task, it turned out that the differences in the achievements of students in the experimental and two control groups were not statistically significant, although based on descriptive statistics (**Figure 6**), it is evident that students who learned in a CSCL environment achieved the best results. However, when examining the results of students who solved triple integrals, things change. Namely, when solving the second task, which required calculating a triple integral and, in the process, analyzing mathematical objects in 3D, their mutual relationships, intersections, defining the given integration domain–tasks that certainly demand a higher level of visual spatial ability–the students in the experimental group significantly outperformed students who learned multiple integrals online without using GeoGebra and without the possibility of collaborative learning. The positive impact of CSCL on the acquisition of theoretical and practical knowledge related to mapping space using the method of substitution through spherical coordinates, as well as the use of advantages resulting from the appropriate substitution for calculating the volume of a body determined by a sphere, is indicated by the analysis of the third task, where students in the experimental group not only outperformed their peers who learned about it independently, online without GeoGebra application, but also surpassed their fellow peers who learned about it in the classroom, using a traditional approach to calculus teaching, where the teacher is a central figure.

Consistent with previous findings, the results of our research indicate the positive effects of CSCL environment on the students' achievements in mathematics (Birgin & Acar, 2022; Lin et al., 2011; Mullins et al., 2011), as well as on the development of their skills (Chen et al., 2018), such as spatial literacy. The findings of our research are in line with previous studies indicating that the students' achievements after learning in a CSCL environment exceed the achievements of students who acquired knowledge and skills independently (Lou, 2004; Lou et al., 2001).

Based on the results of the entire test, we can accept **H1**, confirming that the use of GeoGebra for necessary visualization during the solving process of multiple integrals using MS Teams platform that enables work in small collaborative groups of students leads to better student achievements compared to online learning with less student engagement and without students' collaboration. Furthermore, based on the research results, we can accept **H2** and state that CSCL with the proper use of GeoGebra software leads to results that do not deviate from the results of students who attended traditional in-person teaching they are accustomed to. Moreover, when comparing the success of students in solving the most challenging task, where it is necessary for the student to recognize how the integration domain looks in 3D, to recognize which substitution to introduce, to do it correctly, to determine the projection of the body onto the plane, and then perform the computational part of calculating the integral, students who used GeoGebra for the visualization of mathematical concepts during collaborative learning outperformed their peers who learned about all of this in the classroom without using GeoGebra in an environment that corresponds to behaviorist learning theory. Based on all the results of this research, we can conclude that GeoGebra software application for the visualization of multivariable functions within an online CSCL environment contributes to better student achievements in solving multiple integrals.

Author contributions: Both authors have sufficiently contributed to the study and agreed with the results and conclusions.

Funding: No funding source is reported for this study.

Ethical statement: The authors stated that ethics committee approval was obtained from Ethics Committee of Faculty of Science at University of Kragujevac, where this study was conducted on 1 February 2024 with decision approval code 01-2/24. Written informed consents were obtained from the participants.

Declaration of interest: No conflict of interest is declared by the authors.

Data sharing statement: Data supporting the findings and conclusions are available upon request from the corresponding author.

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APPENDIX A: TEST

1. Calculate $\iint (x+y)\mathrm{d}x\mathrm{d}y$ over region D bounded by parabola $y=3x^2$ and lines $y=x+1$ and $y=3+x$.
2. Calculate $\iiint y\ \mathrm{d}x\mathrm{d}y\mathrm{d}z$ over region V bounded by $2y=x^2, y+z=1$, and $2y+z=2$.
3. Find volume bounded by surfaces $x^2+y^2+z^2=8x$ and $x^2+y^2=z^2$, for $z\ \geq\ 0, z^{\ 2}\geq x^2+y^2, R>0$.